Pythagoras
2021-1-HR01-KA220-HED-000032588

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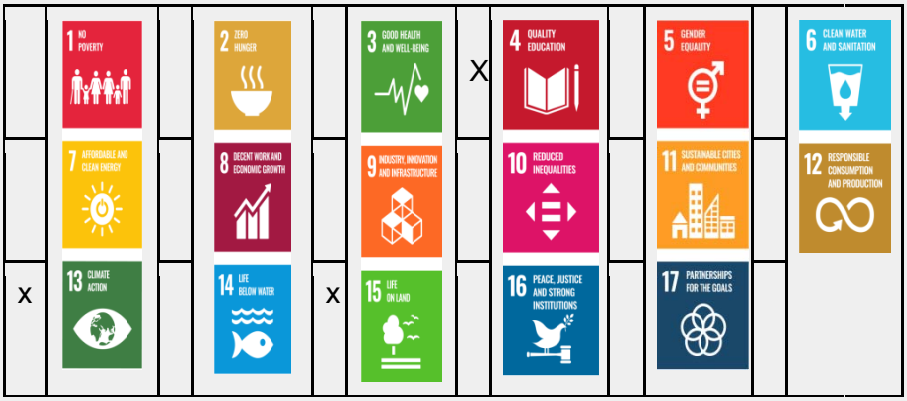
Mini-PBL example

Teaching Guide for Teachers

Mini-PBL project	
Teacher data sheet: Teaching Guide	
Title	The growth of trees
SDG attended	Using this UN graphics, we mark such SDG which this project works. 
Content units	Differential equations
Sessions	1 sessions of 1h
Hours of autonomous work	1h
Competences to be developed	**Reasoning and modelling** • Develop thinking strategies to solve real life problems • Explore, analyse, and apply mathematical ideas • Estimate reasonably and demonstrate fluent, flexible, and strategic thinking about graphs • Model with mathematics in situational contexts • Think creatively and with curiosity and wonder when exploring problems **Understanding and solving** • Develop, demonstrate, and apply conceptual understanding of mathematical ideas through story, inquiry, and problem solving • Visualize to explore and illustrate mathematical concepts and relationships • Apply flexible and strategic approaches to solve problems • Solve problems with persistence and a positive disposition • Engage in problem-solving experiences connected with real-life examples.

	Communicating and representing ● Explain and justify mathematical ideas and decisions in many ways ● Represent mathematical ideas in concrete, pictorial, and symbolic forms ● Use mathematical vocabulary and language to contribute to discussions in the classroom ● Take risks when offering ideas in classroom discourse **Connecting and reflecting** ● Reflect on mathematical thinking ● Connect mathematical concepts with each other, other areas, and personal interests ● Use mistakes as opportunities to advance learning ● Incorporate First Peoples worldviews, perspectives, knowledge, and practices to make connections with mathematical concepts
ICT tools to be used	Available Computer Algebra Systems: Mathematica, Maple, Matlab, GeoGebra, etc.
Context: project statement	The climate changes are among other things caused also by the increasing deforestation and decreasing number of forests, trees and green vegetation on the globe. To benefit from the sustainable development and to support vegetation growth it is necessary to understand the growth of individual trees and be aware of the time necessary for the renewal of their missing numbers. To grow each particular tree in a new forest it isessential to know the time interval required for its cultivation.
Tasks and problems	A specific type of tree will grow under suitable conditions with the speed inversely proportional to its height. The tree can grew up to 1 meter during the first three years after planting. A new forest was planted with the particular tree seedlings that were all about 0,5 meter high. The tree growth can be described by a differential equation with separable variables, while general solution of this equation contains constants that can be expressed with respect to the described height decrease under good environment conditions. Tree can grow in good conditions about 7 years till it will reach the average height, while after this period it might beneficially vegetate for about 50 years and still grow, but quite slowly, with the half of the initial growth speed. **Task 1:** Assemble the differential equation describing the growth of trees in a new forest, assuming favorable conditions are secured for their growth. Answer: Let $y(t) > 0$ be the function representing the tree height depending on the time t of its growth, while derivative $y'(t)$ be the speed of the tree growth. This growth is described by the differential equation with separable variables $$y'(t) = \frac{k}{y(t)}, k > 0$$ that can be rewritten as the differential equation with separated

variables and solved directly by integration.

$$y(t).y'(t) - k = 0$$

$$\int y\,dy - \int k\,dt = c$$

$$\frac{y^2}{2} - kt = c$$

$$y^2 = 2kt + 2c$$

Task 2:
Find general solution of this differential equation and particular solution determined by Cauchy initial conditions.

Answer:
General solution of the equation is in the form

$$y(t) = \sqrt{2kt + 2c}$$

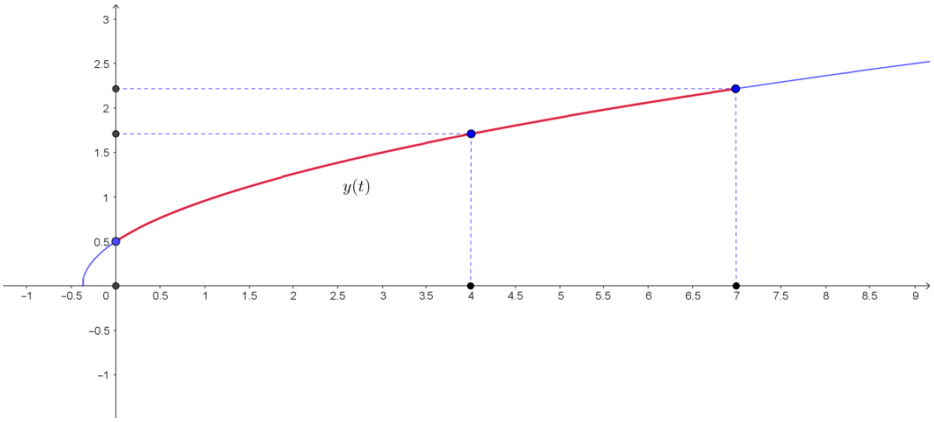
where constants *k* and *c* can be found according to the given initial conditions describing the tree growth:

$$y(0) = 0{,}5 \qquad y(3) = 1{,}5$$

$$y(t) = \sqrt{\frac{2}{3}t + 0{,}25}\ , \qquad t \in \langle 0, T\rangle, T \in R$$

Task 3:
Sketch respective integral curve of the particular solution representing the tree growth during *T* years, until it will reach its average height.

Answer:



Tree will grow until it reaches its average height, which is after*T* = 7 years from its planting. The red curve represents the tree growth.

Task 4:
Calculate the average height of trees in this forest after 4 years and after 7 years, when the trees reach their average height.

Answer:

$$y(4) = \sqrt{\frac{8}{3} + \frac{1}{4}} = \sqrt{\frac{32+3}{12}} = \sqrt{\frac{35}{12}} \doteq 1{,}71\ m$$

$$y(7) = \sqrt{\frac{14}{3} + \frac{1}{4}} = \sqrt{\frac{56+3}{12}} = \sqrt{\frac{59}{12}} \doteq 2{,}22\ m$$

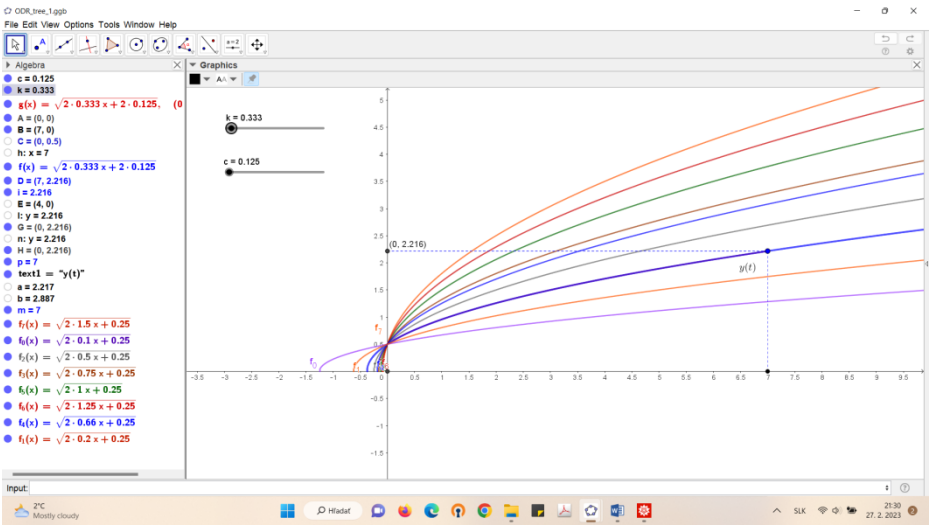
Task 5:
Estimate the height of the tree after 50 years.

Answer:

$$y(50) = \sqrt{\frac{100}{3} + \frac{1}{4}} = \sqrt{\frac{400+3}{12}} = \sqrt{\frac{403}{12}} \doteq 5{,}79\ m$$

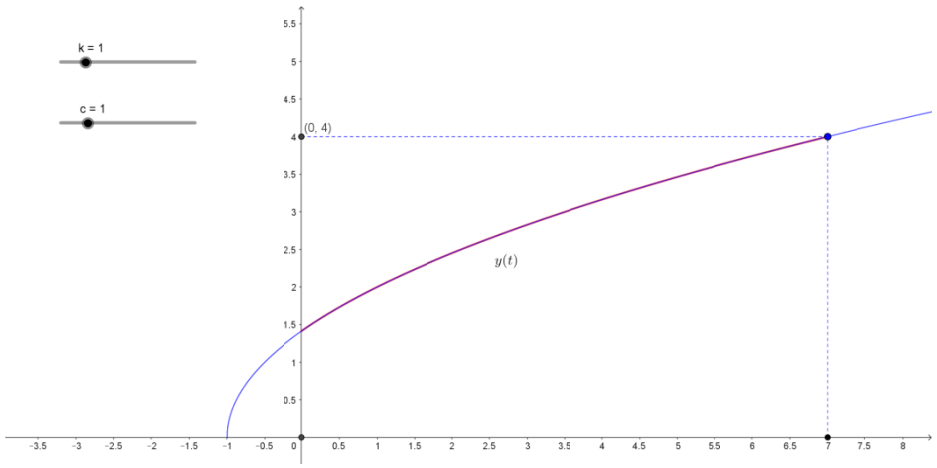
Task 6:
Sketch several integral curves of the general solution and investigate their forms determined by different values of the included constants *c* and *k* representing the tree growth under different conditions.

Answer:



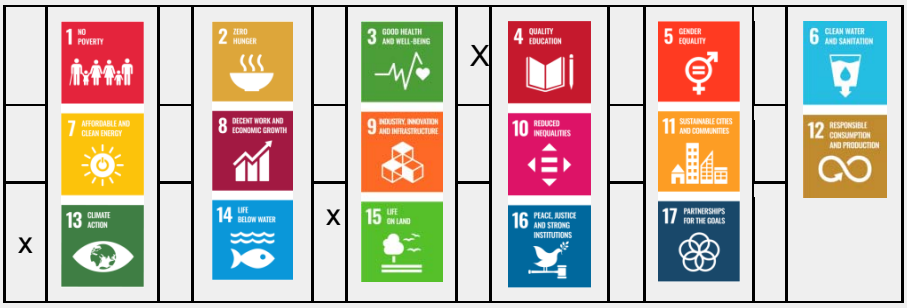
Task 7:
Comment on the obtained results from a sustainable point of view. Investigate how the height of the tree seedlings influences the speed of the trees growth.

Answer:
Animation can be obtained easily in the program GeoGebra, with sliders determining the values of constants *k* and *c*.

	
Outcomes expected	- Graphics fitting the solution; - Numerical results explained and put in context; - Capture of ICT tools solutions used; - Sequence of steps followed; - Remark computations done by hand and done by ICT tools; - Provide complete answer to questions; - All the results must be presented in the context of the problem;
Guide for Learning	At the beginning of the course, the students need guides on new activities, and feel your support on a well-structured pack of suggestions on how to address the problems posted. Namely: - Read carefully the problem statement and the tasks posted. Always maintain a global view of all the projects. - Identify, or try to do a first draft match, the content units of your lecture notes involved in every task. - Take your lecture notes open and review before starting to solve the problems. - Match output expected with the tasks posted, at least as first draft approach. - Follow the order of the tasks, try to increase the knowledge of the problem while you are solving the activities. - Always think that maybe there are different ways to solve a problem. - Use ICT tools to avoid hard computations and check your solutions are correct in different ways if possible. - The solutions are always part of a context, expressing such a final solution totally integrated in the problem posted. - Be sure you answer the complete questions. - Always try to solve the questions by yourself. - If the project can be done in groups, discuss with the groups the proposed problem, to confirm and detect fails or weaknesses, confront strategies, discuss presentation format, etc. Working in groups doesn't mean work less but work better.
Guide for Teaching	Some hints needed to present and launch the mini-PBL to students - Do a small Introduction concerning deforestation of the planet and the impact it is causing to the Climate Change crisis we are currently living in. - Do a small introduction about the relations between the

	struggles to support vegetation growth - namely seeding of new forests, and the time necessary for the growth of individual trees in terms of the renewal of their missing numbers, with the basic equations. - Students will form groups of 4 students and solve the mini-PBL using the eduScrum methodology. - The students should do each exercise in a sequential order, starting from Task 1. - The students should be able to thoroughly read and interpret the numerical results from a mathematical and the real-life example point of view. They should include also a discussion of the climate change crisis and enumerate some strategies they could apply at home or even at university to save resources, namely reduce energy consumption. They should also mention how this mini-PBL helps them identify the Sustainable Development Goals 4, 13 and 15.
Assessment	- Final report; - Oral presentation; - Peer-assessment: students will apply peer-assessment for their periodic performance using online peer assessment tools used and available at the respective institution.
Others: References	Active Learning Calculus I (colorado.edu) https://eduscrum.org/about-us-and-how-we-try-to-make-it-happen/ More refs on active-learning tools: https://scholar.google.com/citations?hl=en&user=Aw39XwEAAAAJ&view_op=list_works&sortby=pubdate https://www.youtube.com/watch?v=mQ_mbDAB1us (there are more examples online)

Learning Guide for Students

Mini-PBL project	
Student data sheet: Learning Guide	
Title	The growth of trees
SDG attended	Using this UN graphic, we mark such SDG which this project works. 
Content units	Ordinary differential equations of order I
Sessions	1 sessions of 1h
Hours of autonomous work	1h
ICT tools to be used	Available Computer Algebra Systems: Mathematica, Maple, Matlab, GeoGebra, etc.
Context: project statement	The climate changes are among other things caused also by the increasing deforestation and decreasing number of forests, trees and green vegetation on the globe. To benefit from the sustainable development and to support vegetation growth it is necessary to understand the growth of individual trees and be aware of the time necessary for the renewal of their missing numbers. To grow each particular tree in a new forest it is essential to know the time interval required for its cultivation.
Tasks and problems	A specific type of tree will grow under suitable conditions with the speed inversely proportional to its height. The tree can grew up to 1 meter during the first three years after planting. A new forest was planted with the particular tree seedlings that were all about 0,5 meter high. The tree growth can be described by a differential equation with separable variables, while general solution of this equation contains constants that can be expressed with respect to the described height decrease under good environment conditions. Tree can grow in good conditions about 7 years till it will reach the average height, while after this period it might beneficially vegetate for about 50 years and still grow, but much slowlier, with respect of the initial growth speed.

	Task 1: Assemble the differential equation describing the growth of trees in a new forest, assuming favorable conditions are secured for their growth. Answer: **Task 2:** Find general solution of this differential equation and particular solution determined by Cauchy initial conditions. Answer: **Task 3:** Sketch respective integral curve of the particular solution representing the tree growth during *T* years, until it will reach its average height. Answer: **Task 4:** Calculate the average high of trees in this forest after 4 years and after 7 years, when the trees reach their average height. Answer: **Task 5:** Estimate the height of the tree after 50 years. Answer: **Task 6**: Sketch several integral curves of the general solution and investigate their forms determined by different values of the included constants *c* and *k* representing the tree growth under different conditions. Answer: **Task 7:** Comment on the obtained results from a sustainable point of view. Investigate how the height of the tree seedlings influences the speed of the trees growth. Answer:
Outcomes expected	- Graphics fitting the solution; - Numerical results explained and put in context; - Capture of ICT tools solutions used; - Sequence of steps followed; - Remark computations done by hand and done by ICT tools; - Provide complete answer to questions; - All the results must be presented in the context of the problem;
Guide for learning	- Read carefully the problem statement and the tasks posted. Always maintain a global view of all the projects.

	- Identify, or try to do a first draft match, the content units of your lecture notes involved in every task. - Take your lecture notes open and review before starting to solve the problems. - Match output expected with the tasks posted, at least as first draft approach. - Follow the order of the tasks; try to increase the knowledge of the problem while you are solving the activities. - Always think that maybe there are different ways to solve a problem. - Use ICT tools to avoid hard computations and check your solutions are correct in different ways if possible. - The solutions are always part of a context, expressing such a final solution totally integrated in the problem posted. - Be sure you answer the complete questions. - Always try to solve the questions by yourself. - If the project can be done in groups, discuss with the groups the proposed problem, to confirm and detect fails or weaknesses, confront strategies, discuss presentation format, etc. Working in groups doesn't mean work less but work better.
Assessment	- Final report; - Oral presentation; - Peer-assessment: students will apply peer-assessment for their periodic performance using online peer assessment tools used and available at the respective institution.
Others: References	Active Learning Calculus I (colorado.edu) https://eduscrum.org/about-us-and-how-we-try-to-make-it-happen/ More refs on active-learning tools: https://scholar.google.com/citations?hl=en&user=Aw39XwEAAAAJ&view_op=list_works&sortby=pubdate https://www.youtube.com/watch?v=mQ_mbDAB1us (there are more examples online)

ANNEX 1: RUBRIC

Category	4=Excellent	3=Good	2=Low	1=Poor
Mathematical Concepts	Explanation shows complete understan- ding of the mathe- matical concepts used to solve the problem(s).	Explanation shows substantial understan- ding of the mathema- tical concepts used to solve the problem(s).	Explanation shows some understan- ding of the mathe- matical concepts needed to solve the problem(s).	Explanation shows very limited unders- tanding of the underlying concepts needed to solve the problem(s) OR is not written.
Mathematical Terminology and Notation	Correct terminology and notation are always used, making it easy to understand what was done.	Correct terminology and notation are usually used, making it fairly easy to understand what was done.	Correct terminolo- gy and notation are used, but it is sometimes not easy to understand what was done.	There is little use, or a lot of inappropriate use, of terminology and notation.
Strategy/Procedure	Typically, uses an efficient and effective strategy to solve the problem(s).	Typically, uses an effective strategy to solve the problem(s).	Sometimes uses an effective strategy to solve problems, but does not do it consistently.	Rarely uses an effective strategy to solve problems.
Completion	All problems are completed.	All but one of the problems are completed.	All but two of the problems are completed.	Several of the problems are not completed.
Mathematical Errors	90-100% of the steps and solutions have no mathematical errors.	Almost all (85-89%) of the steps and solu- tions have no mathematical errors.	Most (75-84%) of the steps and solu- tions have no mathematical errors.	More than 75% of the steps and solu- tions have mathe- matical errors.

Sources Checking				
Working with Others	Student was an engaged partner, listening to suggestions of others and working cooperatively throughout lesson.	Student was an engaged partner but had trouble listening to others and/or working cooperatively.	Student cooperated with others, but needed prompting to stay on-task.	Student did not work effectively with others.
Neatness and Organization	The work is presented in a neat, clear, organized fashion that is easy to read.	The work is presented in a neat and organized fashion that is usually easy to read.	The work is presented in an organized fashion but may be hard to read at times.	The work appears sloppy and unorganized. It is hard to know what information goes together.
Diagrams and Sketches	Diagrams and/or sketches are clear and greatly add to the reader's understanding of the procedure(s).	Diagrams and/or sketches are clear and easy to understand.	The work is presented in an organized fashion but may be hard to read at times.	Diagrams and/or sketches are difficult to understand or are not used.
CT tools used				