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### Teacher data sheet: Teaching Guide

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<p><b>ICT tools to be used</b></p>	
<p><b>Context: project statement</b></p>	<p>The production of autonomous cars is rapidly increasing. Artificial Intelligence might play a considerably important role in this field. It might be very useful for mankind if it could utilise tested algorithms based on deep scientific psychological research. This has to be carried very carefully, keeping in mind intuitive aspects of human behaviour or genetically inherited abilities to predict consequences of certain circumstances. Thus, various reactions of human beings on visual stimuli have to be tested and statistically analysed. Reaction time of a driver to sudden situation on the road presented as visual stimulus is one of such aspects that might lead to a more elaborated autonomous driving systems saving human lives on roads.</p>
<p><b>Tasks and problems</b></p>	<p><b>TASK</b></p> <p>The reaction time of a driver to visual stimulus is normally distributed with a mean of 0,4 sec (second) and a variance of 0,0025 sec<sup>2</sup>.</p> <p><b>Task 1.</b></p> <p>Write a name of the random variable <math>X</math> and its parameters of normal distribution.</p> <p><i>Solution:</i></p> <p><math>X</math> – the reaction time of a driver to visual stimulus</p> $\mu = 0,4 ; \sigma^2 = 0,0025 = 0,05^2 ; \rightarrow X \sim N(0,4; 0,05^2)$ <p><b>Task 2.</b></p> <p>What is the probability that a reaction requires less than 0,5 sec?</p> <p><i>Solution:</i></p> $P(X \leq 0,5) = F(0,5) = P\left(Z \leq \frac{0,5 - 0,4}{0,05}\right) = P(Z < 2) = \Phi(2) = 0,97725 = 97,725 \approx 97,73\%$

### Task 3.

What is the reaction time that is exceeded 90% of the time?  
Present the solution on graphs  $f(x)$ ,  $F(x)$ ,  $\phi(z)$ ,  $\Phi(z)$  too.

**Solution:**

$$P(X > x) = 90\% = 0,9$$

$$1 - P(X \leq x) = 0,9$$

$$1 - F(x) = 0,9$$

$$P\left(Z > \frac{x-0,4}{0,05}\right) = 0,9$$

$$1 - P\left(Z \leq \frac{x-0,4}{0,05}\right) = 0,9$$

$$1 - \Phi\left(\frac{x-0,4}{0,05}\right) = 0,9$$

$$\Phi\left(\frac{x-0,4}{0,05}\right) = 0,1$$

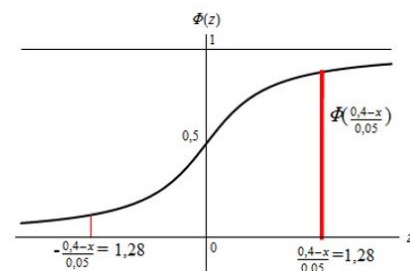
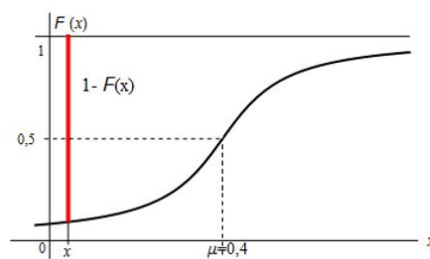
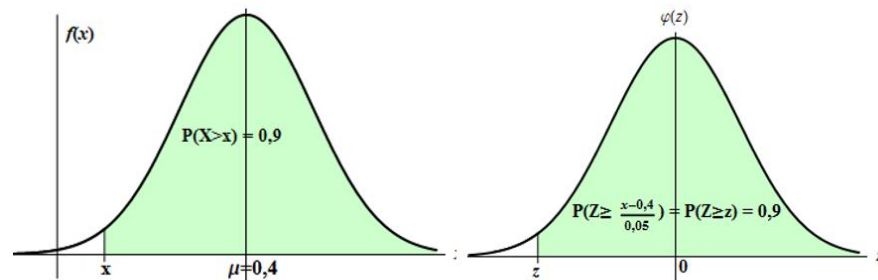
$$\Phi\left(-\frac{0,4-x}{0,05}\right) = 0,1$$

$$1 - \Phi\left(\frac{0,4-x}{0,05}\right) = 0,9$$

$$\Phi\left(\frac{0,4-x}{0,05}\right) = 0,1$$

$$\frac{0,4-x}{0,05} = 1,28 \Rightarrow x = 0,4 - 1,28 \cdot 0,05$$

$$x = 0,336$$



### Task 4.

The reaction time of a driver is required between 0,4 sec and 0,5 sec.  
What is the probability that a reaction time of drivers do not comply this requirement?

**Solution:**

$$\begin{aligned}
1 - P(0,4 < X < 0,5) &= 1 - (F(0,5) - F(0,4)) = \\
&= 1 - P\left(\frac{0,4 - 0,4}{0,05} Z < \frac{0,5 - 0,4}{0,05}\right) = 1 - P(0 < Z < 2) = \\
&= 1 - (\Phi(2) - \Phi(0)) = \\
&= 1 - (0,97725 - 0,5) = 1 - 0,47725 = \\
&= 0,52275 = 52,28\%
\end{aligned}$$

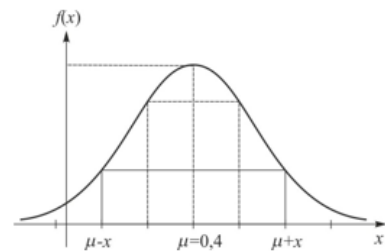
### Task 5.

4500 reaction times of drivers from 5000 must correspond the norm. Determine interval whom bounds are symmetrical around the mean and it contains these the reaction times of drivers.

Present the solution on the graph  $f(x)$  too.

*Solution:*

$$\begin{aligned}
P(0,4 - x \leq X \leq 0,4 + x) &= \frac{4500}{5000} \\
P(0,4 - x \leq X \leq 0,4 + x) &= 0,9 \\
P\left(\frac{(0,4 - x) - 0,4}{0,05} \leq Z \leq \frac{(0,4 + x) - 0,4}{0,05}\right) &= 0,9 \\
P\left(\frac{-x}{0,05} \leq Z \leq \frac{x}{0,05}\right) &= 0,9 \\
\Phi\left(\frac{x}{0,05}\right) - \Phi\left(\frac{-x}{0,05}\right) &= 0,9 \\
\Phi\left(\frac{x}{0,05}\right) - \left(1 - \Phi\left(\frac{x}{0,05}\right)\right) &= 0,9 \\
2\Phi\left(\frac{x}{0,05}\right) &= 1,9 \\
\Phi\left(\frac{x}{0,05}\right) &= 0,95
\end{aligned}$$



$$\begin{aligned}
\frac{x}{0,05} &= 1,65 \\
x &= 1,65 \cdot 0,05 \\
x &= 0,0825
\end{aligned}$$

$$[0,4 - x; 0,4 + x] = [0,3175; 0,4825]$$

### Outcomes expected














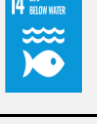
















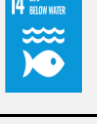
















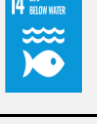



- Graphics fitting the solution;
- Numerical results explained and put in context;
- Capture of ICT tools solutions used;
- Sequence of steps followed;
- Remark computations done by hand and done by ICT tools;
- Provide complete answer to questions;
- All the results must be presented in the context of the problem;

### Guide for Learning

- At the beginning of the course, the students need guides on new activities, and feel your support on a well-structured pack of suggestions on how to address the problems posted. Namely:
- Read carefully the problem statement and the tasks posted. Always maintain a global view of all the projects.
  - Identify, or try to do a first draft match, the content units of your lecture notes involved in every task.
  - Take your lecture notes open and review before starting to solve the problems.
  - Match output expected with the tasks posted, at least as first draft approach.
  - Follow the order of the tasks, try to increase the knowledge of the problem while you are solving the activities.

	<ul style="list-style-type: none"> <li>- Always think that maybe there are different ways to solve a problem.</li> <li>- Use ICT tools to avoid hard computations and check your solutions are correct in different ways if possible.</li> <li>- The solutions are always part of a context, expressing such a final solution totally integrated in the problem posted.</li> <li>- Be sure you answer the complete questions.</li> <li>- Always try to solve the questions by yourself.</li> <li>- If the project can be done in groups, discuss with the groups the proposed problem, to confirm and detect fails or weaknesses, confront strategies, discuss presentation format, etc.</li> <li>- Working in groups doesn't mean work less but work better.</li> </ul>
<b>Guide for Teaching</b>	<p>Some hints needed to present and launch the mini-PBL to students</p> <ul style="list-style-type: none"> <li>- Do a small Introduction concerning Energy consumption, added to the Climate Change crisis we are currently living in.</li> <li>- Do a small introduction about the relations between power and energy, with the basic equations.</li> <li>- Students will form groups of 4 students and solve the mini-PBL using the eduScrum methodology.</li> <li>- The students should do each exercise in a sequential order, starting from Task 1.</li> <li>- The students should be able to thoroughly read and interpret the numerical results from a mathematical and the real-life example point of view. They should include also a discussion of the climate change crisis and enumerate some strategies they could apply at home or even at university to save resources, namely reduce energy consumption. They should also mention how this mini-PBL helps them identify the indicated Sustainable Development Goals.</li> </ul>
<b>Assessment</b>	<ul style="list-style-type: none"> <li>- Final report;</li> <li>- Oral presentation;</li> <li>- Peer-assessment: students will apply peer-assessment for their periodic performance using online peer assessment tools used and available at the respective institution.</li> </ul>
<b>Others: References</b>	<p>Janiga, I., Gabková, J. Základy štatistickej analýzy. Zbierka úloh. Vydavateľstvo STU, Bratislava, 2016.</p>

## Learning Guide for Students

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