Pythagoras
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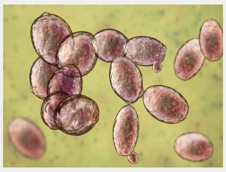
Mini-PBL project	
Student data sheet: Learning Guide	
Title	**Yeast (logistic) growth**
SDG attended	9 INDUSTRY, INNOVATION AND INFRASTRUCTURE; 3 GOOD HEALTH AND WELL-BEING; 7 AFFORDABLE AND CLEAN ENERGY; 12 RESPONSIBLE CONSUMPTION AND PRODUCTION
Content units	Data analysis: regression, ODE: Logistic equation, adjust by functions
Sessions	2
Hours of autonomous work	4
ICT tools to be used	DESMOS, Google Calc, IA
Context: project estatement	**Yeasts** are tiny but mighty organisms that play a **crucial role in many aspects of our lives**, especially in food production. Here's why they are important: ● **Food and Beverage Production:** Yeasts are essential for the fermentation process, which is used to produce a variety of alcoholic beverages like beer, wine, and cider. In these processes, yeast consumes sugars and converts them into alcohol and carbon dioxide. They are also key to baking, where yeast fermentation causes dough to rise, resulting in fluffy breads and pastries.  ● **Nutritional Value:** Some types of yeast, like brewer's yeast, are a good source of vitamins, particularly B vitamins, and protein. ● **Medicine:** Yeasts are used in the production of some medications, such as insulin. ● **Biofuels:** Certain yeast strains can be used to produce biofuels, which are renewable energy sources derived from organic matter.  ● **Scientific Research:** Because of their relatively simple cell structure and rapid reproduction rate, yeasts are widely used as model organisms in scientific research. **Yeasts**, due to their well-defined growth patterns and ease of study,

have become a favorite subject for mathematical modeling of microbial growth. The most common model used to describe yeast growth is:

Logistic Model: This classic model, developed by Pierre-François Verhulst, represents a population's growth initially at an exponential rate, but eventually slowing down and reaching a carrying capacity limited by available resources.

There are more models to be applied in yeast growth. The choice of model depends on the specific aspect of yeast growth being studied. **Simpler models like the Logistic model provide a good starting point**, and this is the center of this project.

Logistic Differential Equation

The **Logistic Differential Equation** is a mathematical tool used to model population growth that takes into account resource limitations. Here are the key fundamentals:

$$\frac{dN}{dt} = r\left(1 - \frac{N}{K}\right)N \qquad N(0) = N_0$$

- **Population Change:** It describes the rate of change of a population (***dN/dt***) over time (dt). This rate of change is represented by a derivative.
- **Growth Proportionality:** The rate of change is proportional (***r***) to the current population size (***N***). This means that larger populations tend to grow faster (but not infinitely).
- **Resource Limitation:** The equation incorporates a factor that slows down growth as the population approaches a carrying capacity (***K***). This carrying capacity represents the maximum population sustainable with available resources (like food or space).
- Logistic Term: The term that considers resource limitation is typically (***1 - N/K***). This term approaches zero as the population (***N***) gets closer to the carrying capacity (***K***), reducing the growth rate.
- **Exponential Growth:** When the population is small relative to the carrying capacity (***N << K***), the (***1 - N/K***) term is close to 1, and the growth rate is approximately proportional to the population size (***rN***), resembling exponential growth.

By solving this differential equation, you can obtain mathematical functions that describe how the population grows over time, reaching an equilibrium at the carrying capacity.

We are going to construct the Logistic Equation of a case of yeast growth directly from research results. Our aim is not the resolution of the equation (classical integration activity) but the determination of the parameters (***r*** and ***K***) from real data, using basic mathematical concepts.

Real industrial/technological situation

Table 1

Time (h)	Pop. size ($\times 10^6$/mL)	Time (h)	Pop. size ($\times 10^6$/mL)
0	0.200	19	209
1	0.330	20	190
2	0.500	21	210
3	1.10	22	200
4	1.40	23	215
5	3.10	24	220
6	3.50	25	200
7	9.00	26	180
8	10.0	27	213
9	25.4	28	210
10	27.0	29	210
11	55.0	30	220
12	76.0	31	213
13	115	32	200
14	160	33	211
15	162	34	200
16	190	35	208
17	193	36	230
18	190		

Source: B. K. Mable et al., "Masking and Purging Mutations following EMS Treatment in Haploid, Diploid, and Tetraploid Yeast (Saccharomyces cerevisiae)," Genetical Research 77 (2001): 9–26.

Consider the growth of a population of yeast. Researchers collected the data in Table 1 from a yeast population grown in liquid culture, measuring the population size (in number of individuals per mL of culture) at different points in time (in hours).

Figure 1 is a scatter plot of these data.

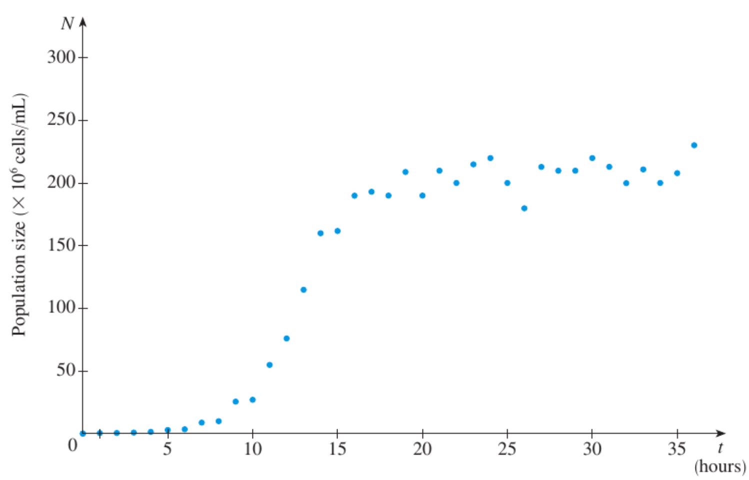


FIGURE 1 A scatter plot of the data in Table 1

Tasks and problems

1. On SDG dimension: **Yeasts** are tiny but mighty organisms that play a **crucial role in many aspects of our lives**,

Activity 1.1: *Why is so relevant the yeast on SDG #9?*

Activity 1.2: *Why is so relevant the yeast on SDG #3?*

Activity 1.3: *Why is so relevant the yeast on SDG #12 and SDG #15?*

Activity 1.4: ……..

2. On mathematical dimension: The goals of these activities is the construction of the ordinary differential equation that fits the experimental data (Table 1 – Figure 1).

***Activity 2.1:* First attempt: Exponential growth.**

Our first approach is to fit the data by the solution of an Ordinary Differential Equation, which one? The most simple model.

a) *Try with the model of exponential growth:*

$$\frac{dN}{dt} = rN$$

Solve this equation analytically.

b) *We may determine the unknown parameter* $\boldsymbol{r}$*, called the rate of growth per capita (why?).*

By taking $N(0)=0.22$ *and* $N(1)=0.33$*, deduce the value of* $\boldsymbol{r}$ *and construct the solution* $N(t)$*.*

c) *Plot the data and the solution together by using DESMOS.*

Activity 2.2: *The previous solution is clearly incomplete for fit all the data. The calculation of* $\boldsymbol{r}$ *has been done with only the first two data. Let's try to improve the estimation.*

a) *Download the data on a spreadsheet to manipulate more elements of the dataset. Open it on Google Calc or any other tool.*

b) *If we see the representation on DESMOS from previous Activity 2.1-(c), the adjustment is going well up to t<14. By using the data in this range (0≤t≤14), try to estimate* ***r*** *by using the regression tool. Why is this a correct method?*

How much is the value of ***r*** *obtained?*

Activity 2.3: *The failure of exponential growth drives us to find another fitting solution. We try another way: the fitting tool of DESMOS* $y_1 \sim f(x_1)$*.*

a) *Try with polynomials*

b) *Try with rational functions*

c) *Do you get the flat behaviour of the data set? Here is where such an exercise of calculation of limits at infinity back to our table.*

***Activity 2.4:* Second attempt: Logistic growth.**

We need to "fold" the exponential obtained, to become flat and approximate to such equilibrium value ***K****. This means that as t increases,* ***dN/dt*** *must go to zero. We need a corrector of the rate of change driving it to zero as* ***N*** *increases to the limit and equilibrium value* ***K****.*

	How do we do it? By introducing the ***Logistic Equation****:* $$\frac{dN}{dt} = rN\,(1 - \frac{N}{K})$$ a) *Study the origin and discoverer of this equation.* b) *Which role does play the new second term?* c) *Solve in this general statement, and present the solution depending of* ***r*** *and* ***K****, with initial data* $N(0)=N_0$. d) *Plot this solution for different values of* N_0, ***r*** *and* ***K.*** ***Hint:*** *By using DESMOS you can define it with mobile bars for the parameters.* *Do you identify the pattern of our dataset?* ***Activity 2.5:*** *Now, we need to determine the parameters* ***r*** *and* ***K*** *for our dataset. We are going to try two strategies.* a) **Direct fitting:** *By using DESMOS, try the general solution* ***N(t)*** *obtained for fitting the dataset.* *We are interested in a first estimation of the parameters* ***r*** *and* ***K****, what values do we get?* b) **Fitting by hand:** *Now we proceed by looking carefully at the dataset.* ***Step 1:*** *Similarly to Activity 2.2-(b), for the general solution* ***N(t)*** *obtained in Activity 2.4-(c), let's try to linearize the expression.* ***Step 2:*** *We fit the data by regression and choose* ***K*** *so that we have the best estimate. This step is where the linear trend of the data materializes (what we were looking to confirm).* ***Step 3:*** *With* ***r*** *obtained in Activity 2.2 and this* ***K****, we can construct the solution* ***N(t)*** *of the logistic equation wich fits the dataset* ***Step 4:*** *Compare dataset with results of solution for these values of* ***r*** *and* ***K****. Plot together all data.*
Outcomes expected	**1. On SDG dimension:** All these answers are obtained by using Gemini Google. ***Activity 1.1:*** *Why is so relevant the yeast on industrial process (SDG #9)?* *Yeast plays a crucial role in various industrial processes, directly aligning with Sustainable Development Goal #9: Industry, Innovation and Infrastructure. Here's how yeast contributes to this SDG:* ● **Biofuel Production:** Yeast can ferment sugars into ethanol, a biofuel that serves as a renewable alternative to fossil fuels [1]. This helps reduce dependence on

	non-renewable resources and combats climate change, a key aspect of SDG #9. • **Efficient and Sustainable:** Yeast fermentation is a relatively clean and efficient process compared to traditional fuel production methods. It requires less land and water compared to growing crops for biofuels, making it a more sustainable option [2]. • **Innovation in Food Production:** Yeast is used in the large-scale production of food and beverages like bread, beer, and wine. Innovation in yeast strains and fermentation processes allows for: 　○ **Improved Food Quality:** Development of strains that enhance flavor, texture, or nutritional content of food products [3]. 　○ **Reduced Waste:** More efficient fermentation processes can minimize food spoilage and waste generation during production [4]. • • **Production of Key Ingredients:** Beyond food and beverages, yeast is used to produce essential industrial ingredients like enzymes and citric acid [1]. These products find applications in various industries, promoting innovation and development. **Overall Impact:** By contributing to biofuel production, efficient food and beverage production, and the creation of valuable products, yeast supports sustainable industrial practices (SDG #9). Its role in innovation and resource efficiency aligns with the goals of building resilient infrastructure and promoting inclusive and sustainable industrialization. **Considerations:** • **Focus on Sustainability:** While industrial yeast production has a positive impact, it's crucial to focus on sustainable practices throughout its lifecycle to minimize environmental impact. • **Continuous Improvement:** Ongoing research and development can further improve the efficiency and sustainability of yeast-based industrial processes. In conclusion, yeast plays a significant role in achieving SDG #9 by promoting sustainable industrial practices, innovation, and resource

efficiency. By focusing on its potential and addressing sustainability concerns, yeast can continue to be a valuable contributor to a more sustainable future.

<u>Activity 1.2:</u> *Why is so relevant the yeast on health (SDG #3)?*

Yeast contributes to good health in several ways, aligning with Sustainable Development Goal #3 (Ensure healthy lives and promote well-being for all at all ages). Here's a breakdown of its relevance:

Nutritional Benefits:

- **Essential Nutrients:** Certain yeasts, particularly baker's yeast (Saccharomyces cerevisiae), are a good source of vitamins, minerals, protein, and fiber [1, 2].
    - **B Vitamins:** Rich in B vitamins like thiamine (B1), riboflavin (B2), niacin (B3), and B6, crucial for energy metabolism and cell function [1].
    - **Minerals:** Contains chromium, selenium, and zinc, important for various bodily functions [2].
    - **Protein and Fiber:** Provides protein and some dietary fiber, beneficial for vegans or those needing dietary supplementation [1].

Probiotic Potential:

- **Gut Health:** Some yeast strains exhibit probiotic properties, similar to beneficial gut bacteria [1, 3]. These strains may help:
    - **Balance Gut Microbiota:** Promote a healthy balance of gut microbiota, potentially aiding digestion and reducing the risk of diarrhea [2, 3].
    - **Immune System Support:** Possibly boost the immune system by interacting with gut immune cells [3].

Food Fermentation:

- **Enhanced Gut Health:** Yeast plays a key role in fermenting various foods like yogurt, kefir, and some cheeses [2]. These fermented foods can improve gut health by promoting the growth of beneficial bacteria [4].

Additional Considerations:

- **Ongoing Research:** While research is promising, more studies are needed to fully understand the extent of yeast's health benefits.

- **Variety Matters:** The specific health benefits of yeast depend on the strain. Baker's yeast and nutritional yeast are the most commonly studied for health purposes.

In summary, yeast contributes to good health (SDG #3) by providing essential nutrients, potentially acting as a probiotic, and supporting gut health through fermentation. While further research is needed, its potential benefits make it a relevant player in promoting overall well-being.

Activity 1.3: Why is so relevant the yeast on SDG #12 and SDG #15?

Yeast's relevance to SDGs #12 (Responsible Consumption and Production) and #15 (Life on Land) stems from its role in promoting sustainable practices and potentially reducing environmental impact. Here's a breakdown:

SDG #12: Responsible Consumption and Production

- **Reduced Waste:** Yeast fermentation processes are known for their efficiency, minimizing waste generation. In food and beverage production, efficient fermentation reduces spoilage and wasted resources [1].

- **Resource Efficiency:** Yeast can be cultivated on simple sugars, requiring less land and water compared to some traditional food production methods [2]. This aligns with the goal of promoting resource efficiency in production systems.

- **Biodegradable Products:** Yeast-based products like biofuels and some bioplastics are biodegradable, contributing to a more sustainable waste management cycle [3].

However, responsible production practices are crucial throughout the yeast lifecycle. Energy used for yeast cultivation and potential environmental impacts of large-scale production need to be considered.

SDG #15: Life on Land

- **Reduced Reliance on Land-intensive Agriculture:** Biofuel production from yeast fermentation offers a

	potential alternative to traditional biofuels derived from crops like corn. This can help reduce pressure on land use for agriculture and potentially protect biodiversity [1, 4]. ● **Limited Land Footprint:** As mentioned earlier, yeast cultivation itself requires less land compared to traditional agriculture. This can help conserve natural habitats and ecosystems (SDG #15). **Important Considerations:** ● **Sustainability Throughout Lifecycle:** While yeast offers potential benefits for both SDGs, ensuring sustainable practices throughout its production and use is crucial. Minimizing energy consumption and waste generation are key considerations. ● **Further Research:** The long-term environmental impact of large-scale yeast production and its use in biofuel production needs further research and development. Overall, yeast presents itself as a tool for promoting aspects of SDGs #12 and #15. However, responsible and sustainable practices are essential to fully realize its potential for a more sustainable future.
Guide for learning	
Assessment	
Others: References	

ANNEX 1: RUBRIC

Category	4=Excellent	3=Good	2=Low	1=Poor
Mathematical Concepts	Explanation shows complete understan- ding of the mathe- matical concepts used to solve the problem(s).	Explanation shows substantial understan- ding of the mathema- tical concepts used to solve the problem(s).	Explanation shows some understan- ding of the mathe- matical concepts needed to solve the problem(s).	Explanation shows very limited unders- tanding of the underlying concepts needed to solve the problem(s) OR is not written.
Mathematical Terminology and Notation	Correct terminology and notation are always used, making it easy to understand what was done.	Correct terminology and notation are usually used, making it fairly easy to understand what was done.	Correct terminolo- gy and notation are used, but it is sometimes not easy to understand what was done.	There is little use, or a lot of inappropriate use, of terminology and notation.
Strategy/Procedure	Typically, uses an efficient and effective strategy to solve the problem(s).	Typically, uses an effective strategy to solve the problem(s).	Sometimes uses an effective strategy to solve problems, but does not do it consistently.	Rarely uses an effective strategy to solve problems.
Completion	All problems are completed.	All but one of the problems are completed.	All but two of the problems are completed.	Several of the problems are not completed.
Mathematical Errors	90-100% of the steps and solutions have no mathematical errors.	Almost all (85-89%) of the steps and solu- tions have no mathematical errors.	Most (75-84%) of the steps and solu- tions have no mathematical errors.	More than 75% of the steps and solu- tions have mathe- matical errors.

Sources Checking				
Working with Others	Student was an engaged partner, listening to sugges- tions of others and working coopera- tively throughout lesson.	Student was an enga- ged partner but had trouble listening to others and/or working cooperatively.	Student coopera- ted with others, but needed prompting to stay on-task.	Student did not work effectively with others.
Neatness and Organization	The work is presen- ted in a neat, clear, organized fashion that is easy to read.	The work is presen- ted in a neat and organized fashion that is usually easy to read.	The work is presented in an organized fashion but may be hard to read at times.	The work appears sloppy and unorga- nized. It is hard to know what information goes together.
Diagrams and Sketches	Diagrams and/or sketches are clear and greatly add to the reader's unders- tanding of the procedure(s).	Diagrams and/or sketches are clear and easy to understand.	The work is presented in an organized fashion but may be hard to read at times.	Diagrams and/or sketches are difficult to understand or are not used.
ICT tools used				

Sources of examples

Special interest for any teacher is to have access to a high variety of sources for examples and problems to supply the mini-PBL projects. In fact, it is enough to give a basic applied problem to generate a bigger list of activities based on that.

Every course, in all universities around the world, graduate students present thousands of degree, master and PhD thesis where you can find for sure a list of examples/applications where a graphic or dataset can be the start point for constructing a mini-PBL project.

We encourage you to find your own sources, ask colleagues from your university for precise use of mathematics in any scientific area. Then connect such applications with SDG and construct your mini-PBL.

For help in such searching, in the following table we group basic scientific areas with the SDG to provide a list of suggestions for search math applications with SDG's interest label. Obviously, this is not a closed list, the items are interchangeable, and anyone can find an application of interest in the thousands of research papers and reports published every year all round the world.

SUSTAINABLE DEVELOPMENT GOALS	**Scientific areas or studies**
1 NO POVERTY	❖ Population studies and databases ❖ Incomes & Costs reduction ❖ Logistic & Transportation ❖ Equality studies
2 ZERO HUNGER	❖ Agriculture ❖ Nutrition ❖ Plagues ❖ Pollution
3 GOOD HEALTH AND WELL-BEING	❖ Medical databases ❖ Epidemiology ❖ Pharmacokinetic ❖ Microbiology ❖ Genetic ❖ Sports ❖ Aging

4 QUALITY EDUCATION	❖ Education databases ❖ Psychology ❖ Neuroscience ❖ Human & childhood behavior ❖ Economy of Education ❖
5 GENDER EQUALITY	❖ Incomes inequalities ❖ Population proportion on labor sectors ❖
6 CLEAN WATER AND SANITATION	❖ Water supply databases ❖ Pollution ❖ Microbiology ❖ Engineering ❖
7 AFFORDABLE AND CLEAN ENERGY	❖ Energy production and consume databases ❖ Engineering ❖ Consume ❖ Optimization of resources ❖ Penetration of renewable energies
8 DECENT WORK AND ECONOMIC GROWTH	❖ Labor databases ❖ Social studies ❖ Share market ❖ Bank products ❖ Economy and enterprise ❖ Tourism ❖ Digital business
9 INDUSTRY, INNOVATION AND INFRASTRUCTURE	❖ Industry and production databases ❖ Engineering ❖ Mobility ❖ Artificial Intelligence ❖ New Materials ❖ Nanotechnology ❖ Connectivity ❖ Logistic & Transportation ❖ 4th Industrial Revolution: Industry 4.0

10 REDUCED INEQUALITIES	❖ Social lacks and gaps databases ❖ Social studies ❖ Population studies ❖ Incomes/costs reduction ❖ Digital access
11 SUSTAINABLE CITIES AND COMMUNITIES	❖ Quality living databases ❖ Population studies ❖ Optimization ❖ Logistic & Transportation ❖ Civil Engineering ❖ Architecture ❖ Baggage management ❖ Energy ❖ New materials for urban furniture: maintenance, cleaning, preventing damage,... ❖ Electric and autonomous mobility ❖ Air pollution ❖ Noise reduction ❖ Mobility ❖
12 RESPONSIBLE CONSUMPTION AND PRODUCTION	❖ Consume and human behavior databases ❖ Marketing ❖ Social networks ❖ Raw and new materials ❖ Energy ❖ Engineering
13 CLIMATE ACTION	❖ Climate change databases ❖ Deforestation ❖ Desertification ❖ Atmospheric physics ❖ Overheating ❖ CO_2 reduction ❖ Carbon footprint
14 LIFE BELOW WATER	❖ Climate change and oceans ❖ Acidification of oceans ❖ Sustainable fishing ❖ Microbiology on oceans ❖ Microplastic and oceans ❖ Waste and biodegradable materials

15 LIFE ON LAND	❖ Climate change and land sustainability ❖ Sustainable farming ❖ Residues and waste processing ❖ Waste and biodegradable materials
16 PEACE, JUSTICE AND STRONG INSTITUTIONS	❖ Social studies databases ❖ Economy ❖ Cooperation for development ❖
17 PARTNERSHIPS FOR THE GOALS	❖ Social studies ❖ Social networks ❖ Civil movements ❖

References

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