Pythagoras
2021-1-RO01-KA220-HED-000032258

Funded by the European Union

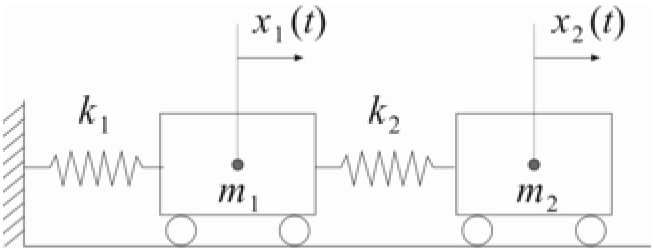
Mini-PBL example

Teaching Guide for Teachers

Mini-PBL project	
Teacher data sheet: Teaching Guide	
Title	Free vibrations
SDG attended	Using this UN graphics, we mark such SDG which this project works. 
Content units	Systems of linear equations
Sessions	2 sessions of 100 min
Hours of autonomous work	1h
Competences to be developed	**Reasoning and modelling** ● Develop thinking strategies to solve real life problems ● Explore, analyse, and apply mathematical ideas ● Estimate reasonably and demonstrate fluent, flexible, and strategic thinking about graphs ● Model with mathematics in situational contexts ● Think creatively and with curiosity and wonder when exploring problems **Understanding and solving** ● Develop, demonstrate, and apply conceptual understanding of mathematical ideas through story, inquiry, and problem solving ● Visualize to explore and illustrate mathematical concepts and relationships ● Apply flexible and strategic approaches to solve problems ● Solve problems with persistence and a positive disposition ● Engage in problem-solving experiences connected with real-life

	examples. **Communicating and representing** ● Explain and justify mathematical ideas and decisions in many ways ● Represent mathematical ideas in concrete, pictorial, and symbolic forms ● Use mathematical vocabulary and language to contribute to discussions in the classroom ● Take risks when offering ideas in classroom discourse **Connecting and reflecting** ● Reflect on mathematical thinking ● Connect mathematical concepts with each other, other areas, and personal interests ● Use mistakes as opportunities to advance learning ● Incorporate First Peoples worldviews, perspectives, knowledge, and practices to make connections with mathematical concepts
ICT tools to be used	
Context: project statement	We can encounter vibrations all around us. Oscillation is sometimes desirable for a person, e.g. music (vibration of musical instruments), sometimes undesirable, e.g. vibration of motor vehicles (uncomfortable ride), swaying of tall buildings due to wind or earthquake. People perceive vibration processes positively - they use their action when moving material, compacting concrete, or feel them as harmful - they reduce the life and reliability of structures and equipment, increase their noise level and worsen the environment. The science of oscillation allows us to penetrate into the essence of oscillation processes - so that we can control them and use them to our advantage.
Tasks and problems	**TASK** Equation of double mass vibration system without damping (2 degrees of freedom, free vibrations) is $M\vec{x}''(t) = -K\vec{x}(t) \Leftrightarrow \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}\begin{pmatrix} x_1''(t) \\ x_2''(t) \end{pmatrix} = -\begin{pmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{pmatrix}\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$ x with initial conditions $\vec{x}(0)=\vec{0}$ $\vec{x}'(0)=\vec{0}$ where *M* – is a matrix of a mass with masses $m_1 = 9\ kg, m_2 = 1\ kg$ *K* – is a matrix of a stiffness with spring stiffnesses $k_1 = 24\ N/m, k_2 = 3\ N/m$ $\vec{x}(t) = \vec{v}e^{i\Omega t}$ – a vector of deviations of carriages (a solution) – Ω – an unknown eigen frequency of a vibration – $\vec{v} \neq \vec{0}$ – a unknown eigenvector (an eigen shape

of a vibration) belonging to Ω.).



Task 1.
After inserting solution in the form $\vec{x}(t) = \vec{v}e^{i\Omega t}$
into equation $M\ddot{\vec{x}} = -K\vec{x}$
solution of the resulting system $K\vec{v} = \Omega^2 M\vec{v}$
will be an eigenvector $\vec{v}$ belonging to Ω.
Algebraically manipulate this system in order to obtain a homogeneous system of equations.

Solution:

$$K\vec{v} = \Omega^2 M\vec{v}$$

$$K\vec{v} - \Omega^2 M\vec{v} = \vec{0}$$

$$(K - \Omega^2 M)\vec{v} = \vec{0}$$

$$\begin{pmatrix} (k_1 + k_2) - \Omega^2 m_1 & -k_2 \\ -k_2 & k_2 - \Omega^2 m_2 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Task 2.

Calculate an eigen frequency Ω of this vibration system.

Solution guide:

First, use substitution $\lambda = \Omega^2$, then calculate all eigenvalues λ of the matrix of the homogeneous system from the requirement for its non-trivial solution ($\vec{v} \neq \vec{0}$).

Solution:

$$(K - \Omega^2 M)\vec{v} = \vec{0} \rightarrow (K - \lambda M)\vec{v} = \vec{0}$$

$$|K - \lambda M| = 0$$

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$$\begin{vmatrix} (k_1+k_2)-\lambda m_1 & -k_2 \\ -k_2 & k_2-\lambda m_2 \end{vmatrix} = 0$$

$$\begin{vmatrix} 27-9\lambda & -3 \\ -3 & 3-\lambda \end{vmatrix} = 0$$

$$\lambda^2-6\lambda+8=0$$

$$(\lambda-2)\cdot(\lambda-4)=0 \Leftrightarrow \begin{array}{l} \lambda_1=2 \rightarrow \Omega_1^2=2 \Rightarrow \Omega_1=\pm\sqrt{2} \\ \lambda_2=4 \rightarrow \Omega_2^2=4 \Rightarrow \Omega_2=\pm 2 \end{array}$$

Task 3.

Calculate the eigenvector $\vec{v} \neq \vec{0}$ (its coordinates v_1, v_2) of this vibration system for the smaller eigenvalue λ.

Solution:

For $\lambda_1 = 2$

$$(K-\lambda_1 M)\vec{v}=\vec{0} \Leftrightarrow \begin{pmatrix} 27-9\cdot 2 & -3 \\ -3 & 3-2 \end{pmatrix}\vec{v}=\vec{0} \Leftrightarrow \begin{pmatrix} 9 & -3 \\ -3 & 1 \end{pmatrix}\cdot\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$-3v_1+v_2=0 \Rightarrow \vec{v}=t\begin{pmatrix} 1 \\ 3 \end{pmatrix}_{t\neq 0} \Rightarrow \vec{v}=\begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

Task 4.

What is the physical interpretation of the vector $\vec{x}\,''(t)$?

Solution:

$\vec{x}\,''(t)$ - the vector of accelerations of carriages

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Outcomes expected

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- Graphics fitting the solution;
- Numerical results explained and put in context;
- Capture of ICT tools solutions used;
- Sequence of steps followed;
- Remark computations done by hand and done by ICT tools;
- Provide complete answer to questions;
- All the results must be presented in the context of the problem;

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Guide for Learning

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At the beginning of the course, the students need guides on new activities, and feel your support on a well-structured pack of suggestions on how to address the problems posted. Namely:

- Read carefully the problem statement and the tasks posted. Always maintain a global view of all the projects.
- Identify, or try to do a first draft match, the content units of your

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	lecture notes involved in every task. - Take your lecture notes open and review before starting to solve the problems. - Match output expected with the tasks posted, at least as first draft approach. - Follow the order of the tasks, try to increase the knowledge of the problem while you are solving the activities. - Always think that maybe there are different ways to solve a problem. - Use ICT tools to avoid hard computations and check your solutions are correct in different ways if possible. - The solutions are always part of a context, expressing such a final solution totally integrated in the problem posted. - Be sure you answer the complete questions. - Always try to solve the questions by yourself. - If the project can be done in groups, discuss with the groups the proposed problem, to confirm and detect fails or weaknesses, confront strategies, discuss presentation format, etc. Working in groups doesn't mean work less but work better.
Guide for Teaching	Some hints needed to present and launch the mini-PBL to students - Do a small Introduction concerning the problem. - Do a small introduction about the relations represented by the basic equations. - Students will form groups of 4 students and solve the mini-PBL using the eduScrum methodology. - The students should do each exercise in a sequential order, starting from Task 1. - The students should be able to thoroughly read and interpret the numerical results from a mathematical and the real-life example point of view. They should include also a discussion of the climate change crisis and enumerate some strategies they could apply at home or even at university to save resources, namely reduce energy consumption. They should also mention how this mini-PBL helps them identify the indicated Sustainable Development Goals.
Assessment	- Final report; - Oral presentation; - Peer-assessment: students will apply peer-assessment for their periodic performance using online peer assessment tools used and available at the respective institution.
Others: References	Starek, L. Kmitanie mechanických sústav. Vydavateľstvo STU, Bratislava, 2006.

Learning Guide for Students

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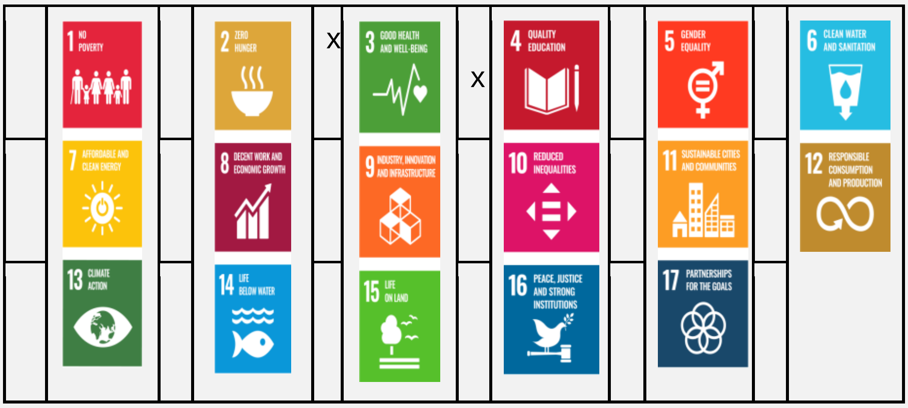
SDG attended	Using this UN graphics, we mark such SDG which this project works. 
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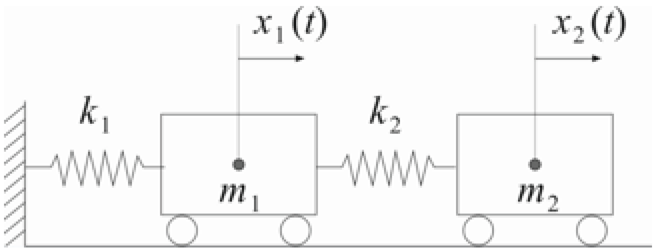

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Guide for Learning

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ANNEX 1: RUBRIC

Category	4=Excellent	3=Good	2=Low	1=Poor
Mathematical Concepts	Explanation shows complete understan- ding of the mathe- matical concepts used to solve the problem(s).	Explanation shows substantial understan- ding of the mathema- tical concepts used to solve the problem(s).	Explanation shows some understan- ding of the mathe- matical concepts needed to solve the problem(s).	Explanation shows very limited unders- tanding of the underlying concepts needed to solve the problem(s) OR is not written.
Mathematical Terminology and Notation	Correct terminology and notation are always used, making it easy to understand what was done.	Correct terminology and notation are usually used, making it fairly easy to understand what was done.	Correct terminolo- gy and notation are used, but it is sometimes not easy to understand what was done.	There is little use, or a lot of inappropriate use, of terminology and notation.
Strategy/Procedure	Typically, uses an efficient and effective strategy to solve the problem(s).	Typically, uses an effective strategy to solve the problem(s).	Sometimes uses an effective strategy to solve problems, but does not do it consistently.	Rarely uses an effective strategy to solve problems.
Completion	All problems are completed.	All but one of the problems are completed.	All but two of the problems are completed.	Several of the problems are not completed.
Mathematical Errors	90-100% of the steps and solutions have no mathematical errors.	Almost all (85-89%) of the steps and solu- tions have no mathematical errors.	Most (75-84%) of the steps and solu- tions have no mathematical errors.	More than 75% of the steps and solu- tions have mathe- matical errors.

Sources Checking				
Working with Others	Student was an engaged partner, listening to suggestions of others and working cooperatively throughout lesson.	Student was an engaged partner but had trouble listening to others and/or working cooperatively.	Student cooperated with others, but needed prompting to stay on-task.	Student did not work effectively with others.
Neatness and Organization	The work is presented in a neat, clear, organized fashion that is easy to read.	The work is presented in a neat and organized fashion that is usually easy to read.	The work is presented in an organized fashion but may be hard to read at times.	The work appears sloppy and unorganized. It is hard to know what information goes together.
Diagrams and Sketches	Diagrams and/or sketches are clear and greatly add to the reader's understanding of the procedure(s).	Diagrams and/or sketches are clear and easy to understand.	The work is presented in an organized fashion but may be hard to read at times.	Diagrams and/or sketches are difficult to understand or are not used.
CT tools used				