Funded by the European Union

Mini-PBL example

Teaching Guide for Teachers

Mini-PBL project	
Teacher data sheet: Teaching Guide	
Title	Water pollution
SDG attended	Using this UN graphics, we mark such SDG which this project works. 
Content units	Ordinary differential equations of order II with constant coefficients
Sessions	1 sessions of 1h
Hours of autonomous work	1h
Competences to be developed	**Reasoning and modelling** ● Develop thinking strategies to solve real life problems ● Explore, analyse, and apply mathematical ideas ● Estimate reasonably and demonstrate fluent, flexible, and strategic thinking about graphs ● Model with mathematics in situational contexts ● Think creatively and with curiosity and wonder when exploring problems **Understanding and solving** ● Develop, demonstrate, and apply conceptual understanding of mathematical ideas through story, inquiry, and problem solving ● Visualize to explore and illustrate mathematical concepts and relationships ● Apply flexible and strategic approaches to solve problems ● Solve problems with persistence and a positive disposition ● Engage in problem-solving experiences connected with real-life

	examples. **Communicating and representing** ● Explain and justify mathematical ideas and decisions in many ways ● Represent mathematical ideas in concrete, pictorial, and symbolic forms ● Use mathematical vocabulary and language to contribute to discussions in the classroom ● Take risks when offering ideas in classroom discourse **Connecting and reflecting** ● Reflect on mathematical thinking ● Connect mathematical concepts with each other, other areas, and personal interests ● Use mistakes as opportunities to advance learning ● Incorporate First Peoples worldviews, perspectives, knowledge, and practices to make connections with mathematical concepts
ICT tools to be used	Available Computer Algebra Systems: Mathematica, Maple, Matlab, GeoGebra, etc.
Context: project statement	The Buriganga River is a river in Bangladesh, which flows past the southwest outskirts of the capital city Dhaka. Its average depth is 7.6 metres and its length is only 18 kilometres, but it is economically very important to Dhaka. Launches and country boats provide connection to other parts of Bangladesh, a largely riverine country. The river is also the source of drinking water. Buriganga is afflicted by the noisome problem of pollution, and it ranks among the most polluted rivers in the world. More than 60,000 cubic metres of toxicchemical waste from mills and textile factories, household waste, medical waste, sewage, dead animals, plastics, and oil are some of the river most dangerous pollutants.  Source: SITA/AP, Buriganga River - Wikipedia https://en.wikipedia.org › *wiki › Buriganga_River* Overall pollution of planet water resources is dangerously increasing

	every year with the increasing speed and acceleration of this process, which can be mathematically modelled as differential equation of order II with constant coefficients. Scientists are urgently pointing to this problem, which can lead to the global lack of clean drinking water resources and endanger the life on the planet. Striking evidence of the rapidly worsening situation can be provided by the mathematical models predicting the unavoidable consequences provided no actions will be taken to completely stop or at least to slow this development.
Tasks and problems	The percentage of the polluted world ocean area is rapidly increasing every year. The latest findings of Canadian oceanographic scientists proved that in the ocean water one can find a higher portion of micro-plastics (six times more!) than is the amount of live plankton. Consider the percentage of the planet polluted water resources to be 10% at the time *t* = 0. What would be the situation in the following years, under the current circumstances, when the process of water pollutionis accelerated by one fourth of the pollution speed? Adopting strong ecological measures resulting in reduction of the water pollution process by half, quite visible and remarkable improvements can be achieved in few coming years. **Task 1:** Assemble differential equation describing the process of increasing percentage of the planet polluted water resources before adaptation of ecological measures and after their imposition. Answer: Let $y(t) > 0$ be the percentage of polluted planet water resources at the time *t*, while acceleration of the pollution speed be the second derivative $y''(t)$ of this percentage (before the impact of the ecological measures) and this equals to one fourth of the pollution speed. Such pollution spreading process can be well represented mathematically by the differential equation od order II $$y''(t) = \frac{1}{4} y'(t)$$ that can be rewritten as the homogeneous differential equation of order IIwith constant coefficients $$y''(t) - \frac{1}{4} y'(t) = 0$$ After impact of the ecological measures, the acceleration of the pollution process will be reduced to one half, $\frac{y''(t)}{2}$, which would be represented by differential equation $$y''(t) - \frac{1}{8} y'(t) = 0$$ **Task 2:** Find general solutions of both differential equations and their particular solutions determined by Cauchy initial conditions. Answer: General solution of the first respective ODR II can be determined from

the characteristic equation

$$r^2 - \frac{1}{4}r = 0 \Rightarrow r_1 = 0, r_2 = \frac{1}{4},$$

$$y(t) = c_1 + c_2 e^{\frac{1}{4}t}, \qquad c_1, c_2 \in R, t \in \langle 0, T\rangle, T \in R$$

General solution of the ODR II describing the pollution process after some ecological restrictions were implied can be determined from the characteristic equation

$$r^2 - \frac{1}{8}r = 0 \Rightarrow r_1 = 0, r_2 = \frac{1}{8}$$

in the form

$$y(t) = c_1 + c_2 e^{\frac{1}{8}t}, \qquad c_1, c_2 \in R, t \in \langle 0, T\rangle, T \in R$$

Values of constants c_1, c_2 for the particular solutions can be calculated from the given initial conditions describing the water pollution as Cauchy initial problem:

$$y(0) = 10, \;\; y'(0) = 1$$

$$c_1 = 6, c_2 = 4$$

$$y_P(t) = 6 + 4e^{\frac{1}{4}t}, \qquad t \in \langle 0, T\rangle, T \in R$$

or in the other circumstances

$$y(0) = 10, \;\; y'(0) = 1$$

$$c_1 = 2, c_2 = 8$$

$$y_P(t) = 2 + 8e^{\frac{1}{8}t}, \qquad t \in \langle 0, T\rangle, T \in R$$

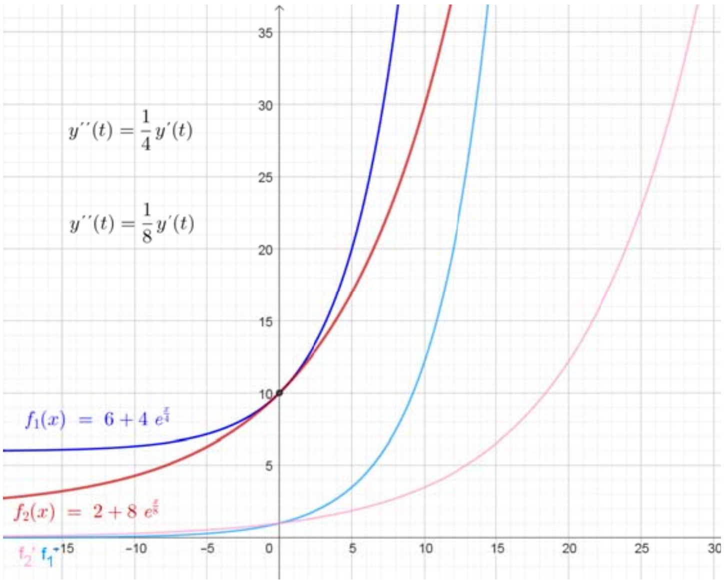
Task 3:
Calculate the amount of polluted planet waters percentage in several next years under both circumstances, compare these values in a table, and estimate the impact of the ecological measures.

Answer:

t	0	2	4	6	8	10	12	**12,5**	14	16	18	**20**
1.	10	12,6	16,9	24	35,5	54,8	86,3	**97**				
2.	10	12,3	15,2	19	23,7	30	37,9	42,6	48	61	78	**99,5**

Achieved impact of the imposed ecological measures is only the reduction of water pollution accumulation in time, while all water resources on the planet will be polluted about 8 years later than in case that no measures will be implied.

Task 4:
Sketch respective integral curves that are particular solutions of ODRs representing the water pollution under both circumstances and compare the pollution growth acceleration during *T* years.
Answer:



Under both circumstancies the higher water pollution is inevoidable, as derivatives of both particular solutions are positive functions.

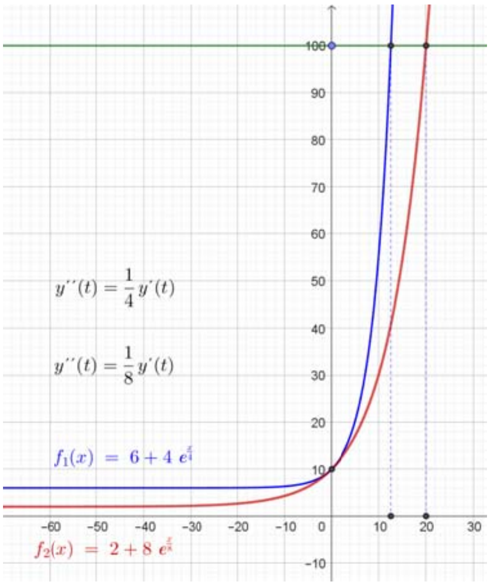
$$y_P´(t) = e^{\frac{1}{4}t}, \qquad t \in \langle 0, T \rangle, T \in R$$

$$y_P´(20) = e^{\frac{1}{8}t}, \ \ t \in \langle 0, T \rangle, T \in R$$

Task 4:
Calculate, how much time is left before 100% of planet water resources are completely polluted in both scenarios, without or with the adoption of strict functioning measures saving the planet clean drinking water.
Answer:

$$y_P´(12{,}5) = 97\%, \qquad t = 12{,}5$$

$$y_P´(20) = 99\%, \ \ t = 20$$



There is left less than 13 years before the total plane water resources are polluted in the first scenario, and about 20 years in the other one.

Task 5:
Find, what would be the necessary measures in decrease of pollution acceleration in order to stop the water pollution in 10 years at 20%.
Answer:

$$y''(t) = \frac{1}{k} y'(t)$$

$$y''(t) - \frac{1}{k} y'(t) = 0$$

$$r^2 - \frac{1}{k} r = 0 \Longrightarrow r_1 = 0, r_2 = \frac{1}{k}$$

$$y_P = c_1 + c_2 e^{\frac{1}{k}t}, \;\; t \in \langle 0, T \rangle, T \in R$$

$$y_P{}' = \frac{1}{k} c_2 e^{\frac{1}{k}t}, \;\; t \in \langle 0, T \rangle, T \in R$$

Initial conditions

$$y(0) = 10, \;\; y'(0) = 1$$

determine the values of constants c_1, c_2

$$c_1 + c_2 = 10, \frac{1}{k} c_2 = 1$$

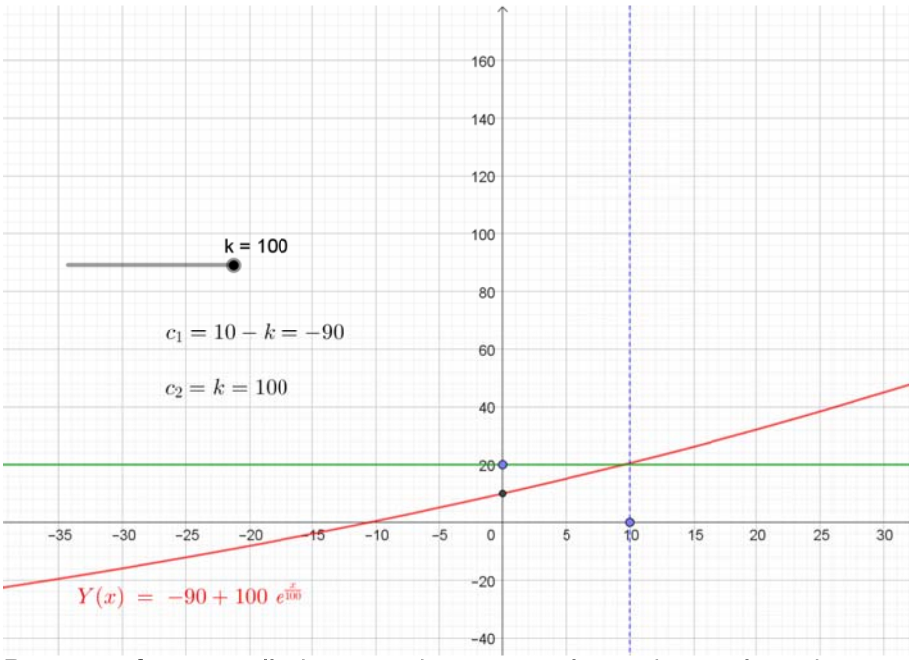
$$c_2 = k$$

$$c_1 = 10 - k$$

$$y_P = 10 - k + k e^{\frac{1}{k}t}, \;\; t \in \langle 0, T \rangle, T \in R$$

$$y(10) = 20 \Longrightarrow 10 - k + k e^{\frac{10}{k}} = 20 \Longrightarrow k \doteq 100$$

$$y_P = -90 + 100 e^{\frac{1}{100}t}, \;\; t \in \langle 0, T \rangle, T \in R$$

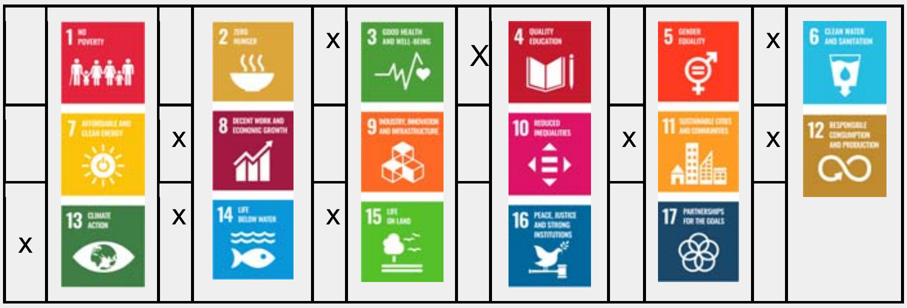


Process of water pollution must be not accelerated, or at least its acceleration must be decreased to one hundred of the pollution speed!

	Task 6: Sketch several integral curves of the general solution of ODR II and investigate their forms determined by different values of the included constants c_1, c_2 representing the ecological restrictions. Answer: Animation can be obtained easily in the program GeoGebra, with sliders determining the values of constants *k* and corresponding c_1, c_2. **Task 7:** Comment on the obtained results from a sustainable point of view. Investigate how the values of the constants c_1, c_2 influence the speed and acceleration of the water pollution. Answer: The amount of polluted water resources could "stop to be increasing" only in case that no toxic materials and waste will be produced, all wastewater would undergo required treatment, the dirty industrial production would be decreased, while all currently polluted water resources would be rapidly and immediately cleaned.
Outcomes expected	- Graphics fitting the solution; - Numerical results explained and put in context; - Capture of ICT tools solutions used; - Sequence of steps followed; - Remark computations done by hand and done by ICT tools; - Provide complete answer to questions; - All the results must be presented in the context of the problem;
Guide for Learning	At the beginning of the course, the students need guides on new activities, and feel your support on a well-structured pack of suggestions on how to address the problems posted. Namely: - Read carefully the problem statement and the tasks posted. Always maintain a global view of all the projects. - Identify, or try to do a first draft match, the content units of your lecture notes involved in every task. - Take your lecture notes open and review before starting to solve the problems. - Match output expected with the tasks posted, at least as first draft approach. - Follow the order of the tasks, try to increase the knowledge of the problem while you are solving the activities. - Always think that maybe there are different ways to solve a problem. - Use ICT tools to avoid hard computations and check your solutions are correct in different ways if possible. - The solutions are always part of a context, expressing such a final solution totally integrated in the problem posted. - Be sure you answer the complete questions. - Always try to solve the questions by yourself. - If the project can be done in groups, discuss with the groups the proposed problem, to confirm and detect fails or weaknesses, confront strategies, discuss presentation format, etc. Working in groups doesn't mean work less but work better.

Guide for Teaching	Some hints needed to present and launch the mini-PBL to students - Do a small Introduction concerning the current water pollution on the planet, and how this is impacting the Climate Change crisis we are currently living in. - Do a small introduction about the relations between increasing water pollution and necessary ecological measures that have to be taken, in terms of the basic equations. - Students will form groups of 4 students and solve the mini-PBL using the eduScrum methodology. - The students should do each exercise in a sequential order, starting from Task 1. - The students should be able to thoroughly read and interpret the numerical results from a mathematical and the real-life example point of view. They should include also a discussion of the climate change crisis and enumerate some strategies they could apply at home or even at university to save resources, namely reduce energy consumption. They should also mention how this mini-PBL helps them identify the Sustainable Development Goals 3, 4, 6, 8 and 11-15.
Assessment	- Final report; - Oral presentation; - Peer-assessment: students will apply peer-assessment for their periodic performance using online peer assessment tools used and available at the respective institution.
Others: References	Active Learning Calculus I (colorado.edu) https://eduscrum.org/about-us-and-how-we-try-to-make-it-happen/ More refs on active-learning tools: https://scholar.google.com/citations?hl=en&user=Aw39XwEAAAAJ&view_op=list_works&sortby=pubdate https://www.youtube.com/watch?v=mQ_mbDAB1us (there are more examples online)

Learning Guide for Students

Mini-PBL project	
Student data sheet: Learning Guide	
Title	The water pollution
SDG attended	Using this UN graphic, we mark such SDG which this project works. 
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Sessions	1 sessions of 1h
Hours of autonomous work	1h
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	 Source: SITA/AP, [Buriganga River - Wikipedia](https://en.wikipedia.org/wiki/Buriganga_River) https://en.wikipedia.org › *wiki › Buriganga_River*
Tasks and problems	The percentage of the polluted world ocean area is rapidly increasing every year. The latest findings of Canadian oceanographic scientists proved that in the ocean water one can find a higher portion of micro-plastics (six times more!) than is the amount of live plankton. Consider the percentage of the planet polluted water resources to be 10% at the time t = 0. What would be the situation in the following years, under the current circumstances, when the process of water pollution is accelerated by one fourth of the pollution speed? Adopting strong ecological measures resulting in reduction of the water pollution process by half, quite visible and remarkable improvements can be achieved in few coming years. **Task 1:** Assemble differential equation describing the process of increasing percentage of the planet polluted water resources before adaptation of ecological measures and after their imposition. Answer: **Task 2:** Find general solutions of both differential equations and their particular solutions determined by Cauchy initial conditions. Answer: **Task 3:** Calculate the amount of polluted planet waters percentage in several next years under both circumstances, compare these values in a table, and estimate the impact of the ecological measures. Answer: **Task 4:** Sketch respective integral curves that are particular solutions of ODRs

	representing the water pollution under both circumstances and compare the pollution growth acceleration during *T* years. Answer: **Task 5**: Find, what would be the necessary measures in decrease of pollution acceleration in order to stop the water pollution in 10 years at 20%. Answer: **Task 6:** Sketch several integral curves of the general solution of ODR II and investigate their forms determined by different values of the included constants c_1, c_2 representing the ecological restrictions. Answer: **Task 7:** Comment on the obtained results from a sustainable point of view. Investigate how the values of the constants c_1, c_2 influence the speed and acceleration of the water pollution. Answer:
Outcomes expected	- Graphics fitting the solution; - Numerical results explained and put in context; - Capture of ICT tools solutions used; - Sequence of steps followed; - Remark computations done by hand and done by ICT tools; - Provide complete answer to questions; - All the results must be presented in the context of the problem;
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ANNEX 1: RUBRIC

Category	4=Excellent	3=Good	2=Low	1=Poor
Mathematical Concepts	Explanation shows complete understan- ding of the mathe- matical concepts used to solve the problem(s).	Explanation shows substantial understan- ding of the mathema- tical concepts used to solve the problem(s).	Explanation shows some understan- ding of the mathe- matical concepts needed to solve the problem(s).	Explanation shows very limited unders- tanding of the underlying concepts needed to solve the problem(s) OR is not written.
Mathematical Terminology and Notation	Correct terminology and notation are always used, making it easy to understand what was done.	Correct terminology and notation are usually used, making it fairly easy to understand what was done.	Correct terminolo- gy and notation are used, but it is sometimes not easy to understand what was done.	There is little use, or a lot of inappropriate use, of terminology and notation.
Strategy/Procedure	Typically, uses an efficient and effective strategy to solve the problem(s).	Typically, uses an effective strategy to solve the problem(s).	Sometimes uses an effective strategy to solve problems, but does not do it consistently.	Rarely uses an effective strategy to solve problems.
Completion	All problems are completed.	All but one of the problems are completed.	All but two of the problems are completed.	Several of the problems are not completed.
Mathematical Errors	90-100% of the steps and solutions have no mathematical errors.	Almost all (85-89%) of the steps and solu- tions have no mathematical errors.	Most (75-84%) of the steps and solu- tions have no mathematical errors.	More than 75% of the steps and solu- tions have mathe- matical errors.

Sources Checking				
Working with Others	Student was an engaged partner, listening to suggestions of others and working cooperatively throughout lesson.	Student was an engaged partner but had trouble listening to others and/or working cooperatively.	Student cooperated with others, but needed prompting to stay on-task.	Student did not work effectively with others.
Neatness and Organization	The work is presented in a neat, clear, organized fashion that is easy to read.	The work is presented in a neat and organized fashion that is usually easy to read.	The work is presented in an organized fashion but may be hard to read at times.	The work appears sloppy and unorganized. It is hard to know what information goes together.
Diagrams and Sketches	Diagrams and/or sketches are clear and greatly add to the reader's understanding of the procedure(s).	Diagrams and/or sketches are clear and easy to understand.	The work is presented in an organized fashion but may be hard to read at times.	Diagrams and/or sketches are difficult to understand or are not used.
CT tools used				