Pythagoras
2021-1-RO01-KA220-HED-000032258

Funded by the European Union

Mini-PBL example

Teaching Guide for Teachers

Mini-PBL project	
Teacher data sheet: Teaching Guide	
Title	Characteristics of magnetron
SDG attended	Using this UN graphics, we mark such SDG which this project works. 
Content units	Numerical methods applied in solution of technical problems.
Sessions	1 sessions of 105 min
Hours of autonomous work	40 min
Competences to be developed	**Reasoning and modelling** • Develop thinking strategies to solve real life problems • Explore, analyse, and apply mathematical ideas • Estimate reasonably and demonstrate fluent, flexible, and strategic thinking about graphs • Model with mathematics in situational contexts • Think creatively and with curiosity and wonder when exploring problems **Understanding and solving** • Develop, demonstrate, and apply conceptual understanding of mathematical ideas through story, inquiry, and problem solving • Visualize to explore and illustrate mathematical concepts and relationships • Apply flexible and strategic approaches to solve problems • Solve problems with persistence and a positive disposition

	● Engage in problem-solving experiences connected with real-life examples. **Communicating and representing** ● Explain and justify mathematical ideas and decisions in many ways ● Represent mathematical ideas in concrete, pictorial, and symbolic forms ● Use mathematical vocabulary and language to contribute to discussions in the classroom ● Take risks when offering ideas in classroom discourse **Connecting and reflecting** ● Reflect on mathematical thinking ● Connect mathematical concepts with each other, other areas, and personal interests ● Use mistakes as opportunities to advance learning ● Incorporate First Peoples worldviews, perspectives, knowledge, and practices to make connections with mathematical concepts
ICT tools to be used	Available Computer Algebra Systems: Mathematica.
Context: project statement	Magnetron is an equipment producing microwaves. Properties of microwaves are used in practise for heating of water and in radars. Usage of radars - in metrology – they provide valuable data to researchers, who investigate developments of weather (rainfall, storms, hail, tornadoes), climate changes and interactions between the atmosphere and the earth's surface, and provide also valuable data to farmers - predicting floods and managing irrigation - in air transport - they help pilots and dispatchers to avoid dangerous turbulence, storms, or icing - in automobile transport – by measuring the safe distance between vehicles, they help to prevent traffic accidents; forecasts of storms and heavy rains help to prevent dangerous aquaplaning - in shipping – they increase safety at reduced visibility. Microwave radiation from a magnetron is used e.g. to protect art monuments from pests (insects, microorganisms, mold parasites), which are disinfected without damage by this radiation.
Tasks and problems	**TASK** Characteristics of the magnetron were measured during in laboratory exercises from Physics. The characteristic of the magnetron is the dependence of the current I_a on the current I_s passing through the solenoid at a constant voltage U_a on the anode. The values of currents I_a and Is were measured at voltages U_a with values of 40V, 50V and 70V. All measurements were measured relatively accurately and are presented in the table. Determine the critical values of the current I_s in the solenoid for each value of voltage U_a.

U_a [V]	i	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.
40	I_S [mA]	20	30	40	50	60	70	78	90	100	110	120
	I_A [mA]	4	4,1	3,85	3,2	2,5	1,65	1,2	1,0	1,1	1,1	1,0
50	I_S [mA]	20	30	40	50	60	70	80	90	100	110	120
	I_A [mA]	5,5	5,6	5,3	4,8	3,8	2,2	1,7	1,55	1,45	1,3	1,25
70	I_S [mA]	20	30	40	50	60	70	80	90	100	110	120
	I_A [mA]	7,4	7,2	7,0	6,8	5,8	4,2	3,0	2,6	2,4	2,2	2,0

Instructions.

- To determine the critical value of the current I_S in the solenoid means to find the inflection point of the function $I_a = I_a\,(I_S)$.
- Solve tasks 1.– 5. using software Mathematica.

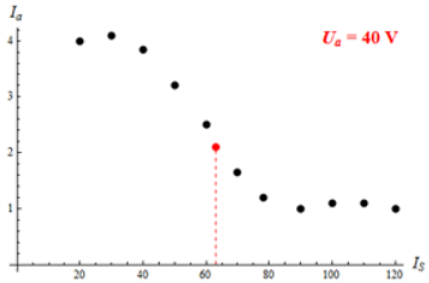
Task 1.

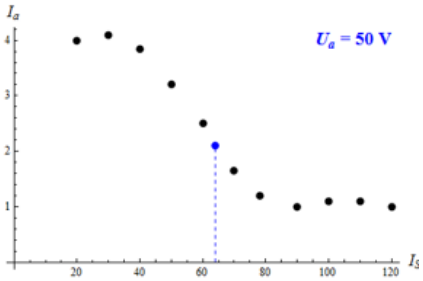
Visualize measurement data and estimate position of inflection points I_S.

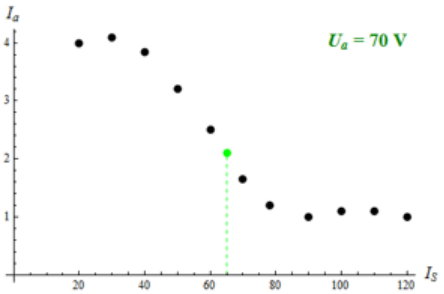
Solution:

Estimation

U_a [V]	I_S [mA]
40	63
50	64
70	65







Task 2.

Choose suitable approximation function $I_a = I_a\,(I_S)$ and find its formula. Explain your choice.
Sketch graph of approximation function into the graph of measurement values.

Solution:

With respect ot the form of the graph of measurement data, the position of inflection point and because all measurements were measured relatively accurately by means of suitable approximation function, the interpolation polynomial will be of degree 3.

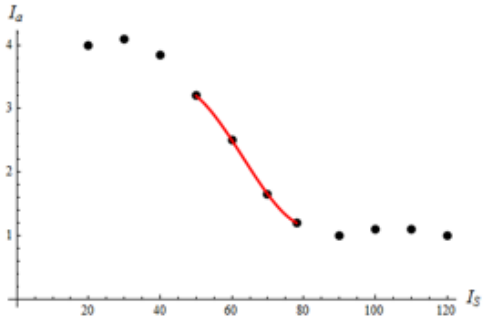
$$N(x) = a_3x^3 + a_2x^2 + a_1x + a_0 \ ; \ a_3, \ a_2, \ a_1, \ a_0 \in R \ ; \ a_3 \neq 0$$

It is necessary to choose 4 measured values from 11 measurements for the polynomial $N(x)$, which are close to the neighbourhood of the estimated inflection point.

U_a = 40 V

$N(x) = -13.1542 + 0.909474\,x - 0.0158393\,x^2 + 0.0000838294\,x^3$

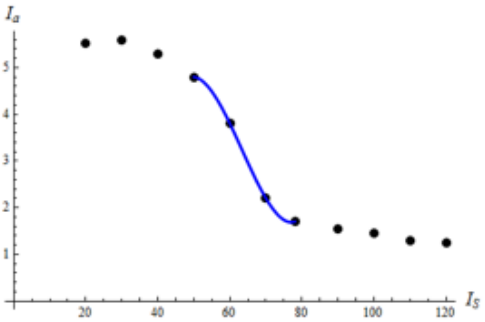
I_S	I_a
20	4
30	4.1
40	3.85
50	3.2
60	2.5
70	1.65
78	1.2
90	1
100	1.1
110	1.1
120	1



U_a = 50 V

$N(x) = -62.325 + 3.44637\,x - 0.0571071\,x^2 + 0.000300595\,x^3$

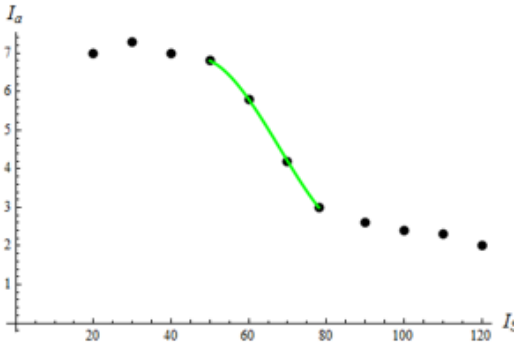
I_S	I_a
20	5.53
30	5.6
40	5.3
50	4.8
60	3.8
70	2.2
78	1.7
90	1.55
100	1.45
110	1.3
120	1.25



U_a = 70 V

$N(x) = -23.8667 + 1.58873\,x - 0.0258571\,x^2 + 0.000126984\,x^3$

I_S	I_a
20	7
30	7.3
40	7
50	6.8
60	5.8
70	4.2
78	3
90	2.6
100	2.4
110	2.3
120	2



Task 3.

Determine critical values of the current I_s in the solenoid.

In the graph sketch all critical values that were calculated.

Solution:

The interpolation polynomial of the 3rd degree – calculation of inflection points:

$$I_a(I_s) \approx N(I_s) \Rightarrow I_a''(I_s) \approx N''(I_s)$$
$$I_a''(I_s) = 0 \approx N''(I_s) = 0$$

U_a = 40 V

$$N(x) = -13.1542 + 0.909474\,x - 0.0158393\,x^2 + 0.0000838294\,x^3$$

$$N''(x)(x) = -0.0316786 + 0.000502976\,x$$

$$N''(x)(x) = 0 \Leftrightarrow x = 62.9822 \longrightarrow I_S = 62.9822$$

U_a = 50 V

$$N(x) = -62.325 + 3.44637\,x - 0.0571071\,x^2 + 0.000300595\,x^3$$

$$N''(x)(x) = -0.114214 + 0.00180357\,x$$

$$N''(x)(x) = 0 \Leftrightarrow x = 63.3267 \longrightarrow I_S = 63.3267$$

U_a = 70 V

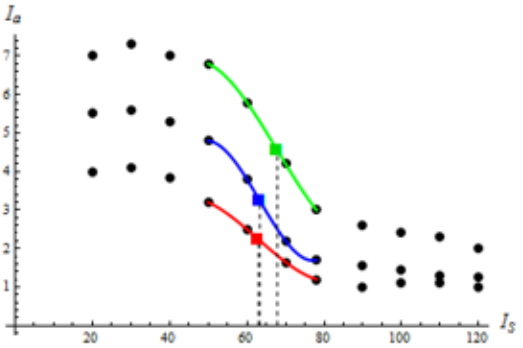
$$N(x) = -23.8667 + 1.58873\,x - 0.0258571\,x^2 + 0.000126984\,x^3$$

$$N''(x)(x) = -0.0517143 + 0.000761905\,x$$

$$N''(x)(x) = 0 \Leftrightarrow x = 67.875 \longrightarrow I_S = 67.875$$

Summary

U_a [V]	I_S [mA]
40	62.9822
50	63.3267
70	67.875



Task 4.

a) Find formula for the interpolation polynomial function whose graph is passing through points representing all measured values.

b) Calculate inflection points of these interpolation polynomic functions.

c) Into one picture sketch graphs of all interpolation polynomial functions and indicate also all calculated inflection points.

Solution:

Graph of interpolation polynomial passing through all (11) measurement values will be the interpolation polynomil of degree 10.

$$N(x)=a_{10}x^{10}+a_9x^9+\ldots+a_1x+a_0 \;;\; a_{10}, a_9, \ldots, a_1, a_0 \in R \;;\; a_{10}\neq 0$$

Calculation of inflection points:

$$I_a(I_s)\approx N_{10}(I_s) \Rightarrow I_a''(I_s)\approx N''(I_s)$$

$$I_a''(I_s)=0 \approx N'(I_s)=0$$

U_a = 40 V

$$N(x) = 454.209 - 86.763\,x + 7.15347\,x^2 - 0.333852\,x^3 + 0.00981146\,x^4 - 0.000190483\,x^5 + 2.48235\times10^{-6}\,x^6 - 2.15057\times10^{-8}\,x^7 + 1.1886\times10^{-10}\,x^8 - 3.79373\times10^{-13}\,x^9 + 5.32163\times10^{-16}\,x^{10}$$

$$N''(x) = 14.3069 - 2.00311\,x + 0.117737\,x^2 - 0.00380966\,x^3 + 0.0000744706\,x^4 - 9.03241\times10^{-7}\,x^5 + 6.65618\times10^{-9}\,x^6 - 2.73148\times10^{-11}\,x^7 + 4.78947\times10^{-14}\,x^8$$

$$N''(x)=0 \Leftrightarrow x = 63.7205 \longrightarrow I_S = 63.7205$$

U_a = 50 V

$$N(x) = 1548.28 - 303.005\,x + 25.5226\,x^2 - 1.21855\,x^3 + 0.0366382\,x^4 - 0.000727061\,x^5 + 9.67062\times10^{-6}\,x^6 - 8.53605\times10^{-8}\,x^7 + 4.79758\times10^{-10}\,x^8 - 1.55412\times10^{-12}\,x^9 + 2.20831\times10^{-15}\,x^{10}$$

$$N''(x) = 51.0452 - 7.31129\,x + 0.439659\,x^2 - 0.0145412\,x^3 + 0.000290119\,x^4 - 3.58514\times10^{-6}\,x^5 + 2.68664\times10^{-8}\,x^6 - 1.11896\times10^{-10}\,x^7 + 1.98748\times10^{-13}\,x^8$$

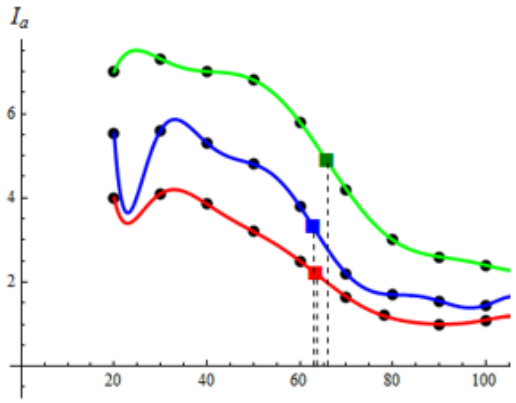
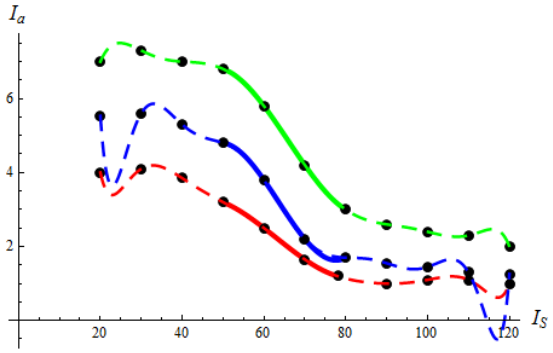
$$N''(x)=0 \Leftrightarrow x = 63.026 \longrightarrow I_S = 63.026$$

U_a = 70 V

$$N(x) = -69.2 + 12.1178\,x - 0.805853\,x^2 + 0.0302361\,x^3 - 0.000734956\,x^4 + 0.0000126971\,x^5 - 1.64139\times10^{-7}\,x^6 + 1.56515\times10^{-9}\,x^7 - 1.01687\times10^{-11}\,x^8 + 3.91314\times10^{-14}\,x^9 - 6.61376\times10^{-17}\,x^{10}$$

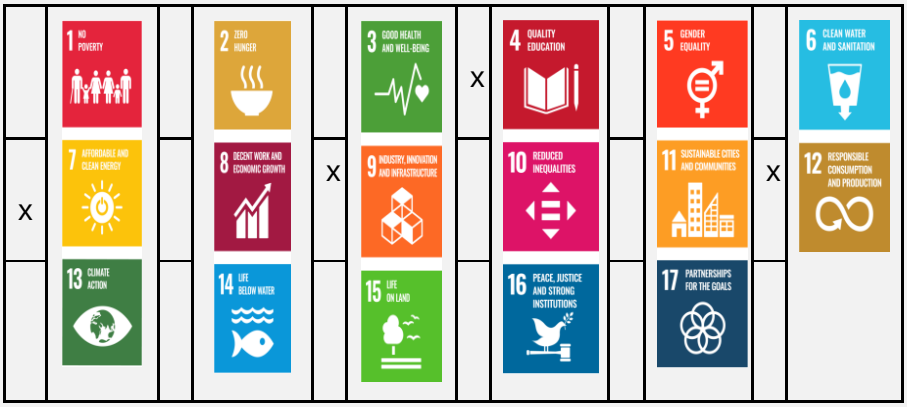
$$N''(x) = -1.61171 + 0.181417\,x - 0.00881947\,x^2 + 0.000253942\,x^3 - 4.92417\times10^{-6}\,x^4 + 6.57361\times10^{-8}\,x^5 - 5.69444\times10^{-10}\,x^6 + 2.81746\times10^{-12}\,x^7 - 5.95238\times10^{-15}\,x^8$$

$$N''(x)=0 \Leftrightarrow x = 66.0101 \longrightarrow I_S = 66.0101$$

	Summary U[V] I_S[mA] 40 63.7205 50 63.026 70 66.0101  **Task 5.** Compare values of inflection points calculated using interpolation polynomials of degree 3. and 10. Into one diagram sketch graphs of all interpolation polynomials of degree 3. a 10. ***Solution***: U[V] $I_S(3^{rd})$ $I_S(10^{rd})$ $\lvert I_S(10^{rd}) - I_S(3^{rd}) \rvert$ 40 62.9822 63.7205 0.738291 50 63.3267 63.026 0.30069 70 67.875 66.0101 1.86488 
Outcomes expected	- Graphics fitting the solution; - Numerical results explained and put in context; - Capture of ICT tools solutions used; - Sequence of steps followed; - Remark computations done by hand and done by ICT tools; - Provide complete answer to questions; - All the results must be presented in the context of the problem;
Guide for Learning	At the beginning of the course, the students need guides on new activities, and feel your support on a well-structured pack of suggestions on how to address the problems posted. Namely: - Read carefully the problem statement and the tasks posted.

	Always maintain a global view of all the projects. - Identify, or try to do a first draft match, the content units of your lecture notes involved in every task. - Take your lecture notes open and review before starting to solve the problems. - Match output expected with the tasks posted, at least as first draft approach. - Follow the order of the tasks, try to increase the knowledge of the problem while you are solving the activities. - Always think that maybe there are different ways to solve a problem. - Use ICT tools to avoid hard computations and check your solutions are correct in different ways if possible. - The solutions are always part of a context, expressing such a final solution totally integrated in the problem posted. - Be sure you answer the complete questions. - Always try to solve the questions by yourself. - If the project can be done in groups, discuss with the groups the proposed problem, to confirm and detect fails or weaknesses, confront strategies, discuss presentation format, etc. Working in groups doesn't mean work less but work better.
Guide for Teaching	Some hints needed to present and launch the mini-PBL to students - Do a small Introduction concerning Energy consumption, added to the Climate Change crisis we are currently living in. - Do a small introduction about the relations between power and energy, with the basic equations. - Students will form groups of 4 students and solve the mini-PBL using the eduScrum methodology. - The students should do each exercise in a sequential order, starting from Task 1. - The students should be able to thoroughly read and interpret the numerical results from a mathematical and the real-life example point of view. They should include also a discussion of the climate change crisis and enumerate some strategies they could apply at home or even at university to save resources, namely reduce energy consumption. They should also mention how this mini-PBL helps them identify the Sustainable Development Goals 4, 7 and 9.
Assessment	- Final report; - Oral presentation; - Peer-assessment: students will apply peer-assessment for their periodic performance using online peer assessment tools used and available at the respective institution.
Others: References	Gabková, J., Richtáriková, D. Základy numerickej matematiky. Teória a riešené príklady. Vydavateľstvo STU, Bratislava, 2012. Benco, P. a kol. Technická fyzika. Návody na laboratórne cvičenia. Vydavateľstvo STU, Bratislava, 2003, ISBN 80-227-1869-6

Learning Guide for Students

Mini-PBL project	
Teacher data sheet: Teaching Guide	
Title	Characteristics of magnetron
SDG attended	Using this UN graphics, we mark such SDG which this project works. 
Content units	Numerical methods applied in solution of technical problems.
Sessions	1 sessions of 105 min
Hours of autonomous work	40 min
Competences to be developed	**Reasoning and modelling** • Develop thinking strategies to solve real life problems • Explore, analyse, and apply mathematical ideas • Estimate reasonably and demonstrate fluent, flexible, and strategic thinking about graphs • Model with mathematics in situational contexts • Think creatively and with curiosity and wonder when exploring problems **Understanding and solving** • Develop, demonstrate, and apply conceptual understanding of mathematical ideas through story, inquiry, and problem solving • Visualize to explore and illustrate mathematical concepts and relationships • Apply flexible and strategic approaches to solve problems • Solve problems with persistence and a positive disposition • Engage in problem-solving experiences connected with real-life examples. **Communicating and representing** • Explain and justify mathematical ideas and decisions in many ways

	● Represent mathematical ideas in concrete, pictorial, and symbolic forms ● Use mathematical vocabulary and language to contribute to discussions in the classroom ● Take risks when offering ideas in classroom discourse **Connecting and reflecting** ● Reflect on mathematical thinking ● Connect mathematical concepts with each other, other areas, and personal interests ● Use mistakes as opportunities to advance learning ● Incorporate First Peoples worldviews, perspectives, knowledge, and practices to make connections with mathematical concepts
ICT tools to be used	Available Computer Algebra Systems: Mathematica.
Context: project statement	Magnetron is an equipment producing microwaves. Properties of microwaves are used in practise for heating of water and in radars. Usage of radars - in metrology – they provide valuable data to researchers, who investigate developments of weather (rainfall, storms, hail, tornadoes), climate changes and interactions between the atmosphere and the earth's surface, and provide also valuable data to farmers - predicting floods and managing irrigation - in air transport - they help pilots and dispatchers to avoid dangerous turbulence, storms, or icing - in automobile transport – by measuring the safe distance between vehicles, they help to prevent traffic accidents; forecasts of storms and heavy rains help to prevent dangerous aquaplaning - in shipping – they increase safety at reduced visibility. Microwave radiation from a magnetron is used e.g. to protect art monuments from pests (insects, microorganisms, mold parasites), which are disinfected without damage by this radiation.
Tasks and problems	**TASK** Characteristics of the magnetron were measured during in laboratory exercises from Physics. The characteristic of the magnetron is the dependence of the current I_a on the current I_s passing through the solenoid at a constant voltage U_a on the anode. The values of currents I_a and Is were measured at voltages U_a with values of 40V, 50V and 70V. All measurements were measured relatively accurately and are presented in the table. Determine the critical values of the current I_s in the solenoid for each value of voltage U_a.

U_a [V]	i	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.
40	I_S [mA]	20	30	40	50	60	70	78	90	100	110	120
	I_A [mA]	4	4,1	3,85	3,2	2,5	1,65	1,2	1,0	1,1	1,1	1,0
50	I_S [mA]	20	30	40	50	60	70	80	90	100	110	120
	I_A [mA]	5,5	5,6	5,3	4,8	3,8	2,2	1,7	1,55	1,45	1,3	1,25
70	I_S [mA]	20	30	40	50	60	70	80	90	100	110	120
	I_A [mA]	7,4	7,2	7,0	6,8	5,8	4,2	3,0	2,6	2,4	2,2	2,0

Instructions.

- To determine the critical value of the current I_s in the solenoid means to find the inflection point of the function $I_a = I_a\,(I_s)$.
- Solve tasks 1.– 5. using software Mathematica.

Task 1.
Visualize measurement data and estimate position of inflection points I_s.

Task 2.
Choose suitable approximation function $I_a = I_a\,(I_s)$ and find its formula. Explain your choice.
Sketch graph of approximation function into the graph of measurement values.

Task 3.
Determine critical values of the current I_s in the solenoid.
In the graph sketch all critical values that were calculated.

Task 4.

a) Find formula for the interpolation polynomial function whose graph is passing through points representing all measured values.

b) Calculate inflection points of these interpolation polynomic functions.

c) Into one picture sketch graphs of all interpolation polynomial functions and indicate also all calculated inflection points.

Task 5.
Compare values of inflection points calculated using interpolation polynomials of degree 3. and 10.
Into one diagram sketch graphs of all interpolation polynomials of degree 3. a 10.

Outcomes expected

- Graphics fitting the solution;
- Numerical results explained and put in context;
- Capture of ICT tools solutions used;
- Sequence of steps followed;
- Remark computations done by hand and done by ICT tools;
- Provide complete answer to questions;
- All the results must be presented in the context of the problem;

Guide for Learning	- Read carefully the problem statement and the tasks posted. Always maintain a global view of all the projects. - Identify, or try to do a first draft match, the content units of your lecture notes involved in every task. - Take your lecture notes open and review before starting to solve the problems. - Match output expected with the tasks posted, at least as first draft approach. - Follow the order of the tasks, try to increase the knowledge of the problem while you are solving the activities. - Always think that maybe there are different ways to solve a problem. - Use ICT tools to avoid hard computations and check your solutions are correct in different ways if possible. - The solutions are always part of a context, expressing such a final solution totally integrated in the problem posted. - Be sure you answer the complete questions. - Always try to solve the questions by yourself. - If the project can be done in groups, discuss with the groups the proposed problem, to confirm and detect fails or weaknesses, confront strategies, discuss presentation format, etc. Working in groups doesn't mean work less but work better.
Assessment	- Final report; - Oral presentation; - Peer-assessment: students will apply peer-assessment for their periodic performance using online peer assessment tools used and available at the respective institution.
Others: References	Gabková, J., Richtáriková, D. Základy numerickej matematiky. Teória a riešené príklady. Vydavateľstvo STU, Bratislava, 2012. Benco, P. a kol. Technická fyzika. Návody na laboratórne cvičenia. Vydavateľstvo STU, Bratislava, 2003, ISBN 80-227-1869-6

ANNEX 1: RUBRIC

Category	4=Excellent	3=Good	2=Low	1=Poor
Mathematical Concepts	Explanation shows complete understan- ding of the mathe- matical concepts used to solve the problem(s).	Explanation shows substantial understan- ding of the mathema- tical concepts used to solve the problem(s).	Explanation shows some understan- ding of the mathe- matical concepts needed to solve the problem(s).	Explanation shows very limited unders- tanding of the underlying concepts needed to solve the problem(s) OR is not written.
Mathematical Terminology and Notation	Correct terminology and notation are always used, making it easy to understand what was done.	Correct terminology and notation are usually used, making it fairly easy to understand what was done.	Correct terminolo- gy and notation are used, but it is sometimes not easy to understand what was done.	There is little use, or a lot of inappropriate use, of terminology and notation.
Strategy/Procedure	Typically, uses an efficient and effective strategy to solve the problem(s).	Typically, uses an effective strategy to solve the problem(s).	Sometimes uses an effective strategy to solve problems, but does not do it consistently.	Rarely uses an effective strategy to solve problems.
Completion	All problems are completed.	All but one of the problems are completed.	All but two of the problems are completed.	Several of the problems are not completed.
Mathematical Errors	90-100% of the steps and solutions have no mathematical errors.	Almost all (85-89%) of the steps and solu- tions have no mathematical errors.	Most (75-84%) of the steps and solu- tions have no mathematical errors.	More than 75% of the steps and solu- tions have mathe- matical errors.

Sources Checking				
Working with Others	Student was an engaged partner, listening to suggestions of others and working cooperatively throughout lesson.	Student was an engaged partner but had trouble listening to others and/or working cooperatively.	Student cooperated with others, but needed prompting to stay on-task.	Student did not work effectively with others.
Neatness and Organization	The work is presented in a neat, clear, organized fashion that is easy to read.	The work is presented in a neat and organized fashion that is usually easy to read.	The work is presented in an organized fashion but may be hard to read at times.	The work appears sloppy and unorganized. It is hard to know what information goes together.
Diagrams and Sketches	Diagrams and/or sketches are clear and greatly add to the reader's understanding of the procedure(s).	Diagrams and/or sketches are clear and easy to understand.	The work is presented in an organized fashion but may be hard to read at times.	Diagrams and/or sketches are difficult to understand or are not used.
CT tools used				