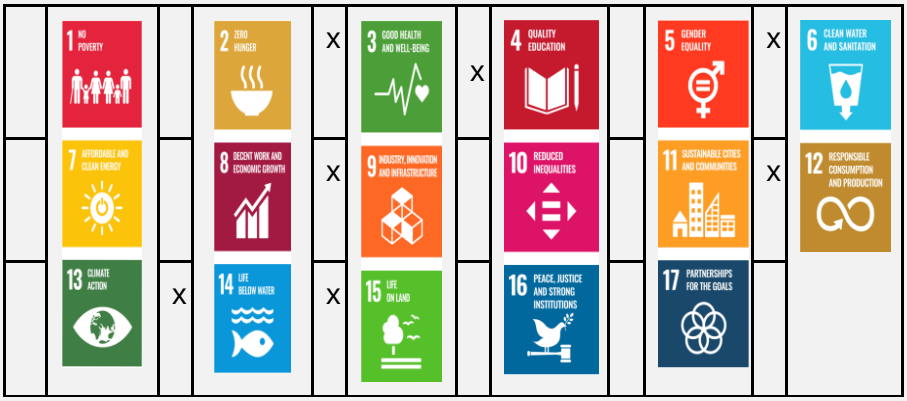
Mini-PBL project	
Teacher data sheet: Teaching Guide	
Title	The quality of water
SDG attended	Using this UN graphics, we mark such SDG which this project works. 
Content units	Tests of hypotheses for a Single Sample
Sessions	2 sessions of 100 min
Hours of autonomous work	30 min
Competences to be developed	**Reasoning and modelling** ● Develop thinking strategies to solve real life problems ● Explore, analyse, and apply mathematical ideas ● Estimate reasonably and demonstrate fluent, flexible, and strategic thinking about graphs ● Model with mathematics in situational contexts ● Think creatively and with curiosity and wonder when exploring problems **Understanding and solving** ● Develop, demonstrate, and apply conceptual understanding of mathematical ideas through story, inquiry, and problem solving ● Visualize to explore and illustrate mathematical concepts and relationships ● Apply flexible and strategic approaches to solve problems ● Solve problems with persistence and a positive disposition ● Engage in problem-solving experiences connected with real-life examples. **Communicating and representing** ● Explain and justify mathematical ideas and decisions in many ways ● Represent mathematical ideas in concrete, pictorial, and

	symbolic forms ● Use mathematical vocabulary and language to contribute to discussions in the classroom ● Take risks when offering ideas in classroom discourse **Connecting and reflecting** ● Reflect on mathematical thinking ● Connect mathematical concepts with each other, other areas, and personal interests ● Use mistakes as opportunities to advance learning ● Incorporate First Peoples worldviews, perspectives, knowledge, and practices to make connections with mathematical concepts
ICT tools to be used	Available Computer Algebra Systems: Statgraphics,, MiniTab, Mathematica, etc.
Context: project statement	The production of waste is increasing with the speed directly proportional to its quantity, due to increasing industrial production and poor environmental measures. Increasing waste accumulation can be detected also on the pollution of the planet water resources. Various methods and parameters might be adopted in order to measure the extent of the water pollution. On the basis of the statistical analysis of the measurement data, appropriate ecological measures might be adopted, in order to considerably improve the planet pollution and have beneficial effect on the climate changes.
Tasks and problems	**TASK** An important quality characteristic of water is the concentration of undissolved particle solids. Following are 60 measurements on undissolved particle solids from a certain lake. **Task 1.** Test the hypotheses $H_0: \mu = 55$ versus, $H_1: \mu \neq 55$ at the level of significance $\alpha = 0{,}05$. Express hypotheses in verbal notation. Test the hypothesis also using: a) the confidence interval

i	x_i	*i*	x_i	*i*	x_i	*i*	x_i	*i*	x_i	*i*	x_i
1.	42,4	**11.**	29,8	**21.**	52,1	**31.**	57,0	**41.**	67,3	**51.**	54,3
2.	73,1	**12.**	59,9	**22.**	62,2	**32.**	66,9	**42.**	56,3	**52.**	57,4
3.	80,1	**13.**	42,8	**23.**	59,6	**33.**	61,4	**43.**	64,2	**53.**	72,5
4.	53,1	**14.**	67,2	**24.**	42,6	**34.**	54,7	**44.**	77,3	**54.**	76,4
5.	51,1	**15.**	61,4	**25.**	77,3	**35.**	89,8	**45.**	52,0	**55.**	66,1
6.	65,7	**16.**	58,7	**26.**	55,8	**36.**	68,7	**46.**	67,3	**56.**	54,0
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10.	73,8	**20.**	73,1	**30.**	48,5	**40.**	50,7	**50.**	59,6	**60.**	31,6

b) *P–value*
c) graph. (Sketcht the critical value, α and *P–value* on the graph.)

Instructions. Find necessary data via statistical software.

Solution:

Note. The statistical software *Statgraphics* was used to test the articulated hypotheses.

X – the concentration of undissolved particle solids

The one random sample is summarized as follows:

$X \sim N(\mu;\sigma^2)$, n = 60 , $\bar{x} = 59{,}867$, $s^2 = 12{,}498^2$

TEST PROCEDURE:

Hypothesis test on μ , σ^2 – unknown; two–sided ***t* – test** at the level of significance $\alpha = 0{,}05$.

$H_0: \mu = 55$	$\rightarrow$	$H_1: \mu \neq 55$
$(H_0: \mu = \mu_0)$		$(H_1: \mu \neq \mu_0)$
The mean of the concentration of undissolved particle solids is 55.		*The mean of the concentration of undissolved particle solids is not 55.*

The test statistic:

$$t_0 = \frac{\bar{x} - \mu_0}{s}\sqrt{n} = \frac{59{,}867 - 55}{12{,}498}\sqrt{60} = 3{,}01645$$

Reject H_0 at $\alpha = 0{,}05$, when

$|t_0| > t_{kr}$

$|3{,}01645| > 2$

$3{,}01645 > 2 \quad \Rightarrow true$

the critical value:

$t_{kr} = t_{kr}(n-1;\ \alpha) = t_{kr}(59;\ 0{,}05) = 2$

Conclusion: ***Reject*** $H_0: \mu = 55$ *at the level of significance* $\alpha = 0{,}05$. : *Stated more completely we concluded at the level* $\alpha = 0{,}05$. *that the mean of the concentration of undissolved particle solids is not 55.*

a) Testing hypotheses using the confidence interval

Reject H_0at $\alpha = 0{,}05$., when

$$(\mu_0 = 55) \notin \left[\bar{x} - t(n-1,\alpha)\frac{s}{\sqrt{n}};\ \bar{x} + t(n-1,\alpha)\frac{s}{\sqrt{n}}\right]$$

$$(\mu_0 = 55) \notin [59{,}867 - 3{,}22858\ ;\ 59{,}867 + 3{,}22858]$$

$$(\mu_0 = 55) \notin [\ 56{,}6384\ ;\ 63{,}0956]$$

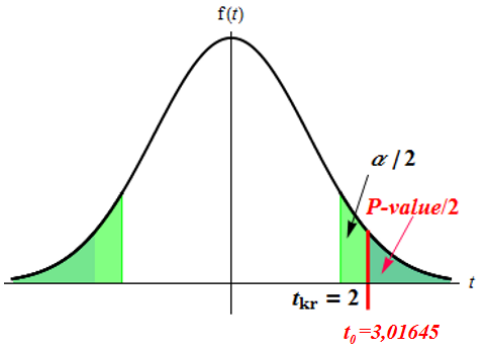
– true**, *reject*** H_0

b) Testing hypotheses using *P*-value

H_0 reject at $\alpha = 0{,}05.$, when P-value $\leq \alpha$

$0{,}00376995 \leq 0{,}05$ – true, ***reject*** H_0

c) Testing hypotheses using graph



Task 2.

Calculate the P-value of the test statistic from Task 1.

Solution:

$$P\text{–}value = 2(1–F(t_0)) = 2(1 – 0{,}9981) = 2.0{,}0019 = 0{,}0038$$

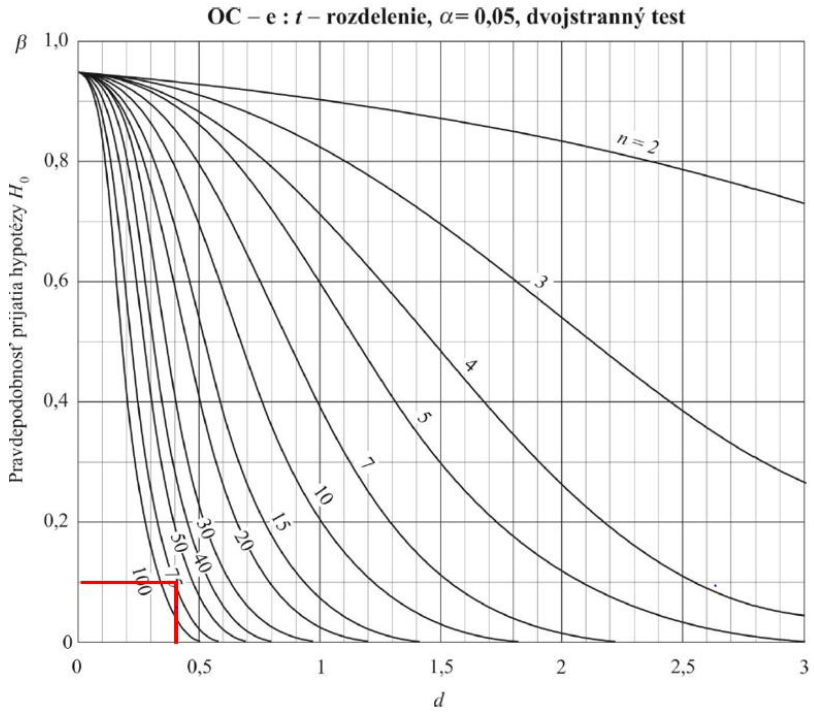
Task 3.

What sample size would be required to detect a true mean of the concentration of undissolved particle solids as low as 50 if we wanted the power of the test to be at least 0,9?

Solution:

The sample size n will be estimated from OC–e (Operating characteristic curve) for two-sided t – test at the level of significance $\alpha = 0{,}05$ in Task 1.

$$\left.\begin{array}{l}\mu = 50\\ \mu_0 = 55\end{array}\right\} \Rightarrow \left.\begin{array}{l} d = \dfrac{|\mu - \mu_0|}{\hat{\sigma}} = \dfrac{|\delta|}{s} = \dfrac{5}{12{,}498} \doteq 0{,}4 \\ 1-\beta = 0{,}9 \Rightarrow \beta = 0{,}1 \end{array}\right\} \Rightarrow OC-e:\ n \doteq 70$$



Conclusion: *That is if the true mean of the concentration of undissolved particle solids is 50, there is approximately a 90% chance that this will be detected by the test with n = 70 measurements.*

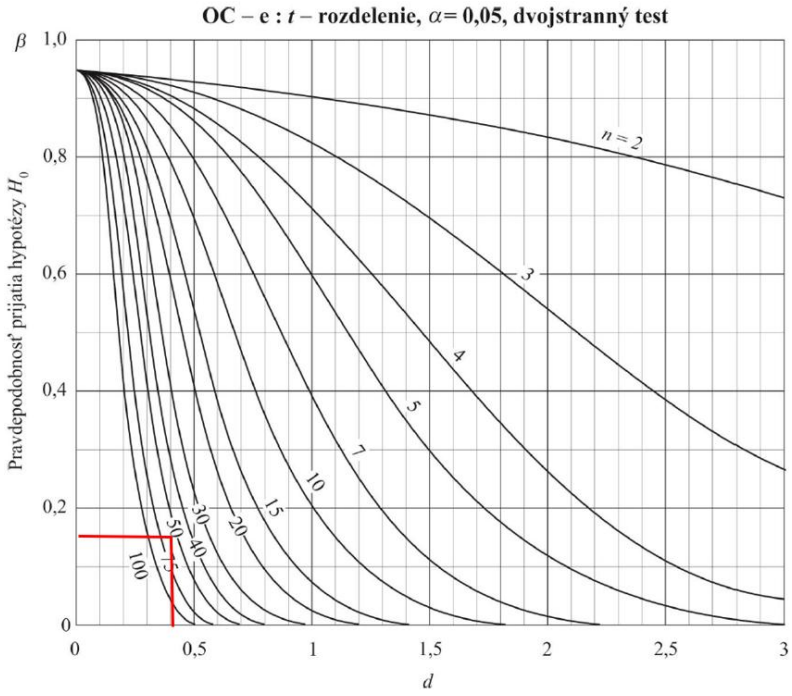
Task 4.

Calculate the power of the test if the true mean of the concentration of undissolved particle solids is as low as 50.

Solution:

The power of the test will be estimated from *OC–e* (Operating characteristic curve) for two-sided *t* – test at the level of significance $\alpha = 0{,}05$ in Task 1.

$$\left.\begin{array}{l}\mu = 50\\ \mu_0 = 55\\ n = 60\end{array}\right\} \Rightarrow d = \frac{|\mu - \mu_0|}{\hat{\sigma}} = \frac{|\delta|}{s} = \frac{5}{12{,}498} \doteq 0{,}4 \quad \Rightarrow OC - e : \beta = 0{,}15 \Rightarrow 1 - \beta = 0{,}85$$

	
Outcomes expected	- Graphics fitting the solution; - Numerical results explained and put in context; - Capture of ICT tools solutions used; - Sequence of steps followed; - Remark computations done by hand and done by ICT tools; - Provide complete answer to questions; - All the results must be presented in the context of the problem;
Guide for Learning	At the beginning of the course, the students need guides on new activities, and feel your support on a well-structured pack of suggestions on how to address the problems posted. Namely: - Read carefully the problem statement and the tasks posted. Always maintain a global view of all the projects. - Identify, or try to do a first draft match, the content units of your lecture notes involved in every task. - Take your lecture notes open and review before starting to solve the problems. - Match output expected with the tasks posted, at least as first draft approach. - Follow the order of the tasks, try to increase the knowledge of the problem while you are solving the activities. - Always think that maybe there are different ways to solve a problem. - Use ICT tools to avoid hard computations and check your solutions are correct in different ways if possible. - The solutions are always part of a context, expressing such a final solution totally integrated in the problem posted. - Be sure you answer the complete questions. - Always try to solve the questions by yourself. - If the project can be done in groups, discuss with the groups the proposed problem, to confirm and detect fails or weaknesses, confront strategies, discuss presentation format, etc.

	- Working in groups doesn’t mean work less but work better.
Guide for Teaching	Some hints needed to present and launch the mini-PBL to students - Do a small Introduction concerning Energy consumption, added to the Climate Change crisis we are currently living in. - Do a small introduction about the relations between extreme industrial production, accumulation of waste and water pollution, leadung to overall planet pollution. - Students will form groups of 4 students and solve the mini-PBL using the eduScrum methodology. - The students should do each exercise in a sequential order, starting from Task 1. - The students should be able to thoroughly read and interpret the numerical results from a mathematical and the real-life example point of view. They should include also a discussion of the climate change crisis and enumerate some strategies they could apply at home or even at university to save resources, namely reduce energy consumption. They should also mention how this mini-PBL helps them identify the indicated Sustainable Development Goals.
Assessment	- Final report; - Oral presentation; - Peer-assessment: students will apply peer-assessment for their periodic performance using online peer assessment tools used and available at the respective institution.
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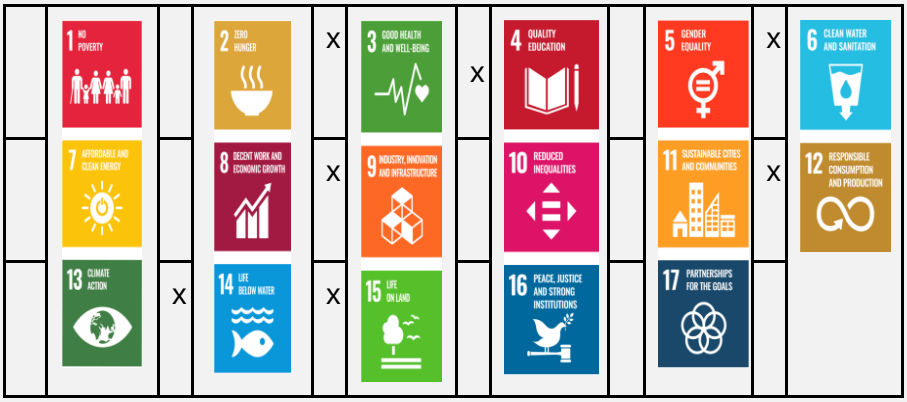
SDG attended	Using this UN graphics, we mark such SDG which this project works. 
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Following are 60 measurements on undissolved particle solids from a certain lake.

<table>
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Express hypotheses in verbal notation.

Test the hypothesis also using:
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