Pythagoras
2021-1-RO01-KA220-HED-000032258

Funded by the European Union

Mini-PBL example

Teaching Guide for Teachers

Mini-PBL project	
Teacher data sheet: Teaching Guide	
Title	Safe driving of passengers traveling on uneven road
SDG attended	Using this UN graphics, we mark such SDG which this project works. <table><tr><td></td><td>1 NO POVERTY</td><td></td><td>2 ZERO HUNGER</td><td>X</td><td>3 GOOD HEALTH AND WELL-BEING</td><td>X</td><td>4 QUALITY EDUCATION</td><td></td><td>5 GENDER EQUALITY</td><td></td><td>6 CLEAN WATER AND SANITATION</td></tr><tr><td></td><td>7 AFFORDABLE AND CLEAN ENERGY</td><td>X</td><td>8 DECENT WORK AND ECONOMIC GROWTH</td><td>X</td><td>9 INDUSTRY, INNOVATION AND INFRASTRUCTURE</td><td></td><td>10 REDUCED INEQUALITIES</td><td></td><td>11 SUSTAINABLE CITIES AND COMMUNITIES</td><td>X</td><td>12 RESPONSIBLE CONSUMPTION AND PRODUCTION</td></tr><tr><td></td><td>13 CLIMATE ACTION</td><td></td><td>14 LIFE BELOW WATER</td><td></td><td>15 LIFE ON LAND</td><td></td><td>16 PEACE, JUSTICE AND STRONG INSTITUTIONS</td><td>X</td><td>17 PARTNERSHIPS FOR THE GOALS</td><td></td><td></td></tr></table>
Content units	Systems of linear differential equations
Sessions	4 sessions of 100 min
Hours of autonomous work	100 min
Competences to be developed	**Reasoning and modelling** ● Develop thinking strategies to solve real life problems ● Explore, analyse, and apply mathematical ideas ● Estimate reasonably and demonstrate fluent, flexible, and strategic thinking about graphs ● Model with mathematics in situational contexts ● Think creatively and with curiosity and wonder when exploring problems **Understanding and solving** ● Develop, demonstrate, and apply conceptual understanding of mathematical ideas through story, inquiry, and problem solving ● Visualize to explore and illustrate mathematical concepts and relationships ● Apply flexible and strategic approaches to solve problems ● Solve problems with persistence and a positive disposition

	• Engage in problem-solving experiences connected with real-life examples. **Communicating and representing** • Explain and justify mathematical ideas and decisions in many ways • Represent mathematical ideas in concrete, pictorial, and symbolic forms • Use mathematical vocabulary and language to contribute to discussions in the classroom • Take risks when offering ideas in classroom discourse **Connecting and reflecting** • Reflect on mathematical thinking • Connect mathematical concepts with each other, other areas, and personal interests • Use mistakes as opportunities to advance learning • Incorporate First Peoples worldviews, perspectives, knowledge, and practices to make connections with mathematical concepts
ICT tools to be used	Available Computer Algebra Systems: Mathematica
Context: project statement	Travellers in cars are exposed to fatigue and unwanted vibrations that affect their health. Driver fatigue affects his performance and reactions, which increases the risk of a traffic accident. Undesirable vibrations, the frequency of which depends on the speed of the car, occur when the car drives on an uneven road (a road with a different profile) in the vertical direction (causing floating), in the longitudinal direction (causing rocking) and in the transverse direction (causing swinging). They act on a person through the floor, seat and steering wheel of the car. In cars, we consider oscillation in the vertical direction (vertical acceleration), which affects the driver and passengers, to be a decisive factor. For this direction of oscillation, it is necessary to avoid the frequency band in the range of 4-8 Hz (the natural frequency of the human organism in the abdomen), which causes nausea. The safety of the car and the comfort of passengers are improved by the suspension system of the car and properly designed damping. Their task is to capture and dampen vertical forces when the car drives over an uneven road. From the practical point of view - the safe driving of passengers on an uneven road - it is important that the movement of the driver and passengers in the car seats when driving on an uneven road stabilizes as soon as possible, and adapts to the shape of the road so that the dangerous acceleration of the car's suspended masses is dampened as soon as possible. Several models of the car - quarter, half and full are used in order to simulate the oscillation of the car, it's non spring-loaded and spring-loaded masses when driving on an uneven road at different speeds. Each model works with an excitation signal from the road (kinematic excitation). Non spring-loaded masses of a car are wheel, tire, axle, brakes, ... Spring-loaded masses of a car are body, driver, passengers, seats, cargo, ...

	The behavior of the car when driving on an uneven road allows us to know the solution of the equations of motion for the corresponding model of the car in the physical and state space.
Tasks and problems	**TASK** A car is moving along an uneven road in time *T* at a constant speed. Mathematical model of the uneven road is given by function *u*(*t*). Movement of the non spring-loaded masses and spring-loaded masses of the car on the uneven road represents a real system with forced oscillation. The behavior of the car (oscillating system) when driving on an uneven road allows us to know the solution of the motion equations for a quarter model of a car with 2 degrees of freedom (Fig.) in the physical space $m_1\ddot{x}_1 + k_1(x_1 - u) - k_2(x_2 - x_1) - b_2(\dot{x}_2 - \dot{x}_1) = 0$ $m_2\ddot{x}_2 + k_2(x_2 - x_1) + b_2(\dot{x}_2 - \dot{x}_1) = 0$ $x_1(0) = 0 \ , \ \dot{x}_1(0) = 0$ $x_2(0) = 0 \ , \ \dot{x}_2(0) = 0$ where • $x_i = x_i\,(t)\ ,\ x_i' = x_i'\,(t)\ ,\ x_i'' = x_i''(t)\ ,\ u = u(t)$ • the mathematical model of an uneven road $u(t) = 0{,}05\sin(t)$ – the amplitude $A = 0{,}05\ m$ • non spring-loaded masses of a car $x_1\,(t)$ – a deflection $m_1 = 386{,}48\ [kg]$ – a mass $k_1 = 1{,}2528.106\ [N/m]$ – a radial stiffness coefficient of the tyre • spring-loaded masses of a car $x_2(t)$ – a deflection $m_2 = 2628{,}7\ [kg]$ – a mass $k_2 = 2{,}0597.105\ [N/m]$ – a coefficient of the spring stiffness $b_2 = 6{,}722\ .103\ [Ns/m]$ – a damping coefficient of oil shock absorber

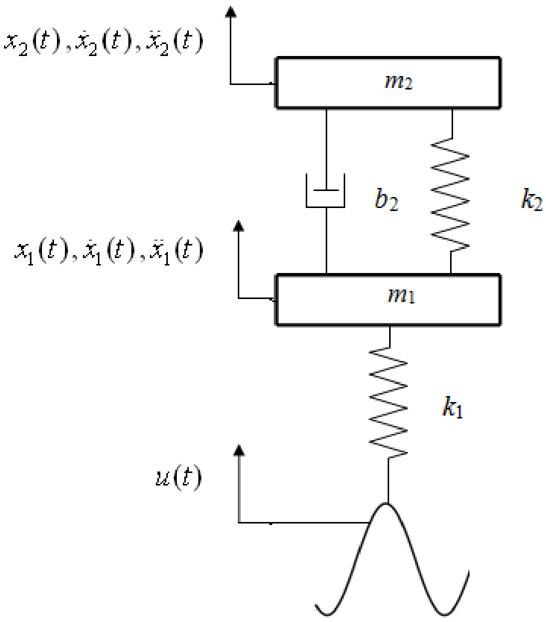


Fig. Quarter model with 2 degrees of freedom
of a car moving on uneven road

Task 1.

Write equations of motion in matrix form.

Solution:

$$m_1\ddot{x}_1 + k_1(x_1 - u) - k_2(x_2 - x_1) - b_2(\dot{x}_2 - \dot{x}_1) = 0$$
$$m_2\ddot{x}_2 + k_2(x_2 - x_1) + b_2(\dot{x}_2 - \dot{x}_1) = 0$$
$$\downarrow$$
$$m_1\ddot{x}_1 + b_2\dot{x}_1 - b_2\dot{x}_2 + (k_1 + k_2)x_1 - k_2x_2 = k_1u$$
$$m_2\ddot{x}_2 - b_2\dot{x}_1 + b_2\dot{x}_2 - k_2x_1 + k_2x_2 = 0$$

$$\begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \cdot \begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} + \begin{pmatrix} b_2 & -b_2 \\ -b_2 & b_2 \end{pmatrix} \cdot \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} + \begin{pmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} k_1u \\ 0 \end{pmatrix}$$

$$M\bar{x}'' + B\bar{x}' + K\bar{x} = \bar{F}(t)\ ,\ \bar{x}(0) = \bar{0}\ ,\ \bar{x}'(0) = \bar{0}$$

$\bar{x}(t)$ – a deflection vector
$\bar{x}'(t)$ – *a* velocity vector
$\bar{x}''(t)$ – an acceleration vector
M – *a* mass matrix
B – *a* damping matrix
K – *a* stiffness matrix
$\bar{F}(t)$ – *a* driven force vector

	Task 2. Transform the motion equations into the state space. Solve next tasks in this space. ***Solution guide*:** Transform the motion equation into a system of linear differential equations of first order $\bar{\boldsymbol{z}}'(\boldsymbol{t}) = \boldsymbol{A}\,\bar{\boldsymbol{z}}(\boldsymbol{t}) + \bar{\boldsymbol{f}}(\boldsymbol{t})$ along with Cauchy initial conditions. First adjust the motion equations to the form $x_1'' = f_1(t, x_1, x_1', x_2, x_2')$ $x_2'' = f_2(t, x_1, x_1', x_2, x_2')$ Then realize their transformation into the state space by reducing the order of the differential equation using $x_1(t) = z_1(t)$, $x_1'(t) = z_2(t)$, $x_2(t) = z_3(t)$, $x_2'(t) = z_4(t)$ ***Solution*:** $$\ddot{x}_1 = \frac{-k_1 - k_2}{m_1} x_1 - \frac{b_2}{m_1}\dot{x}_1 + \frac{k_2}{m_1} x_2 + \frac{b_2}{m_1}\dot{x}_2 + \frac{k_1}{m_1} u$$ $$\ddot{x}_2 = \frac{k_2}{m_2} x_1 + \frac{b_2}{m_2}\dot{x}_1 - \frac{k_2}{m_2} x_2 - \frac{b_2}{m_2}\dot{x}_2$$ Transformation into the state space $$z_1' = z_2 \quad ; \quad z_1(0) = 0$$ $$z_2' = \frac{-k_1 - k_2}{m_1} z_1 - \frac{b_2}{m_1} z_2 + \frac{k_2}{m_1} z_3 + \frac{b_2}{m_1} z_4 + \frac{k_1}{m_1} u \quad ; \quad z_2(0) = 0$$ $$z_3' = z_4 \quad ; \quad z_3(0) = 0$$ $$z_4' = \frac{k_2}{m_2} z_1 + \frac{b_2}{m_2} z_2 - \frac{k_2}{m_2} z_3 - \frac{b_2}{m_2} z_4 \quad ; \quad z_4(0) = 0$$ and in the matrix form $$\begin{pmatrix} z_1' \\ z_2' \\ z_3' \\ z_4' \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_1 - k_2}{m_1} & \frac{-b_2}{m_1} & \frac{k_2}{m_1} & \frac{b_2}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{m_2} & \frac{b_2}{m_2} & \frac{-k_2}{m_2} & \frac{-b_2}{m_2} \end{pmatrix} \cdot \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{k_1}{m_1} u(t) \\ 0 \\ 0 \end{pmatrix} \; ; \; \begin{pmatrix} z_1(0) \\ z_2(0) \\ z_3(0) \\ z_4(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$ $$\bar{z}'(t) = A \cdot \bar{z}(t) + \bar{f}(t) \; ; \; \bar{z}(0) = \bar{0}$$ Insert input values and solve system of two linear differential equations with constant coefficients

$$\begin{pmatrix} z_1' \\ z_2' \\ z_3' \\ z_4' \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -3774.5 & -17.3929 & 532.938 & 17.3929 \\ 0 & 0 & 0 & 1 \\ 78.3543 & 2.55716 & -78.3543 & -2.55716 \end{pmatrix} \cdot \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} + \begin{pmatrix} 0 \\ 162.78\sin(t) \\ 0 \\ 0 \end{pmatrix} ; \ \bar{z}(0) = \bar{0}$$

Task 3.
What is the difference between solutions of motion equations in the physical space and in the state space?

Solution:
Solutions of motion equations

- in the physical space

  $x_1(t)$, $x_2(t)$ – only deflections of the non spring-loaded and spring-loaded masses of a car
- in the state space

  $z_1(t) = x_1(t)$, $z_3(t) = x_2(t)$ – deflections of the non spring-loaded and spring-loaded masses of a ca

  $z_2(t) = x_1'(t)$, $z_4(t) = x_2'(t)$ – velocities of the non spring-loaded and spring-loaded masses of a car

Task 4.
Investigate the stability of the system with forced oscillations.
(Use the eigenvalues of matrix A of the system $\bar{\boldsymbol{z}}'(\boldsymbol{t}) = \boldsymbol{A}\,\bar{\boldsymbol{z}}(\boldsymbol{t}) + \bar{\boldsymbol{f}}(\boldsymbol{t})$.)

Solution guide:

The forced oscillation system is stable, i. e. forced oscillations of the non spring-loaded and spring-loaded masses of a car are dampened), when all eigenvalues of matrix A are $\lambda \in C \wedge a < 0, \lambda = a + ib$.
After inserting solution in the form $\bar{z}(t) = \bar{v}e^{\lambda t}$ into the system $\bar{z}'(t) = A\,\bar{z}(t)$, solution of the resulting system $A\,\bar{v} = \lambda\,\bar{v}$ will be an eigenvector $\bar{v}$ (an eigen shape of an oscillation) belonging to λ.
Algebraically manipulate this system in order to obtain a homogeneous system of equations.
Calculate all eigenvalues λ ($\lambda = \Omega^2$, where Ω is an eigen frequency of an oscillation)) of the matrix of the homogeneous system from the requirement for its non-trivial solution ($\bar{v} \neq \bar{0}$).

Solution:

$$A \cdot \bar{v} = \lambda \cdot \bar{v} \Leftrightarrow (A - \lambda E) \cdot \bar{v} = \bar{0} \rightarrow \det(A - \lambda E) = 0$$

$$(A - \lambda E) = \begin{pmatrix} -\lambda & 1 & 0 & 0 \\ -3774.5 & -17.3929 - \lambda & 532.938 & 17.3929 \\ 0 & 0 & -\lambda & 1 \\ 78.3543 & 2.55716 & -78.3543 & -2.55716 - \lambda \end{pmatrix}$$

$$\det(A - \lambda E) = 0 \Leftrightarrow 253991. + 8289.19\,\lambda + 3852.86\,\lambda^2 + 19.95\,\lambda^3 + \lambda^4 = 0$$

$\lambda \to -9.03318 - 60.5765\, i$
$\lambda \to -9.03318 + 60.5765\, i$
$\lambda \to -0.941841 - 8.17457\, i$
$\lambda \to -0.941841 + 8.17457\, i$

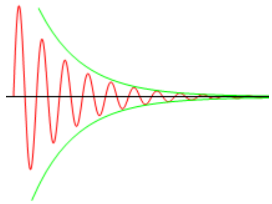
Conclusion.

The forced oscillations system is stable, i. e. forced oscillations of individual parts of a car will be dampened, because all $\lambda \in C \wedge a < 0$.

Comment.

Look at the solution

$\bar{z}(t) = \bar{v}e^{\lambda t} = \bar{v}e^{(a+ib)t} = \bar{v}e^{at}\big(cos(bt) + isin(bt)\big).$



Task 5.

System $\bar{z}'(t) = A\,\bar{z}(t) + \bar{f}(t)\,, \bar{z}(0) = \bar{0}$ does not have an analytical solution. To solve the system we will use numerical methods.

Analyse conditions of the numerical solvability of this system (the existence and the uniqueness of its solution) under which this system is solvable on the interval [0;10].

Solution:

Individual text inserted by students.

INSTRUCTION. Solve tasks 6.– 9. using software Mathematica.

Task 6.

Display the deflection, velocity, and acceleration of the non spring-loaded and spring-loaded masses of a car at time t ∈ [0,10] in sec using the **NDSolve** command.

Can we say, based on the graphs, that after a certain time the acceleration of the non spring-loaded and spring-loaded masses of the car are dampened, i.e. the car's movement on the uneven road will be stabilized?

Solution:

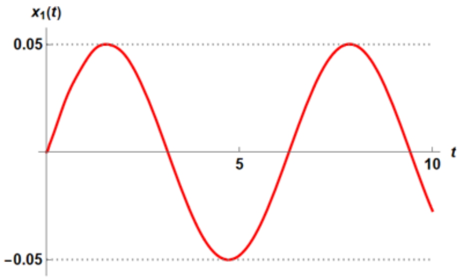
NDSolve[eqns, {y_1, y_2, … },{x, x_{MIN}, x_{MAX}}] – finds a numerical solution of the ordinary differential equations (*eqns)* for the functions y_1, y_2, …, with the independent variable *x* in the range x_{min} to x_{max}.

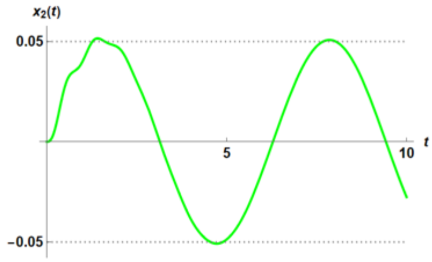
NDSolve gives results in terms of InterpolatingFunction objects (InterpolatingFunction[{{x_{min}, x_{max}.}}, <>][x]).

```
z1(t) = InterpolatingFunction[{{0., 10.}}, <>][t]
z2(t) = InterpolatingFunction[{{0., 10.}}, <>][t]
z3(t) = InterpolatingFunction[{{0., 10.}}, <>][t]
z4(t) = InterpolatingFunction[{{0., 10.}}, <>][t]
```

Non *spring-loaded masses of a car* | ***Spring-loaded masses of a car***

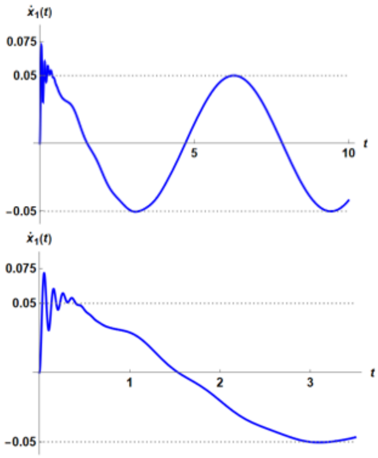
the deflection

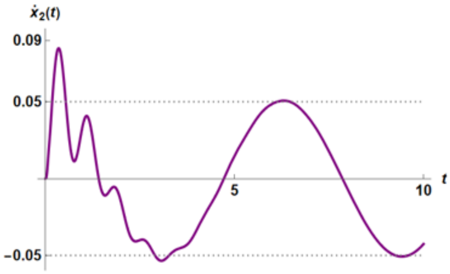




Non *spring-loaded masses of a car* | ***Spring-loaded masses of a car***

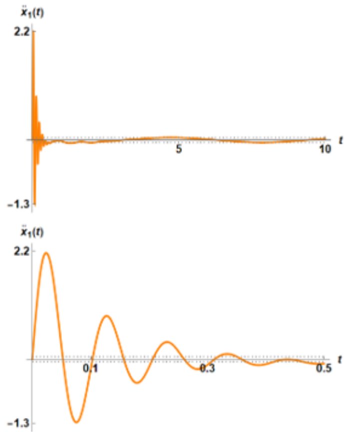
the velocity

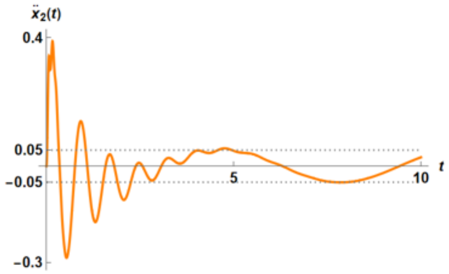




Non *spring-loaded masses of a car* | ***Spring-loaded masses of a car***

the acceleration

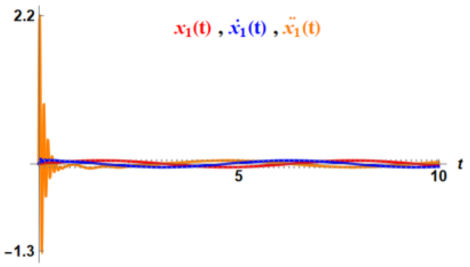




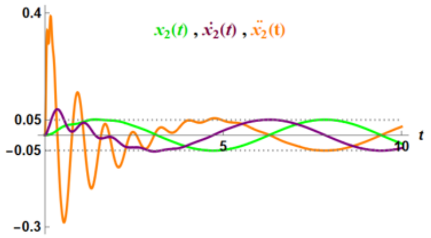
Conclusion.

Accelerations of the non spring-loaded masses and spring-loaded masses of a car are dampened after a certain time, it means a motion of a car will be stable on an uneven road.

Non *spring-loaded masses of a car*



Spring-loaded masses of a car



Task 7.

Find and display deflection and velocity of the non spring-loaded and spring-loaded masses of a car at time $t \in [0,10]$ in sec using Euler´s method with step h = 0.01.

If the method is not stable for the given step h, modify step h so that it was stabled.

Solution:

The iteration scheme for Euler´s method

$$\bar{k} = A \cdot \bar{z}_i + \bar{g}(t_i)$$
$$\bar{z}_{i+1} = \bar{z}_i + h \cdot \bar{k}$$
$$t_{i+1} = t_i + h$$
$$i = 0, 1, \ldots, n$$

The approximate solutions using step h = 0.01.
(We only list every 100th solution.)

i	t_i	$z_1 = x_1$	$z_2 = \dot{x}_1$	$z_3 = x_2$	$z_4 = \dot{x}_2$
0	0	0	0	0	0
100	1.	-4.8216528	142.45075	0.10323332	-13.966853
200	2.	30139.254	1.028814×10^{6}	-1564.6171	48633.257
300	3.	-2.6361025×10^{6}	$-1.6155318 \times 10^{10}$	9.9674546×10^{6}	3.1659682×10^{8}
400	4.	-1.547458×10^{12}	8.361022×10^{13}	-3.1498093×10^{9}	$-5.2218036 \times 10^{12}$
500	5.	1.3278946×10^{16}	1.3142161×10^{17}	$-4.9242005 \times 10^{14}$	2.7855262×10^{16}
600	6.	$-3.2168563 \times 10^{19}$	$-5.4710466 \times 10^{21}$	4.345893×10^{18}	3.5427139×10^{19}
700	7.	$-4.1846491 \times 10^{23}$	3.9617056×10^{25}	$-1.1258663 \times 10^{22}$	$-1.7517438 \times 10^{24}$
800	8.	5.2293204×10^{27}	$-5.1476601 \times 10^{28}$	$-1.3074909 \times 10^{26}$	1.3031243×10^{28}
900	9.	$-2.2613148 \times 10^{31}$	$-1.6263524 \times 10^{33}$	1.6967982×10^{30}	$-1.9424451 \times 10^{31}$
1000	10.	$-8.0336814 \times 10^{34}$	1.6492878×10^{37}	$-7.5996394 \times 10^{33}$	$-5.1379114 \times 10^{35}$

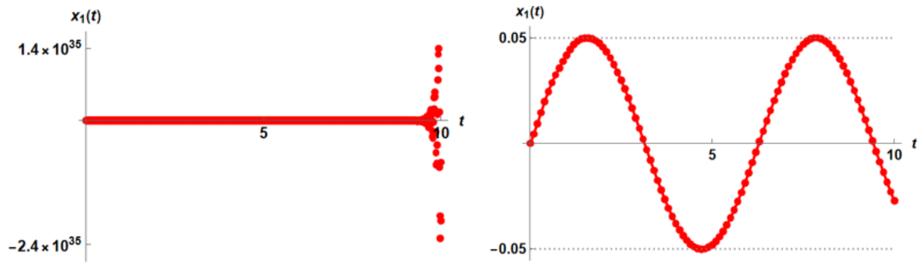
The approximate solutions using step h = 0.001.
(We only list every 100th solution.)

i	t_i	$z_1=x_1$	$z_2=\dot{x}_1$	$z_3=x_2$	$z_4=\dot{x}_2$
0	0	0	0	0	0
100	0.1	0.0044315129	0.02716347	0.00080629195	0.02274988
200	0.2	0.0093245361	0.042712682	0.0047636469	0.058011377
300	0.3	0.014564984	0.049488039	0.011904953	0.082293592
400	0.4	0.019781983	0.04927819	0.020330743	0.082246217
500	0.5	0.024591327	0.044683303	0.027612362	0.060770491
600	0.6	0.028797255	0.038869683	0.032273834	0.032605602
700	0.7	0.032448803	0.034331343	0.034471375	0.013727904
800	0.8	0.035744642	0.031892377	0.035606021	0.011796715
900	0.9	0.038868192	0.03067624	0.037273608	0.023064485
1000	1.	0.041861431	0.028992102	0.040277192	0.036484144
			....		
9900	9.9	-0.02293263	-0.044557411	-0.023209894	-0.045146911
10000	10.	-0.027267524	-0.04204301	-0.027602317	-0.042602217

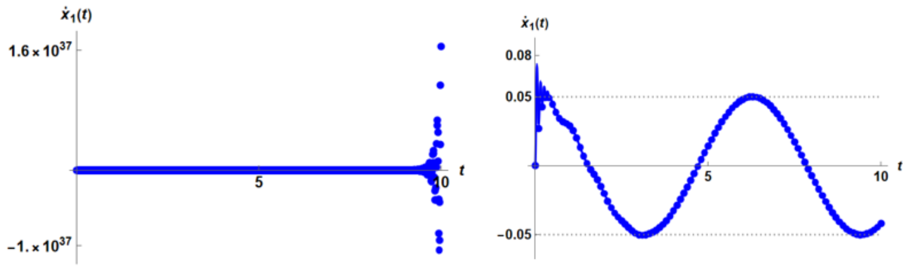
the step ***h* = 0,01** — the **unstable** method – the given step is too large

the step ***h* = 0,001** — the **stable** method – the selected step is suitable

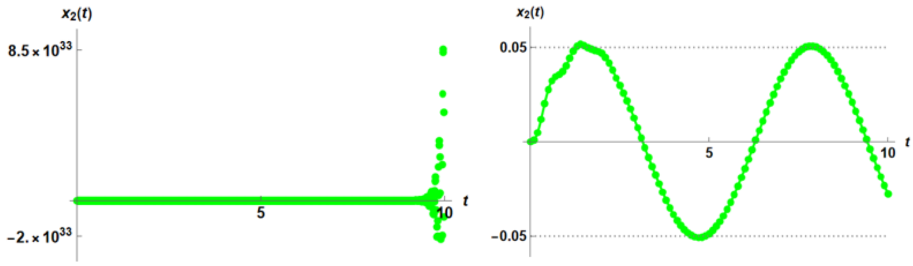
Non spring-loaded masses of a car **–** the deflection



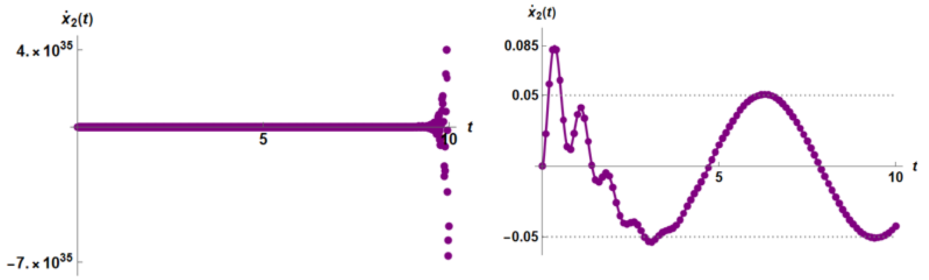
Non spring-loaded of a car **–** the velocity



Spring-loaded masses of a car **–** the deflection



Spring-loaded masses of a car – the velocity



Task 8.

Calculate and compare approximate values of accelerations at the time *t* = 1 sec. Use Non-symmetric formula backward of the order $O(h^2)$, *h* = 0.001.

Solution:

Non-symmetric formula backward of the order $O(h^2)$

$$f''(x) \approx \frac{2f(x) - 5f(x-h) + 4f(x-2h) - f(x-3h)}{h^2}$$

NDSolve command

i	t_i	$x_1(t)$	$x_2(t)$
997	0.997	0.041782851	0.040242023
998	0.998	0.041811839	0.040277838
999	0.999	0.041840801	0.040313755
1000	1.	0.041869739	0.040349774

Euler´s method

i	t_i	$x_1(t)$	$x_2(t)$
997	0.997	0.041774298	0.040168379
998	0.998	0.041803369	0.040204543
999	0.999	0.041832413	0.040240814
1000	1.	0.041861431	0.040277192

Comparison

$f''(x)$	*masses of a car*	
	non spring-loaded	*spring-loaded*
NDSolve	– 0,022	0,102
Euler´s method	– 0,025	0,107
the difference	0,003	0,005

Task 9.

What is the greatest value of time $t \in [0,10]$ for which it is possible to use the Non-symmetric forward formula of the order $O(h^2)$, *h* = 0.001 to calculate an approximate value of the acceleration?

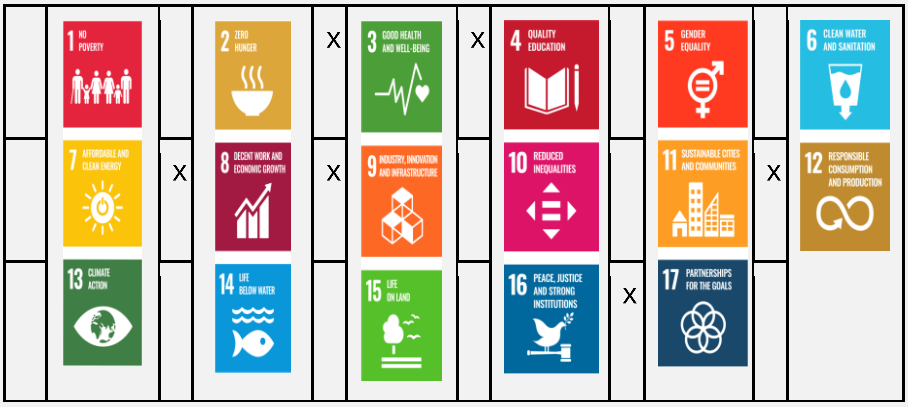
Give your reasons.

Solution guide:

	Non-symmetric formula forward of the order $O(h^2)$ $$f''(x) \approx \frac{2f(x) - 5f(x+h) + 4f(x+2h) - f(x+3h)}{h^2}$$ ***Solution:*** *t* = 9,997 sec
Outcomes expected	- Graphics fitting the solution; - Numerical results explained and put in context; - Capture of ICT tools solutions used; - Sequence of steps followed; - Remark computations done by hand and done by ICT tools; - Provide complete answer to questions; - All the results must be presented in the context of the problem;
Guide for Learning	At the beginning of the course, the students need guides on new activities, and feel your support on a well-structured pack of suggestions on how to address the problems posted. Namely: - Read carefully the problem statement and the tasks posted. Always maintain a global view of all the projects. - Identify, or try to do a first draft match, the content units of your lecture notes involved in every task. - Take your lecture notes open and review before starting to solve the problems. - Match output expected with the tasks posted, at least as first draft approach. - Follow the order of the tasks, try to increase the knowledge of the problem while you are solving the activities. - Always think that maybe there are different ways to solve a problem. - Use ICT tools to avoid hard computations and check your solutions are correct in different ways if possible. - The solutions are always part of a context, expressing such a final solution totally integrated in the problem posted. - Be sure you answer the complete questions. - Always try to solve the questions by yourself. - If the project can be done in groups, discuss with the groups the proposed problem, to confirm and detect fails or weaknesses, confront strategies, discuss presentation format, etc. Working in groups doesn't mean work less but work better.
Guide for Teaching	Some hints needed to present and launch the mini-PBL to students - Do a small Introduction concerning Energy consumption, added to the Climate Change crisis we are currently living in. - Do a small introduction about the relations between power and energy, with the basic equations. - Students will form groups of 4 students and solve the mini-PBL using the eduScrum methodology. - The students should do each exercise in a sequential order, starting from Task 1. - The students should be able to thoroughly read and interpret

	the numerical results from a mathematical and the real-life example point of view. They should include also a discussion of the climate change crisis and enumerate some strategies they could apply at home or even at university to save resources, namely reduce energy consumption. They should also mention how this mini-PBL helps them identify the Sustainable Development Goals 4, 7 and 9.
Assessment	- Final report; - Oral presentation; - Peer-assessment: students will apply peer-assessment for their periodic performance using online peer assessment tools used and available at the respective institution.
Others: References	Starek, L. Kmitanie s riadením. Vydavateľstvo STU, Bratislava, 2009. Gabková, J. Riešenie pohybových rovníc pre štvrtinový model automobilu v programovom systéme Mathematica. In *26th International Colloquium on the Management of Educational Process : Proceedings of abstracts and electronic version of reviewed contributions on CD-Rom. - Brno, May 22, 2007*. Brno : University of Defence, 2008, s.CD Rom. ISBN 978-80-7231-511-6

Learning Guide for Students

Mini-PBL project	
Student data sheet: Teaching Guide	
Title	Safe driving of passengers traveling on uneven road
SDG attended	Using this UN graphics, we mark such SDG which this project works. 
Content units	Systems of linear differential equations
Sessions	4 sessions of 100 min
Hours of autonomous work	100 min
Competences to be developed	**Reasoning and modelling** ● Develop thinking strategies to solve real life problems ● Explore, analyse, and apply mathematical ideas ● Estimate reasonably and demonstrate fluent, flexible, and strategic thinking about graphs ● Model with mathematics in situational contexts ● Think creatively and with curiosity and wonder when exploring problems **Understanding and solving** ● Develop, demonstrate, and apply conceptual understanding of mathematical ideas through story, inquiry, and problem solving ● Visualize to explore and illustrate mathematical concepts and relationships ● Apply flexible and strategic approaches to solve problems ● Solve problems with persistence and a positive disposition ● Engage in problem-solving experiences connected with real-life examples. **Communicating and representing** ● Explain and justify mathematical ideas and decisions in many ways

	• Represent mathematical ideas in concrete, pictorial, and symbolic forms • Use mathematical vocabulary and language to contribute to discussions in the classroom • Take risks when offering ideas in classroom discourse **Connecting and reflecting** • Reflect on mathematical thinking • Connect mathematical concepts with each other, other areas, and personal interests • Use mistakes as opportunities to advance learning • Incorporate First Peoples worldviews, perspectives, knowledge, and practices to make connections with mathematical concepts
ICT tools to be used	Available Computer Algebra Systems: Mathematica.
Context: project statement	Travellers in cars are exposed to fatigue and unwanted vibrations that affect their health. Driver fatigue affects his performance and reactions, which increases the risk of a traffic accident. Undesirable vibrations, the frequency of which depends on the speed of the car, occur when the car drives on an uneven road (a road with a different profile) in the vertical direction (causing floating), in the longitudinal direction (causing rocking) and in the transverse direction (causing swinging). They act on a person through the floor, seat and steering wheel of the car. In cars, we consider oscillation in the vertical direction (vertical acceleration), which affects the driver and passengers, to be a decisive factor. For this direction of oscillation, it is necessary to avoid the frequency band in the range of 4-8 Hz (the natural frequency of the human organism in the abdomen), which causes nausea. The safety of the car and the comfort of passengers are improved by the suspension system of the car and properly designed damping. Their task is to capture and dampen vertical forces when the car drives over an uneven road. From the practical point of view - the safe driving of passengers on an uneven road - it is important that the movement of the driver and passengers in the car seats when driving on an uneven road stabilizes as soon as possible, and adapts to the shape of the road so that the dangerous acceleration of the car's suspended masses is dampened as soon as possible. Several models of the car - quarter, half and full are used in order to simulate the oscillation of the car, it’s non spring-loaded and spring-loaded masses when driving on an uneven road at different speeds. Each model works with an excitation signal from the road (kinematic excitation). Non spring-loaded masses of a car are wheel, tire, axle, brakes, ... Spring-loaded masses of a car are body, driver, passengers, seats, cargo, ... The behavior of the car when driving on an uneven road allows us to know the solution of the equations of motion for the corresponding model of the car in the physical and state space.
Tasks and problems	**TASK** A car is moving along an uneven road in time T at a constant speed. Mathematical model of the uneven road is given by function $u(t)$.

Movement of the non spring-loaded masses and spring-loaded masses of the car on the uneven road represents a real system with forced oscillation.

The behavior of the car (oscillating system) when driving on an uneven road allows us to know the solution of the motion equations for a quarter model of a car with 2 degrees of freedom (Fig.) in the physical space

$$m_1\ddot{x}_1 + k_1\left(x_1 - u\right) - k_2\left(x_2 - x_1\right) - b_2\left(\dot{x}_2 - \dot{x}_1\right) = 0$$

$$m_2\ddot{x}_2 + k_2\left(x_2 - x_1\right) + b_2\left(\dot{x}_2 - \dot{x}_1\right) = 0$$

$$x_1(0) = 0 \;,\; \dot{x}_1(0) = 0$$

$$x_2(0) = 0 \;,\; \dot{x}_2(0) = 0$$

where

- $x_i = x_i\,(t)$, $x_i' = x_i'\,(t)$, $x_i'' = x_i''(t)$, $u = u(t)$.
- $u(t) = 0{,}05\,\sin(t)$ – an unevenness of a road
- non spring-loaded masses of a car

  $x_1\,(t)$ – a deflection

  $m_1 = 386{,}48\;[kg]$ – a mass

  $k_1 = 1{,}2528.106\;[N/m]$ – a radial stiffness coefficient of the tyre
- spring-loaded masses of a car

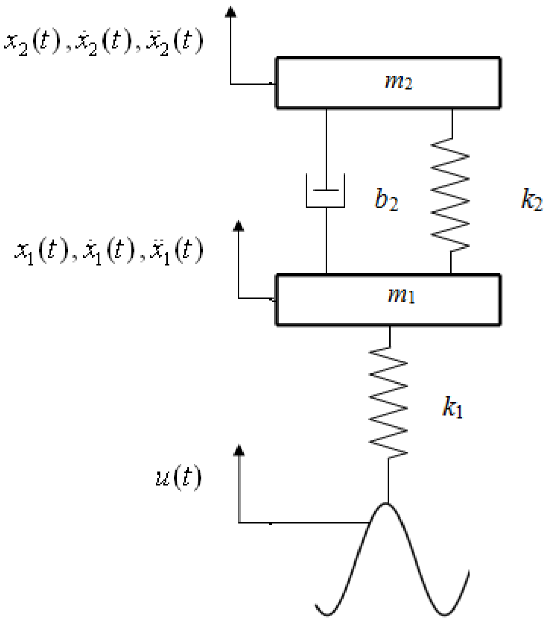
  $x_2(t)$ – a deflection

  $m_2 = 2628{,}7\;[kg]$ – a mass

  $k_2 = 2{,}0597.105\;[N/m]$ – a coefficient of the spring stiffness

  $b_2 = 6{,}722\;.103\;[Ns/m]$ – a damping coefficient of oil shock absorber
- the model of an uneven road

  $u(t) = 0{,}05\sin(t)$ – the amplitude $A = 0{,}05\;m$

	 Fig. Quarter model with 2 degrees of freedom of a car moving on uneven road **Task 1.** Write equations of motion in matrix form. **Task 2.** Transform the motion equations into the state space. Solve next tasks in this space. ***Solution guide*:** First adjust the motion equations to the form $x_1'' = f_1(t, x_1, x_1', x_2, x_2')$ $x_2'' = f_2(t, x_1, x_1', x_2, x_2')$ Then realize their transformation into the state space by reducing the order of the differential equation using $x_1(t) = z_1(t)$, $x_1'(t) = z_2(t)$, $x_2(t) = z_3(t)$, $x_2'(t) = z_4(t)$ **Task 3.** What is the difference between solutions of motion equations in the physical space and in the state space? **Task 4.** Investigate the stability of the system with forced oscillations. (Use the eigenvalues of matrix of system.) ***Solution guide:*** This system is stable, i. e. oscillations of the non spring-loaded and spring-loaded masses of a car are dampened, when all eigenvalues of matrix *A* are

	$\lambda \in C \wedge a < 0, \lambda = a + ib$. After inserting solution in the form $\bar{z}(t) = \bar{v}e^{\lambda t}$ into the system $\bar{z}'(t) = A\,\bar{z}(t)$, solution of the resulting system $A\,\bar{v} = \lambda\,\bar{v}$ will be an eigenvector $\bar{v}$ (an eigen shape of a vibration) belonging to λ. Algebraically manipulate this system in order to obtain a homogeneous system of equations. Calculate all eigenvalues λ ($\lambda = \Omega^2$, where Ω is an eigen frequency of a vibration) of the matrix of the homogeneous system from the requirement for its non-trivial solution ($\bar{v} \neq \bar{0}$). **Task 5.** System $\bar{z}'(t) = A\,\bar{z}(t) + \bar{f}(t), \bar{z}(0) = \bar{0}$ does not have an analytic solution. To solve the system we will use numerical methods. Analyse conditions of the numerical solvability of this system (the existence and the uniqueness of its solution) under which this system is solvable on the interval [0;10]. ***Solution:*** Individual text inserted by students. **INSTRUCTION.** Solve tasks 6.– 9. using software Mathematica. **Task 6.** Display the deflection, velocity, and acceleration of the non spring-loaded and spring-loaded masses of a car at time t ∈ [0,10] in sec using the **NDSolve** command. Can we say, based on the graphs, that after a certain time the acceleration of the non spring-loaded and spring-loaded masses of the car are dampened, i.e. the car's movement on the uneven road will be stabilized? **Task 7.** Find and display deflection and velocity of the non spring-loaded and spring-loaded masses of a car at time $t \in [0,10]$ in sec using Euler method with step *h* = 0.01. If the method is not stable for the given step *h*, modify step *h* so that it was stabled. **Task 8.** Calculate and compare approximate values of accelerations at the time *t* = 1 sec. Use Non-symmetric formula backward of the order $O(h^2)$, *h* = 0.001. **Task 9.** What is the greatest value of time $t \in [0,10]$ for which it is possible to use the Non-symmetric forward formula of the order $O(h^2)$, *h* = 0.001 to calculate an approximate value of the acceleration?

	Give your reasons. ***Solution guide:*** Non-symmetric formula forward of the order $O(h^2)$ $f''(x) \approx \frac{2f(x) - 5f(x+h) + 4f(x+2h) - f(x+3h)}{h^2}$
Outcomes expected	- Graphics fitting the solution; - Numerical results explained and put in context; - Capture of ICT tools solutions used; - Sequence of steps followed; - Remark computations done by hand and done by ICT tools; - Provide complete answer to questions; - All the results must be presented in the context of the problem;
Guide for Learning	- Read carefully the problem statement and the tasks posted. Always maintain a global view of all the projects. - Identify, or try to do a first draft match, the content units of your lecture notes involved in every task. - Take your lecture notes open and review before starting to solve the problems. - Match output expected with the tasks posted, at least as first draft approach. - Follow the order of the tasks, try to increase the knowledge of the problem while you are solving the activities. - Always think that maybe there are different ways to solve a problem. - Use ICT tools to avoid hard computations and check your solutions are correct in different ways if possible. - The solutions are always part of a context, expressing such a final solution totally integrated in the problem posted. - Be sure you answer the complete questions. - Always try to solve the questions by yourself. - If the project can be done in groups, discuss with the groups the proposed problem, to confirm and detect fails or weaknesses, confront strategies, discuss presentation format, etc. Working in groups doesn't mean work less but work better.
Assessment	- Final report; - Oral presentation; - Peer-assessment: students will apply peer-assessment for their periodic performance using online peer assessment tools used and available at the respective institution.
Others: References	Starek, L. Kmitanie s riadením. Vydavateľstvo STU, Bratislava, 2009. Gabková, J. Riešenie pohybových rovníc pre štvrtinový model automobilu v programovom systéme Mathematica. In *26th International Colloquium on the Management of Educational Process : Proceedings of abstracts and electronic version of reviewed contributions on CD-Rom. - Brno, May 22, 2007*. Brno : University of Defence, 2008, s.CD Rom. ISBN 978-80-7231-511-6

ANNEX 1: RUBRIC

Category	4=Excellent	3=Good	2=Low	1=Poor
Mathematical Concepts	Explanation shows complete understan- ding of the mathe- matical concepts used to solve the problem(s).	Explanation shows substantial understan- ding of the mathema- tical concepts used to solve the problem(s).	Explanation shows some understan- ding of the mathe- matical concepts needed to solve the problem(s).	Explanation shows very limited unders- tanding of the underlying concepts needed to solve the problem(s) OR is not written.
Mathematical Terminology and Notation	Correct terminology and notation are always used, making it easy to understand what was done.	Correct terminology and notation are usually used, making it fairly easy to understand what was done.	Correct terminolo- gy and notation are used, but it is sometimes not easy to understand what was done.	There is little use, or a lot of inappropriate use, of terminology and notation.
Strategy/Procedure	Typically, uses an efficient and effective strategy to solve the problem(s).	Typically, uses an effective strategy to solve the problem(s).	Sometimes uses an effective strategy to solve problems, but does not do it consistently.	Rarely uses an effective strategy to solve problems.
Completion	All problems are completed.	All but one of the problems are completed.	All but two of the problems are completed.	Several of the problems are not completed.
Mathematical Errors	90-100% of the steps and solutions have no mathematical errors.	Almost all (85-89%) of the steps and solu- tions have no mathematical errors.	Most (75-84%) of the steps and solu- tions have no mathematical errors.	More than 75% of the steps and solu- tions have mathe- matical errors.

Sources Checking				
Working with Others	Student was an engaged partner, listening to suggestions of others and working cooperatively throughout lesson.	Student was an engaged partner but had trouble listening to others and/or working cooperatively.	Student cooperated with others, but needed prompting to stay on-task.	Student did not work effectively with others.
Neatness and Organization	The work is presented in a neat, clear, organized fashion that is easy to read.	The work is presented in a neat and organized fashion that is usually easy to read.	The work is presented in an organized fashion but may be hard to read at times.	The work appears sloppy and unorganized. It is hard to know what information goes together.
Diagrams and Sketches	Diagrams and/or sketches are clear and greatly add to the reader's understanding of the procedure(s).	Diagrams and/or sketches are clear and easy to understand.	The work is presented in an organized fashion but may be hard to read at times.	Diagrams and/or sketches are difficult to understand or are not used.
CT tools used				