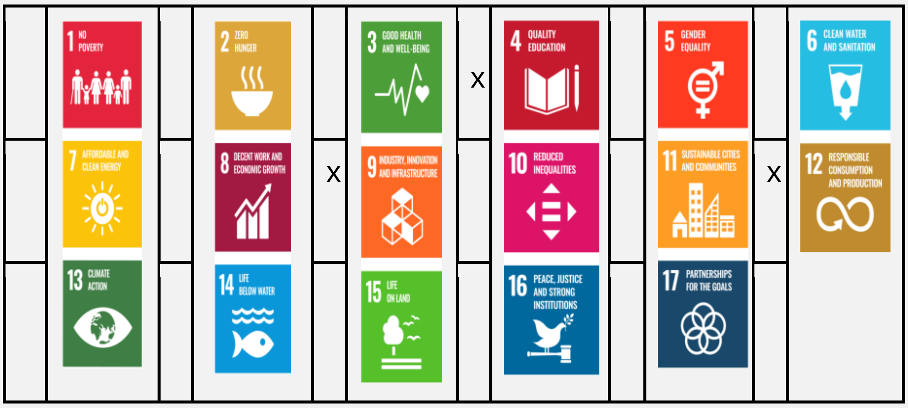
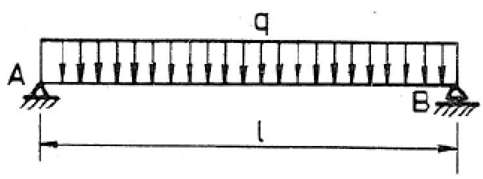
2021-1-RO01-KA220-HED-000032258
Funded by the European Union

Mini-PBL example

Teaching Guide for Teachers

Mini-PBL project	
Teacher data sheet: Teaching Guide	
Title	A bending momentum and a translation force of the support
SDG attended	Using this UN graphics, we mark such SDG which this project works. 
Content units	Functions of one real variable
Sessions	10 sessions of 100 min
Hours of autonomous work	20 min
Competences to be developed	**Reasoning and modelling** ● Develop thinking strategies to solve real life problems ● Explore, analyse, and apply mathematical ideas ● Estimate reasonably and demonstrate fluent, flexible, and strategic thinking about graphs ● Model with mathematics in situational contexts ● Think creatively and with curiosity and wonder when exploring problems **Understanding and solving** ● Develop, demonstrate, and apply conceptual understanding of mathematical ideas through story, inquiry, and problem solving ● Visualize to explore and illustrate mathematical concepts and relationships ● Apply flexible and strategic approaches to solve problems ● Solve problems with persistence and a positive disposition

	• Engage in problem-solving experiences connected with real-life examples. **Communicating and representing** • Explain and justify mathematical ideas and decisions in many ways • Represent mathematical ideas in concrete, pictorial, and symbolic forms • Use mathematical vocabulary and language to contribute to discussions in the classroom • Take risks when offering ideas in classroom discourse **Connecting and reflecting** • Reflect on mathematical thinking • Connect mathematical concepts with each other, other areas, and personal interests • Use mistakes as opportunities to advance learning • Incorporate First Peoples worldviews, perspectives, knowledge, and practices to make connections with mathematical concepts
ICT tools to be used	
Context: project statement	Beams are used to span large distances without using the necessary support. Steel beams are used in the field of steel structures, mechanical engineering, in ground and underground constructions (exceptional works). They contribute to protection against noise. Even the builders of many monumental buildings (New York skyscrapers or the Eiffel Tower) relied on steel beams. These beams are popular products in the construction industry thanks to the possibilities of combining them with wood, glass and concrete. Beams in construction are part of the roof, ceiling and perimeter structure. For example ceramic-concrete ceiling beams maintain a healthy microclimate in the rooms. They are suitable structural elements for residential and civil construction building in terms of fire resistance, thermal insulation and acoustic parameters.
Tasks and problems	**TASK** The support of the constant length, with the same strength has a rectangular section of constant width (Fig. 1).  Fig.1 **Task 1.** When the removed constraints will be replaced by reactions R_A and R_B (Fig. 2) after releasing of support, then the bending momentum is

$$M(x) = \frac{ql}{2}x - \frac{q}{2}x^2 \,, x \in [0, l]$$

where *q* – is a relative load of the legth unit (constant).

Sketch a graph of the momentum *M*(*x*) in the section *x*.

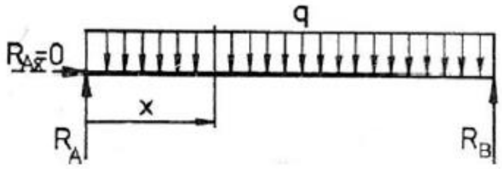


Fig.2

Solution:

Adjustment of the function:

$$M(x) = \frac{ql}{2}x - \frac{q}{2}x^2 = -\frac{q}{2}(x^2 - lx) = -\frac{q}{2}\left(x - \frac{l}{2}\right)^2 + \frac{ql^2}{8}$$

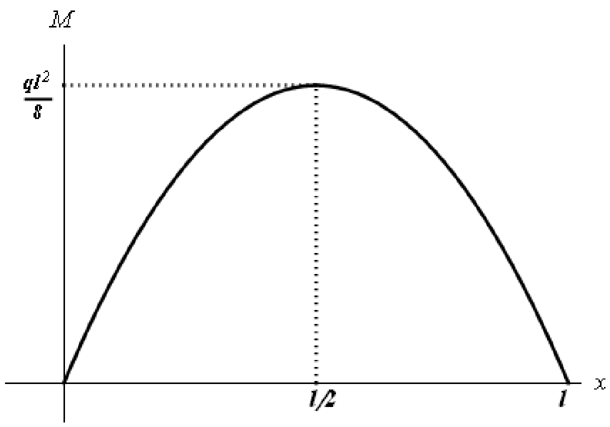
– the quadratic function, its graph is the parabola

– the parabola is turned around y⁻ – axis, vertex is

the point $V = \left[\frac{l}{2}\,; \frac{ql^2}{8}\right]$

$M(x) = \frac{ql}{2}x - \frac{q}{2}x^2 = x\left(\frac{ql}{2} - \frac{q}{2}x\right) \xrightarrow{M(x)=0} [0; 0]\,, \left[\frac{l}{2}; l\right]$,

$\left\{x = 0\,; x = \frac{l}{2}\right\} \in [0; l]$



Task 2.

When the removed constraints will be replaced by reactions R_A and R_B (Fig. 2) after releasing of support, then the translation force is,

$$T(x) = \frac{ql}{2} - qx \,, x \in [0, l]$$

where *q* – is a relative load of the legth unit (constant).

<table>
<tr>
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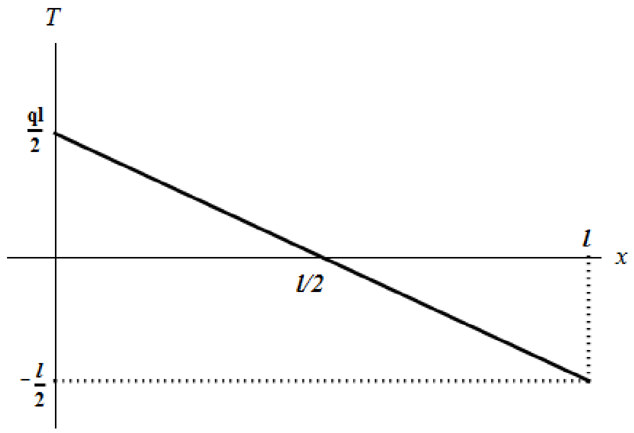
<td>
Sketch a graph of the translation force T(x).

Solution:

Modification of the function: $T(x) = \frac{ql}{2} - qx = -qx + \frac{ql}{2}$

– the linear function, its graph is a descending straight line with the slope $k = -q$

– the points for $x \in [0\,;l]$: $\left[0\,;\frac{ql}{2}\right]$, $\left[\frac{l}{2};0\right]$, $\left[l;-\frac{l}{2}\right]$



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<tr>
<td>Outcomes expected</td>
<td>
- Graphics fitting the solution;
- Numerical results explained and put in context;
- Capture of ICT tools solutions used;
- Sequence of steps followed;
- Remark computations done by hand and done by ICT tools;
- Provide complete answer to questions;
- All the results must be presented in the context of the problem;
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<td>Guide for Learning</td>
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At the beginning of the course, the students need guides on new activities, and feel your support on a well-structured pack of suggestions on how to address the problems posted. Namely:
- Read carefully the problem statement and the tasks posted. Always maintain a global view of all the projects.
- Identify, or try to do a first draft match, the content units of your lecture notes involved in every task.
- Take your lecture notes open and review before starting to solve the problems.
- Match output expected with the tasks posted, at least as first draft approach.
- Follow the order of the tasks, try to increase the knowledge of the problem while you are solving the activities.
- Always think that maybe there are different ways to solve a problem.
- Use ICT tools to avoid hard computations and check your solutions are correct in different ways if possible.
- The solutions are always part of a context, expressing such a final solution totally integrated in the problem posted.
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	- Be sure you answer the complete questions. - Always try to solve the questions by yourself. - If the project can be done in groups, discuss with the groups the proposed problem, to confirm and detect fails or weaknesses, confront strategies, discuss presentation format, etc. Working in groups doesn’t mean work less but work better.
Guide for Teaching	Some hints needed to present and launch the mini-PBL to students - Do a small Introduction concerning Energy consumption, added to the Climate Change crisis we are currently living in. - Do a small introduction about the relations between power and energy, with the basic equations. - Students will form groups of 4 students and solve the mini-PBL using the eduScrum methodology. - The students should do each exercise in a sequential order, starting from Task 1. - The students should be able to thoroughly read and interpret the numerical results from a mathematical and the real-life example point of view. They should include also a discussion of the climate change crisis and enumerate some strategies they could apply at home or even at university to save resources, namely reduce energy consumption. They should also mention how this mini-PBL helps them identify the Sustainable Development Goals 4 And 7.
Assessment	- Final report; - Oral presentation; - Peer-assessment: students will apply peer-assessment for their periodic performance using online peer assessment tools used and available at the respective institution.
Others: References	Syč-Milý, J. Pružnosť a pevnosť. Vydavateľstvo ALFA, Bratislava, 1988.

Learning Guide for Students

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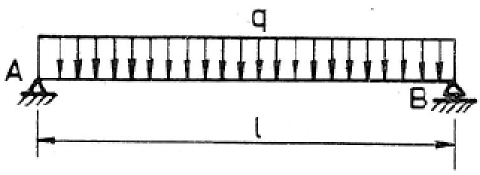
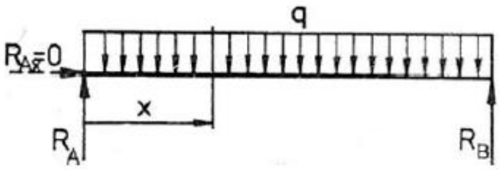
Tasks and problems	**TASK** The support of the constant length, with the same strength has a rectangular section of constant width (Fig. 1).  Fig.1 **Task 1.** When the removed constraints will be replaced by reactions R_A and R_B (Fig. 2) after releasing of support, then the bending momentum is $$M(x) = \frac{ql}{2}x - \frac{q}{2}x^2 , x \in [0, l]$$ where *q* – is a relative load of the legth unit (constant). Sketch a graph of the momentum *M*(*x*) in the section *x*.

	 Fig.2 **Task 2.** When the removed constraints will be replaced by reactions R_A and R_B (Fig. 2) after releasing of support, then the translation force is, $$T(x) = \frac{ql}{2} - qx\,, x \in [0, l]$$ where *q* – is a relative load of the legth unit (constant). Sketch a graph of the translation force *T*(*x*).
Outcomes expected	- Graphics fitting the solution; - Numerical results explained and put in context; - Capture of ICT tools solutions used; - Sequence of steps followed; - Remark computations done by hand and done by ICT tools; - Provide complete answer to questions; - All the results must be presented in the context of the problem;
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ANNEX 1: RUBRIC

Category	4=Excellent	3=Good	2=Low	1=Poor
Mathematical Concepts	Explanation shows complete understan- ding of the mathe- matical concepts used to solve the problem(s).	Explanation shows substantial understan- ding of the mathema- tical concepts used to solve the problem(s).	Explanation shows some understan- ding of the mathe- matical concepts needed to solve the problem(s).	Explanation shows very limited unders- tanding of the underlying concepts needed to solve the problem(s) OR is not written.
Mathematical Terminology and Notation	Correct terminology and notation are always used, making it easy to understand what was done.	Correct terminology and notation are usually used, making it fairly easy to understand what was done.	Correct terminolo- gy and notation are used, but it is sometimes not easy to understand what was done.	There is little use, or a lot of inappropriate use, of terminology and notation.
Strategy/Procedure	Typically, uses an efficient and effective strategy to solve the problem(s).	Typically, uses an effective strategy to solve the problem(s).	Sometimes uses an effective strategy to solve problems, but does not do it consistently.	Rarely uses an effective strategy to solve problems.
Completion	All problems are completed.	All but one of the problems are completed.	All but two of the problems are completed.	Several of the problems are not completed.
Mathematical Errors	90-100% of the steps and solutions have no mathematical errors.	Almost all (85-89%) of the steps and solu- tions have no mathematical errors.	Most (75-84%) of the steps and solu- tions have no mathematical errors.	More than 75% of the steps and solu- tions have mathe- matical errors.

Sources Checking				
Working with Others	Student was an engaged partner, listening to suggestions of others and working cooperatively throughout lesson.	Student was an engaged partner but had trouble listening to others and/or working cooperatively.	Student cooperated with others, but needed prompting to stay on-task.	Student did not work effectively with others.
Neatness and Organization	The work is presented in a neat, clear, organized fashion that is easy to read.	The work is presented in a neat and organized fashion that is usually easy to read.	The work is presented in an organized fashion but may be hard to read at times.	The work appears sloppy and unorganized. It is hard to know what information goes together.
Diagrams and Sketches	Diagrams and/or sketches are clear and greatly add to the reader's understanding of the procedure(s).	Diagrams and/or sketches are clear and easy to understand.	The work is presented in an organized fashion but may be hard to read at times.	Diagrams and/or sketches are difficult to understand or are not used.
CT tools used				