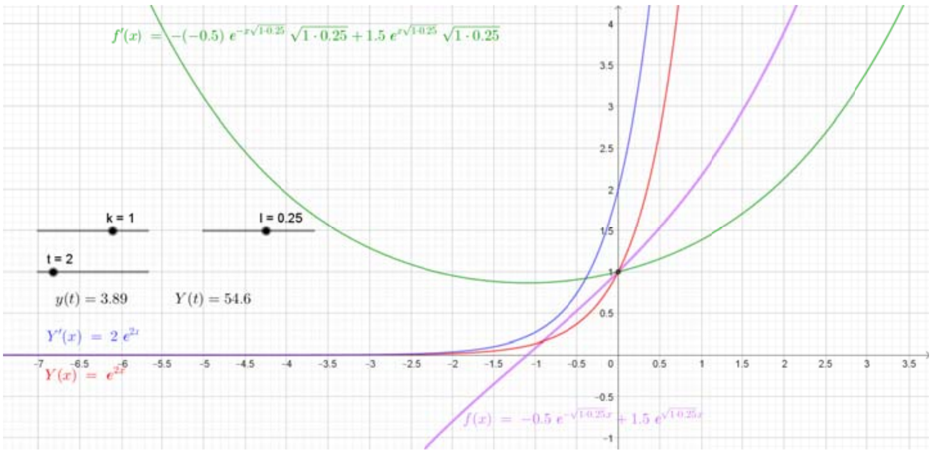
Mini-PBL example

Teaching Guide for Teachers

Mini-PBL project	
Teacher data sheet: Teaching Guide	
Title	The waste reduction
SDG attended	Using this UN graphics, we mark such SDG which this project works. 
Content units	Ordinary differential equations of order II with constant coefficients
Sessions	1 sessions of 1h
Hours of autonomous work	1h
Competences to be developed	**Reasoning and modelling** ● Develop thinking strategies to solve real life problems ● Explore, analyse, and apply mathematical ideas ● Estimate reasonably and demonstrate fluent, flexible, and strategic thinking about graphs ● Model with mathematics in situational contexts ● Think creatively and with curiosity and wonder when exploring problems **Understanding and solving** ● Develop, demonstrate, and apply conceptual understanding of mathematical ideas through story, inquiry, and problem solving ● Visualize to explore and illustrate mathematical concepts and relationships ● Apply flexible and strategic approaches to solve problems ● Solve problems with persistence and a positive disposition ● Engage in problem-solving experiences connected with real-life

	examples. **Communicating and representing** ● Explain and justify mathematical ideas and decisions in many ways ● Represent mathematical ideas in concrete, pictorial, and symbolic forms ● Use mathematical vocabulary and language to contribute to discussions in the classroom ● Take risks when offering ideas in classroom discourse **Connecting and reflecting** ● Reflect on mathematical thinking ● Connect mathematical concepts with each other, other areas, and personal interests ● Use mistakes as opportunities to advance learning ● Incorporate First Peoples worldviews, perspectives, knowledge, and practices to make connections with mathematical concepts
ICT tools to be used	Available Computer Algebra Systems: Mathematica, Maple, Matlab, GeoGebra, etc.
Context: project statement	The production of waste is increasing with the speed directly proportional to its quantity, due to increasing industrial production and poor environmental measures. To decrease the waste accumulation it is necessary to adopt various ecological measures, as recycling of the produced waste, decrease in the production, etc. leading to the decreased speed of its accumulation and decrease of its growth acceleration. These adopted environmental measures cannot lead to the complete diminishing of the accumulated waste on the planet, but they can considerably improve the planet pollution and have beneficial effect on the climate changes.
Tasks and problems	The accumulation of waste is increasing with the speed equal to twice its actual quantity. After adopting strong ecological measures, the speed of the accumulation was decreased to be equal to the actual waste quantity, while acceleration of this growth was decreased to one quarter of the former speed. Consider the amount of the waste to be 1 unit at the time, when the measures were imposed, *t* = 0. The process of waste accumulation can be represented by ordinary differential equation, of order 1 before measures, and order II after the regulations. Already after 2 years there can be visible remarkable improvements and considerable decrease in the accumulated waste. **Task 1:** Assemble both differential equations describing the accumulation of waste before measures and after their imposition. Answer: Let $y(t) > 0$ be the amount of accumulated waste in the time *t*, while derivative $y'(t)$ be the speed of the waste growth before impact of the ecological measures. This growth is described by the differential equation od order I $$y'(t) = 2y(t)$$ that can be rewritten as the homogeneous differential equation of order I and solved directly.

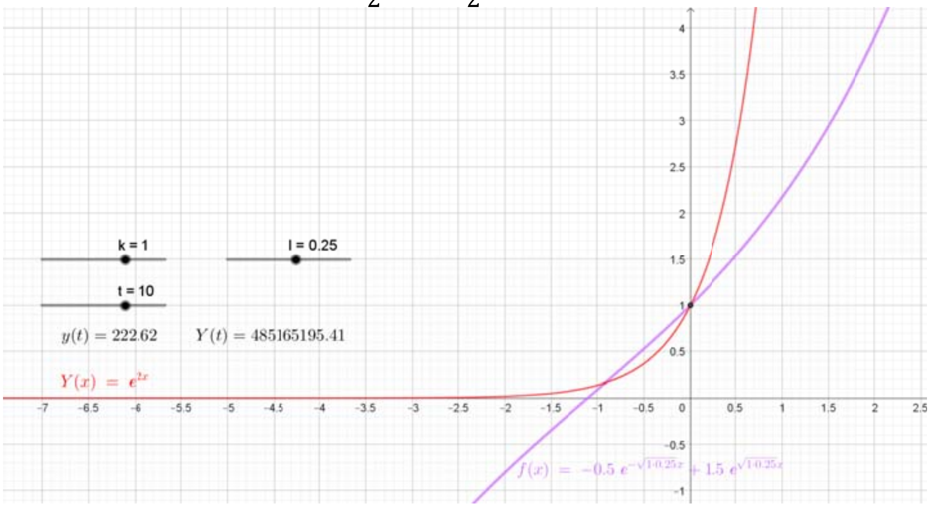
	$$y'(t) - 2y(t) = 0$$ Acceleration of the waste accumulation is then expressed as $$y''(t) = 2y'(t) = 4y(t)$$ therefore the solution can be obtained from the differential equation of order II with constant coefficients $$y''(t) - 4y(t) = 0$$ leading to the same general solution as differential equation of order I. After impact of the ecological measures, derivative $y'(t)$ would be represented as $$y'(t) = y(t)$$ while the acceleration of this speed will be reduced to $$y''(t) = \frac{1}{4}y'(t) = \frac{1}{4}y(t)$$ Such waste accumulation process can be described by the differential equation od order II with constant coefficients $$y''(t) - \frac{1}{4}y(t) = 0$$ **Task 2:** Find general solutions of both differential equations and their particular solutions determined by Cauchy initial conditions. Answer: General solution of theODR I is in the form $$Y(t) = ce^{2t}, c \in R, t \in \langle 0, T\rangle$$ where value of constant *c* in the particular solution can be determined from the given initial condition describing the waste accumulation as Cauchy initial problem: $$Y(0) = 1 \Longrightarrow 1 = ce^{0} \Longrightarrow c = 1$$ $$Y_P(t) = e^{2t} \quad , \qquad t \in \langle 0, T \rangle, T \in R$$ General solution of the respective ODR II can be determined from the characteristic equation $$r^2 - 4 = 0 \Longrightarrow r_1 = -2, r_2 = 2$$ in the form $$y(t) = c_1e^{-2t} + c_2e^{+2t}, \qquad c_1, c_2 \in R, t \in \langle 0, T\rangle$$ Values of constants c_1, c_2 for the particular solution can be calculated from the given initial conditions describing the waste accumulation as Cauchy initial problem: $$y(0) = 1, \;\; y'(0) = 2$$ $$Y_P(t) = 0.e^{-2t} + 1.e^{+2t} = \; e^{2t}, \qquad t \in \langle 0, T\rangle, T \in R$$ General solution of the ODR II describing the accumulation process after some ecological restrictions are implied can be determined from

	the characteristic equation $$r^2 - \frac{1}{4} = 0 \Longrightarrow r_1 = -\frac{1}{2} r_2 = \frac{1}{2}$$ in the form $$y(t) = c_1 e^{-\frac{1}{2}t} + c_2 e^{+\frac{1}{2}t}, \qquad c_1, c_2 \in R, t \in \langle 0, T \rangle, T \in R$$ Values of constants c_1, c_2 for the particular solution can be calculated from the given initial conditions describing the waste accumulation as Cauchy initial problem: $$y(0) = 1,\ \ y'(0) = 1$$ $$y_P(t) = -\frac{1}{2} e^{-\frac{1}{2}t} + \frac{3}{2} e^{+\frac{1}{2}t}, \qquad t \in \langle 0, T \rangle, T \in R$$ **Task 3:** Calculate the amount of accumulated waste after 2 years under both circumstances, compare these values, and estimate the impact of the ecological measures. Answer: ODR I $$Y_P(2) = e^4 \doteq 54{,}6\ units$$ ODR II $$y_P(2) = -\frac{1}{2} e^{-1} + \frac{3}{2} e^{1} \doteq -\frac{1}{6} + \frac{9}{2} \doteq \frac{-1+27}{6} \doteq \frac{13}{3} \doteq 3{,}9\ units$$ Achieved reduction of accumulated waste can be up to $50{,}7\ units$. **Task 4:** Sketch respective integral curves that are particular solutions of ODRs representing the waste accumulation under both circumstances and compare the waste growth during *T* years. Answer:  Both functions are increasing, as their derivatives are positive functions for $t \geq 0$. $$Y_P' = 2e^{2t}, \qquad t \in \langle 0, T \rangle, T \in R$$

$$y_P{}' = \frac{1}{4}e^{-\frac{1}{2}t} + \frac{3}{4}e^{+\frac{1}{2}t}, \;\; t \in \langle 0, T\rangle, T \in R$$

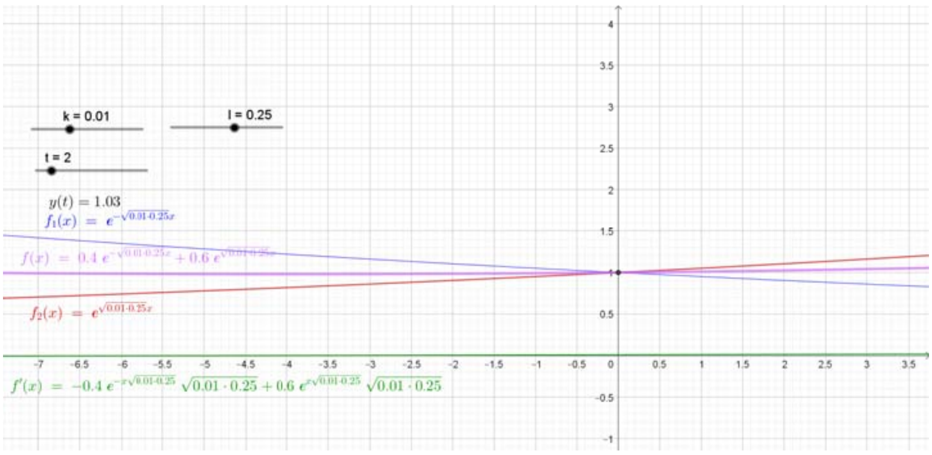
Task 4:
Calculate the amount of waste after 10 years of functioning measures saving the planet ecology.
Answer:

$$y_P(10) = -\frac{1}{2}e^{-5} + \frac{3}{2}e^{5} \doteq 222{,}62\; units$$



Task 5:
Estimate, what would be the necessary measures in order not to increase the amount of accumulated waste on the planet.
Answer:
Function $y_P(t)$ is increasing for $t \in \langle 0, T\rangle, T \in R$, as its derivative is positive function for $t \geq 0$.

$$y_P{}' = \frac{1}{4}e^{-\frac{1}{2}t} + \frac{3}{4}e^{+\frac{1}{2}t}, \;\; t \in \langle 0, T\rangle, T \in R$$



None of the chosen values of constants $k, l \in R$ representing adopted measures that determine the growth speed and acceleration

$$y'(t) = k.y(t), y''(t) = l.y(t)$$

$$y''(t) = k.y'(t) = k.l.y(t)$$

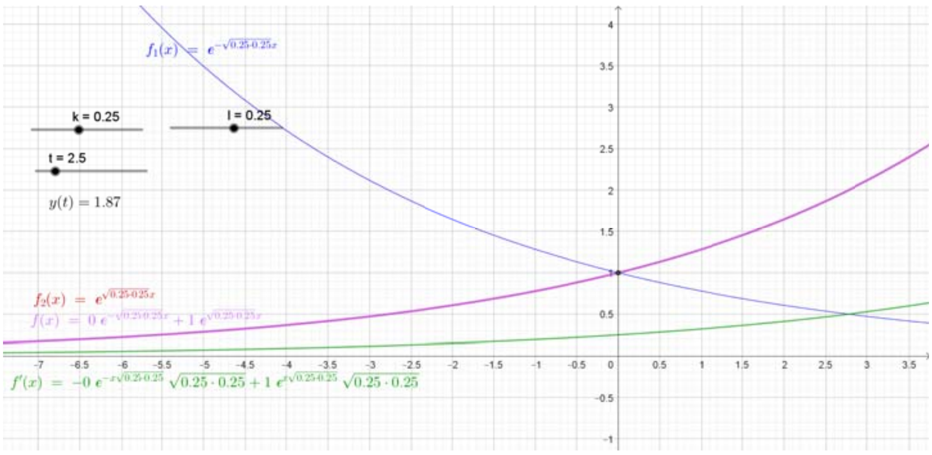
leading to the differential equation

$$y'' - k - l - y = 0$$

could result in the waste accumulation decrease.

The amount of waste could "stop to be increasing" only in case that the industrial production will be decreased, while all accumulated waste will be rapidly and immediately recycled.

Task 6:
Sketch several integral curves of the general solution of ODR II and investigate their forms determined by different values of the included constants c_1, c_2 representing the ecological restrictions.
Answer:

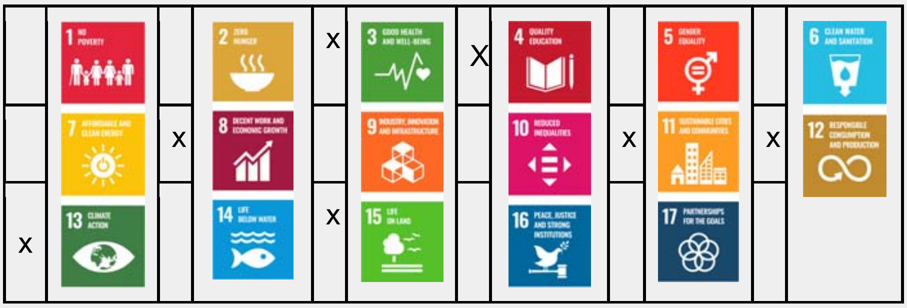


Task 7:
Comment on the obtained results from a sustainable point of view. Investigate how the values of the constants c_1, c_2 influence the speed of the waste decrease.
Answer:
Animation can be obtained easily in the program GeoGebra, with sliders determining the values of constants *k* and *c*.

Outcomes expected	- Graphics fitting the solution; - Numerical results explained and put in context; - Capture of ICT tools solutions used; - Sequence of steps followed; - Remark computations done by hand and done by ICT tools; - Provide complete answer to questions; - All the results must be presented in the context of the problem;
Guide for Learning	At the beginning of the course, the students need guides on new activities, and feel your support on a well-structured pack of suggestions on how to address the problems posted. Namely: - Read carefully the problem statement and the tasks posted. Always maintain a global view of all the projects. - Identify, or try to do a first draft match, the content units of your lecture notes involved in every task. - Take your lecture notes open and review before starting to solve the problems. - Match output expected with the tasks posted, at least as first draft approach. - Follow the order of the tasks, try to increase the knowledge of the problem while you are solving the activities. - Always think that maybe there are different ways to solve a problem. - Use ICT tools to avoid hard computations and check your solutions are correct in different ways if possible. - The solutions are always part of a context, expressing such a final solution totally integrated in the problem posted. - Be sure you answer the complete questions. - Always try to solve the questions by yourself. - If the project can be done in groups, discuss with the groups the proposed problem, to confirm and detect fails or weaknesses, confront strategies, discuss presentation format, etc. Working in groups doesn't mean work less but work better.
Guide for Teaching	Some hints needed to present and launch the mini-PBL to students - Do a small Introduction concerning accumulation of waste and its impact to the Climate Change crisis we currently live in. - Do a small introduction about the relations between production and accumulation of waste and its effective recycling, with the basic equations. - Students will form groups of 4 students and solve the mini-PBL using the eduScrum methodology. - The students should do each exercise in a sequential order, starting from Task 1. - The students should be able to thoroughly read and interpret the numerical results from a mathematical and the real-life example point of view. They should include also a discussion of the climate change crisis and enumerate some strategies they could apply at home or even at university to reduce waste production and its accumulation applying effective ecological measures, as recycling, etc. - They should also mention how this mini-PBL helps them identify the Sustainable Development Goals, 3, 4, 8, 11, 12, 13 and 15.

Assessment	- Final report; - Oral presentation; - Peer-assessment: students will apply peer-assessment for their periodic performance using online peer assessment tools used and available at the respective institution.
Others: References	Active Learning Calculus I (colorado.edu) https://eduscrum.org/about-us-and-how-we-try-to-make-it-happen/ More refs on active-learning tools: https://scholar.google.com/citations?hl=en&user=Aw39XwEAAAAJ&view_op=list_works&sortby=pubdate https://www.youtube.com/watch?v=mQ_mbDAB1us (there are more examples online)

Learning Guide for Students

Mini-PBL project	
Student data sheet: Learning Guide	
Title	The waste reduction
SDG attended	Using this UN graphic, we mark such SDG which this project works. 
Content units	Ordinary differential equations of order II with constant coefficients
Sessions	1 sessions of 1h
Hours of autonomous work	1h
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Tasks and problems	The accumulation of waste is increasing with the speed equal to twice its actual quantity. After adopting strong ecological measures, the speed of the accumulation was decreased to be equal to the actual waste quantity, while acceleration of this growth was decreased to one quarter of the former speed. Consider the amount of the waste to be 1 unit at the time, when the measures were imposed, t = 0. The process of waste accumulation can be represented by ordinary differential equation, of order 1 before measures, and order II after the regulations. Already after 2 years there can be visible remarkable improvements and considerable decrease in the accumulated waste.

	Task 1: Assemble both differential equations describing the accumulation of waste before measures and after their imposition. Answer: **Task 2:** Find general solutions of both differential equations and their particular solutions determined by Cauchy initial conditions. Answer: **Task 3:** Calculate the amount of accumulated waste after 2 years under both circumstances, compare these values, and estimate the impact of the ecological measures. Answer: **Task 4:** Calculate the amount of waste after 10 years of functioning measures saving the planet ecology. Answer: **Task 5**: Estimate, what would be the necessary measures in order not to increase the amount of accumulated waste on the planet. Answer: **Task 6:** Sketch several integral curves of the general solution of ODR II and investigate their forms determined by different values of the included constants c_1, c_2 representing the ecological restrictions. Answer: **Task 7:** Comment on the obtained results from a sustainable point of view. Investigate how the values of the constants c_1, c_2 influence the speed of the waste decrease. Answer:
Outcomes expected	- Graphics fitting the solution; - Numerical results explained and put in context; - Capture of ICT tools solutions used; - Sequence of steps followed; - Remark computations done by hand and done by ICT tools; - Provide complete answer to questions; - All the results must be presented in the context of the problem;
Guide for learning	- Read carefully the problem statement and the tasks posted. Always maintain a global view of all the projects. - Identify, or try to do a first draft match, the content units of your lecture notes involved in every task. - Take your lecture notes open and review before starting to solve the problems. - Match output expected with the tasks posted, at least as first

	draft approach. - Follow the order of the tasks, try to increase the knowledge of the problem while you are solving the activities. - Always think that maybe there are different ways to solve a problem. - Use ICT tools to avoid hard computations and check your solutions are correct in different ways if possible. - The solutions are always part of a context, expressing such a final solution totally integrated in the problem posted. - Be sure you answer the complete questions. - Always try to solve the questions by yourself. - If the project can be done in groups, discuss with the groups the proposed problem, to confirm and detect fails or weaknesses, confront strategies, discuss presentation format, etc. Working in groups doesn't mean work less but work better.
Assessment	- Final report; - Oral presentation; - Peer-assessment: students will apply peer-assessment for their periodic performance using online peer assessment tools used and available at the respective institution.
Others: References	Active Learning Calculus I (colorado.edu) https://eduscrum.org/about-us-and-how-we-try-to-make-it-happen/ More refs on active-learning tools: https://scholar.google.com/citations?hl=en&user=Aw39XwEAAAAJ&view_op=list_works&sortby=pubdate https://www.youtube.com/watch?v=mQ_mbDAB1us (there are more examples online)

ANNEX 1: RUBRIC

Category	4=Excellent	3=Good	2=Low	1=Poor
Mathematical Concepts	Explanation shows complete understan- ding of the mathe- matical concepts used to solve the problem(s).	Explanation shows substantial understan- ding of the mathema- tical concepts used to solve the problem(s).	Explanation shows some understan- ding of the mathe- matical concepts needed to solve the problem(s).	Explanation shows very limited unders- tanding of the underlying concepts needed to solve the problem(s) OR is not written.
Mathematical Terminology and Notation	Correct terminology and notation are always used, making it easy to understand what was done.	Correct terminology and notation are usually used, making it fairly easy to understand what was done.	Correct terminolo- gy and notation are used, but it is sometimes not easy to understand what was done.	There is little use, or a lot of inappropriate use, of terminology and notation.
Strategy/Procedure	Typically, uses an efficient and effective strategy to solve the problem(s).	Typically, uses an effective strategy to solve the problem(s).	Sometimes uses an effective strategy to solve problems, but does not do it consistently.	Rarely uses an effective strategy to solve problems.
Completion	All problems are completed.	All but one of the problems are completed.	All but two of the problems are completed.	Several of the problems are not completed.
Mathematical Errors	90-100% of the steps and solutions have no mathematical errors.	Almost all (85-89%) of the steps and solu- tions have no mathematical errors.	Most (75-84%) of the steps and solu- tions have no mathematical errors.	More than 75% of the steps and solu- tions have mathe- matical errors.

Sources Checking				
Working with Others	Student was an engaged partner, listening to suggestions of others and working cooperatively throughout lesson.	Student was an engaged partner but had trouble listening to others and/or working cooperatively.	Student cooperated with others, but needed prompting to stay on-task.	Student did not work effectively with others.
Neatness and Organization	The work is presented in a neat, clear, organized fashion that is easy to read.	The work is presented in a neat and organized fashion that is usually easy to read.	The work is presented in an organized fashion but may be hard to read at times.	The work appears sloppy and unorganized. It is hard to know what information goes together.
Diagrams and Sketches	Diagrams and/or sketches are clear and greatly add to the reader's understanding of the procedure(s).	Diagrams and/or sketches are clear and easy to understand.	The work is presented in an organized fashion but may be hard to read at times.	Diagrams and/or sketches are difficult to understand or are not used.
CT tools used				