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A review of state-of-the-art gamification theories and trends for online and hybrid mathematics in higher education

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Introduction and motivation

Games embody well-established principles and models of learning and allow placing the learning activity into a meaningful context. Gaming elements have been introduced in the classroom in a variety of forms: serious games, digital game-based learning, and gamification.

One of the recurrent topics on mathematics in HE is the lack of student motivation and attitudes towards the subject (Triantafyllou et al., 2016). Gamification could help support student motivation and engagement with mathematical activities. However, initiatives on gamifying mathematics in STEM courses in HE are still scarce (Ortiz Rojas et al., 2016).

In this result, the consortium will investigate successful applications of gamification in online mathematics with the aim to identify opportunities and challenges. A study of the existing conditions in the gamification domain will be conducted, with a focus on environments and tools utilized in mathematics education. Towards this goal, a detailed literature review and desktop research will be carried out that will result in a report highlighting successful initiatives and trends of gamification utilized in mathematics with a special focus on online and hybrid education.

Literature Review

Online and hybrid education

Online education, also known as e-learning or virtual learning, refers to the delivery of educational content and instruction through digital platforms and the internet (Sun & Chen, 2016). This mode of education allows students to access courses, lectures, and learning materials remotely, providing flexibility in terms of time and location. Online education can take various forms, including fully online courses, degree programs, or individual modules that supplement traditional in-person instruction.

Hybrid education, often referred to as blended learning, combines elements of both traditional face-to-face instruction and online learning (Kazu & Yalçin, 2022). In a hybrid model, students engage in a mix of in-person and virtual activities. This approach offers the benefits of both physical classroom interactions and the flexibility of online resources. Hybrid education can involve scheduled in-person classes supplemented with online content, discussions, or assessments.

Online and hybrid education in mathematics can help leveraging gamification benefits by incorporating various digital tools derived from the game industry to motivate students.

Gamification and mathematics

Scholars such as Alt emphasize the significance of gamification through the use of gaming in teaching mathematics, attributing its effectiveness to the engagement, practice, feedback, and enjoyment it offers, creating an optimal environment for knowledge-building (Partovi & Razavi, 2019). Technology-based games, as suggested by Behnamnia et al., are believed to motivate students, foster creativity, and enhance their enjoyment of mathematics (Behnamnia et al., 2020). The primary objective is to augment students' curiosity, motivation, and involvement in mathematical learning. Key principles for effective gamified learning programs in mathematics can include repetition, immediate feedback, adaptation based on difficulty levels, conciseness in assignment structures, and freedom of choice in exercises (Alt, 2023). Additionally, the gamification approach should align with some fundamental aspects: motivating behavior, stimulating learning, and designing game structures to achieve educational goals (Torres-Toukoumidis et al., 2021). We will now dive deeper into what gamification actually means, and its corelated elements.

Gamification

The most referenced definition of gamification is the one created by Deterding & Dixon et al. which states that gamification is “the use of game design elements in non-game contexts” (Deterding, Dixon, et al., 2011, p. 2). However, there are numerous other definitions and uses of this term, and it is important to understand that there is not one agreed upon, unified understanding of this concept. Figure 1 shows some commonly used definitions of gamification.

Figure 1: different definition of gamification

Definition of gamification	**Reference**
“The use of game design elements in non-game contexts.”	(Deterding, Dixon, et al., 2011, p. 2)
“Gamification is the process of adding game mechanics to processes, programs and platforms that wouldn't traditionally use such concepts. The goal is to create incentives and a more engaging experience. In other words, it's about fun.”	(Swan, 2012, p. 13)

"Using game-based mechanics, aesthetics, and game thinking to engage people, motivate action, promote learning, and solve problems. "	(Kapp, 2012, p. 10)
"Implementing design concepts from games, loyalty programs, and behavioral economics to drive user engagement"	(Zichermann & Linder, 2013, p. 12)
"The use of game mechanics and experience design to digitally engage and motivate people to achieve their goals."	(Burke, 2014, p. 6)

These definitions all focus on applying game elements or mechanics in a certain context (physical or digital), often to promote motivation and engagement to reach an end goal. By primarily focusing on the definition from Deterging & Dixon et al., we might understand gamification as the process of adding game elements to a context that does not already have them – a transformational act. To have a more in-depth understanding of the term, especially in relation to the Pythagoras project, it is also helpful to include other concepts related to gamification.

Including other concepts related to gamification

Like gamification, concepts and terminology related to gamification has a myriad of different definitions and applications in research. In the book "The Gamification of Learning and Instruction" by Kapp, the author argues for a unified understanding of "serious games" and "gamification" (Kapp, 2012, p. 15-18). In another article, Grey & Gordon actively include the terms "games", "game-based learning", "serious games" and "playful learning" when trying to explain engagement connected to gamification in higher education (Oscar Bernardes et al., 2022, p. 664-668). In contrast to this, some literature reviews try to specifically separate and exclude these concepts when exploring gamification by e.g. removing the concepts of "serious games" and "game-based learning" entirely from the gamification context (Alzahrani & Alhalafawy, 2022; Khaldi et al., 2023). In this section, we therefore introduce related concepts to gamification, some of their definitions, and try to argue for their relevance and interconnectedness specifically in context of the Pythagoras project.

The game

An important part of gamification is the game. The game part of gamification is explained by Salen and Zimmerman as "a system that enables the player to participate in a digital conflict according to specific rules that produce quantifiable results" (Salen & Zimmerman, 2005). Here, the game is placed in a digital context and is related to measuring the outcomes of solving a conflict. Schell includes this same conflict and problem solving in their definition "a game is a problem-solving activity, approached with a playful attitude" (Schell, 2008, p. 47), but puts the main focus in playfulness instead. A third definition coined by McGonigal present a more pragmatic approach, defining games as anything that has four defining traits; a goal, rules, a feedback system and voluntary participation (McGonigal, 2011, p. 21). This last definitions is in an article by Grey & Gordon used as framework for supporting the notion that higher education in itself can be seen as a game, where the goal is to get an academic qualification, the rules are provided in the specific learning activity requirements, feedback is present in the form of summative and formative assessments, and voluntary participation comes from the fact that Higher Education is normally seen as a voluntary level of education (Oscar Bernardes et al., 2022, p. 664-668). With this perspective, one could look at the education context and some of its sub elements as an already gamified environment. This is useful to understand that even if we want to increase the use of gamification in mathematics, gamification already exists to some degree and is not something completely novel. What a "game" is can vary immensely.

Cezar et al. suggests that educational digital games can be classified into eight categories that usually overlap as seen in table 2 (Cezar et al., 2022). These categories can be used to choose a game genre when creating an educational digital game.

Table 2: categories of educational digital games

Category	**Characteristic**
Action	When the player needs to shoot falling things or enemies, car races and chases.
Adventure	Where unknown worlds are explored, objects are picked up, and problems solved.
Sports	Most are action games where the player controls his character within a sport. There are sports with less action that the game is more statistical.
Strategy	Generally, the player is responsible for evolving an army or a civilization to progress and in most cases defeat their opponents.
Fight	Two or more characters fight a battle until one side is defeated.
Puzzle game	The plot is simply a pretext for problems to be solved.
RPG (Role Playing Game)	They often feature characters known as dwarfs, hobbits, humans, orcs, elves, wizards, etc., who interact with medieval images involving quest tasks to save someone or something.

Simulation	The player pilots air, land, or water machines, usually military. Worlds are built and companies are run that are very connected to reality.

Adapted from "Table 1" in Cezar, V. L., Botelho, V. R., Garcia, P. V., & Miletto, E. M. (2022). The Use of an Educational Digital Game in Higher Education: Design and Application to Increase the Motivation in Calculus Learning. I Handbook of Research on the Influence and Effectiveness of Gamification in Education (s. 360–382). IGI Global. https://doi.org/10.4018/978-1-6684-4287-6.ch018

Game-based learning and serious games

Many studies on the topic of gamification makes a clear distinction between "gamification" and "game-based learning", and between "gamification" and "serious games" (Alzahrani & Alhalafawy, 2022; Khaldi et al., 2023). Where gamification is defined as the use of game design elements in non-game contexts, "game based learning" is often defined as "a learning method that uses games specifically designed to assist the learning process" (Alzahrani & Alhalafawy, 2022), and "serious games" as "an experience designed using game mechanics and game thinking to educate individuals in a specific content domain" (Kapp, 2012, p. 15-18). When putting these definitions up against each other, it is hard to see a clear distinction. In the context of the PYTHAGORAS project, it might also not be useful to exclude one or the other. The goal of the project is to investigate successful applications of gamification in online mathematics with the aim to identify opportunities and challenges. Therefore, the perspective of all three terms, as well as their fluidity and interconnectedness are all relevant and interesting. Gamification, game-based learning, and serious games can all work together towards achieving a better learning experience for students. One does not have to exclude games from the learning experience, for it to be considered gamification, since many learning situations have been incorporating games and game elements for a long time already as stated in the chapter about the game.

Torres-Toukoumidis et al. argues that "gamification is a transformative educational tool adapted to the 21st century capable of fulfilling the aim of Game based Learning with greater pragmatism and accessibility.» (Torres-Toukoumidis et al., 2021). In the context of PYTHAGORAS, it is therefore possible, and useful, to see "gamification" as the process of developing a "serious game", and then be able to apply this serious game in a "game-based learning" context.

Gameful design and gameful experience

«Gameful design», as advocated by Jane McGonigal, involves crafting experiences that tap into the spirit of gamers in real-life scenarios (McGonigal, 2011). McGonigal identifies key characteristics of the gamer mindset, including "Urgent Optimism", where individuals zealously work toward goals; "Social Fabric", fostering collaboration and competition for shared objectives; "Epic Meaning", integrating storylines and goals to lend context to students' actions for a better world; and "Blissful Productivity", acknowledging the hard work and dedication gamers invest in achieving objectives. When educators and facilitators design learning experiences, they must align them with meaningful real-world goals, emphasizing collaboration and engagement in both digital and non-digital learning environments (Schaaf & Mohan, 2014).

Complementing this perspective is the notion of a "gameful experience," conceptualized by Ermi & Mäyrä as "an ensemble of sensations, thoughts, feelings, actions, and meaning-making in a gameplay setting" (Ermi & Mäyrä, 2005). In the context of mathematics education, a gameful experience would involve creating an environment that not only challenges students intellectually but also provide a sense of playfulness, the thrill of overcoming challenges, the satisfaction of accomplishments, and immersion in the learning process. By integrating gameful design principles and fostering a gameful experience, educators can enhance student engagement, collaboration, and motivation in the pursuit of mathematical knowledge and problem-solving. The incorporation of Urgent Optimism, Social Fabric, Epic Meaning, and Blissful Productivity aligns with the goals of promoting active and enthusiastic participation in the learning process for mathematics.

Playfulness and playful learning

Playfulness can be considered a sub-category of the experience of playing games (Högberg et al., 2019). A successful application of gamification can result in "hedonic outcomes" such as playfulness, fun and enjoyment (Patrício et al., 2018). The concept of playful learning encompasses various approaches such as playfulness, games, game-based learning, and serious games, all adopting a lusory attitude towards teaching and learning (Oscar Bernardes et al., 2022. p. 668). This inclusive term seeks to avoid restrictive definitions and alleviate concerns related to extrinsic motivation, which can sometimes lead to unintended negative consequences. Playful learning encourages low-risk experimentation, reflection, and immersion in the learning process. It rejects the potential downsides of metric-driven systems, emphasizing the importance of safe spaces for learning. The notion of a "lusory attitude" is crucial for engaging with learning in a playful way (López et al., 2021). It involves maintaining an open mind, which can be a conflict between the perceived frivolity of play and the seriousness of the task.

The term "Serious Games" highlights the tension between playfulness, often associated with children, and the seriousness of the learning task. "The magic circle" is a concept in gamification and game design introduced by Johan Huizinga in his book "Homo Ludens: A Study of the Play Element in Culture." that tries to alleviate this tension (Huizinga, 1955). The term refers to the idea that when individuals engage in a game or play, there is a symbolic boundary or mental space created – the magic circle – within which the rules, goals, and dynamics of the game apply. Inside this circle, participants willingly suspend disbelief and embrace a playful mindset.

In the context of the gamification of mathematics in higher education, the concept of playfulness and the magic circle is relevant because it helps establish a mental stage where play and playfulness are not only accepted but encouraged. By framing the learning environment as a "magic circle," educators can create a space where students feel free to explore mathematical concepts with a sense of curiosity and enjoyment. This approach can foster a positive attitude towards learning, reduce anxiety associated with challenging subjects like mathematics, and promote a more immersive and engaging educational experience.

Gamified learning environment

The gamified learning environment (GLE) is described by Giráldez et al. as an intentional fusion of teaching and learning processes, involving the deliberate incorporation of game elements to enhance motivation, engagement, and problem-solving skills in students (Bernardes et al., 2022a). Their research introduces a model with the factors that needs to be in place for creating a GLE (see figure 1), categorizing elements into educational, game, recommended knowledge, and motivational components. The educational elements involve curriculum alignment, evaluation methods, and instructional planning. Game elements encompass dynamics, mechanics, narrative forms, aesthetics, and technological support. The model also touches upon the importance of multidisciplinary knowledge for the creation of GLEs in higher education. The authors suggests that a rich gamification experience requires expertise in various domains, including video games, audiovisual and multimedia resources, story creation, and graphic design. This multidisciplinary knowledge is seen as a critical factor contributing to the depth and effectiveness of gamified educational experiences. Motivational elements focus on reward systems such as badges, virtual currency, and leaderboards to sustain student engagement and commitment throughout the gamified learning experience. The importance of careful planning and evaluation of the gamification system is highlighted, acknowledging the need for a supportive approach to challenges in game thinking, emotion building, and motivational influences. This model creates a good overview of the different elements that needs to be a part in a gamification process to create a gamified learning environment.

Figure 1: Factors for creating a Gamified Learning Environment

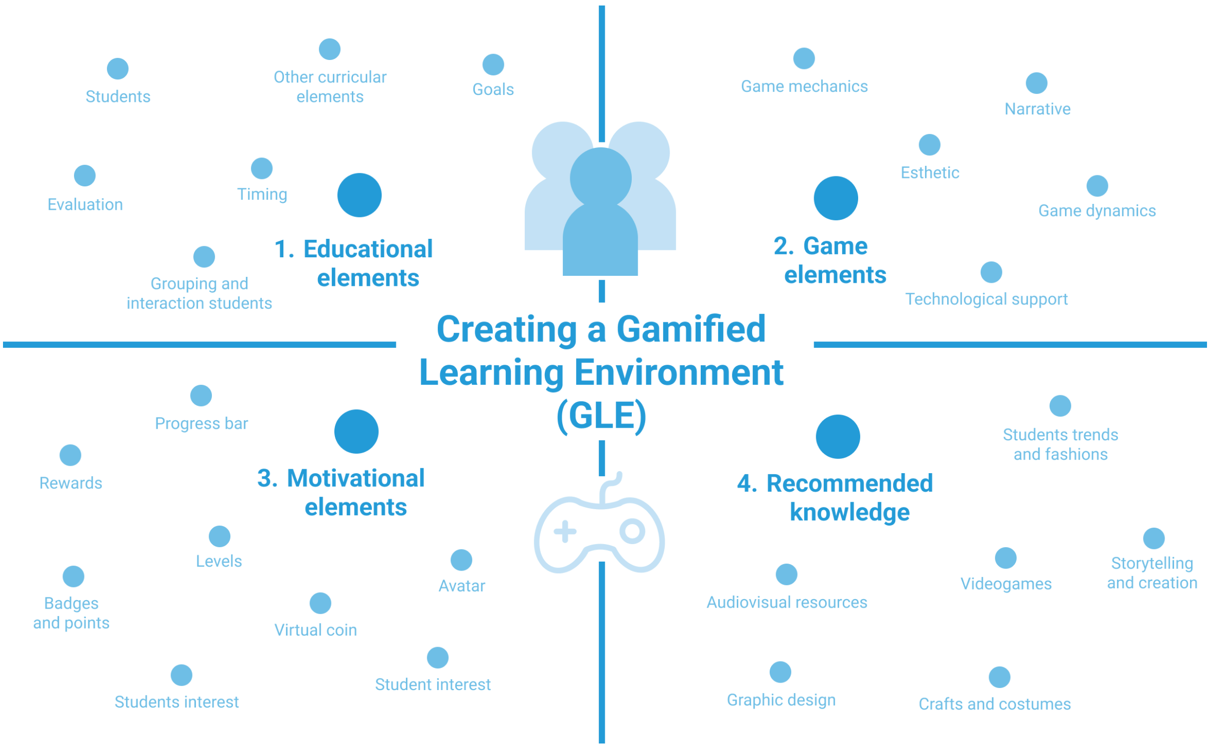


Figure adopted from "figure 1" in Bernardes, O., Amorim, V., & Moreira, A. C. (2022a). Gamification in Higher Education: Analysis of Your Strengths and Weaknesses. I Crossref (s. 63–84). IGI Global. https://doi.org/10.4018/978-1-6684-4287-6.ch004

The game elements

Gamification can be implemented by adding different "parts" from games to the learning situation. These parts are often referred to as gamification "elements", "mechanics", "components", "classifications", or "categories" and include things such as PBL (points, badges, leaderboards), avatars, narratives, storylines, competition, challenges and cooperation (Alzahrani & Alhalafawy, 2022; Khaldi et al., 2023; Manzano-León et al., 2021; Oscar Bernardes et al., 2022). By defining and comparing some of the models that define these components we can get an idea of what elements should be including in the process of gamifying mathematics.

The MDA model – Mechanics, Dynamics, Aesthetics

The most studied gamification system model is the MDA (Mechanics, Dynamics, Aesthetics) system proposed by Hunicke, LeBlanc, and Zubek which is a comprehensive approach to game design that tries to go beyond the use of PBL (points, badges, leaderboards) (Hunicke et al., 2004; Manzano-León et al., 2021). Here, mechanics encompass actions and control mechanisms like draw cards, gamble, trade, attack, compete, cooperate. Dynamics involve behaviors during the gameplay of the mechanics like socializing, bluffing, reflection, status, attention.

Aesthetics focus on the emotional responses evoked in players like sensation, fantasy, narrative, challenge, companionship, discovery, expression, and entertainment.

The DMC model – Dynamics, Mechanics, Components

Another gamification model made by Werbach & Hunter tries to categorize gamification scenarios in a hierarchical structure (Werbach & Hunter, 2012). At the top level are Dynamics, which include limitations, emotions, narration, progress, teams, and relationships. Mechanics are sets of rules that dictate the outcomes of interactions and are the game elements that drive the action forward like challenges, chance, competition, cooperation, feedback, resource gathering and rewards. Components form the basic level, incorporating elements like achievements, avatars, leaderboards, badges, collections, content unlocks, donations, levels, progress bars, and virtual goods. This model provides a layered framework for understanding how different elements contribute to the overall gamified experience.

The MAT model – Mechanics, Aesthetics, Thoughts

A third gamification model proposed by Kapp emphasizes three main gamification elements—Mechanics, Aesthetics, and Thoughts (Kapp, 2012) . Mechanics include elements like levels, badges, ranking, score, and time control, playing a crucial role in the gamification process, but are not enough to transform a boring experience into something engaging. Aesthetics focus on the pleasantness of the gamification experience, incorporating graphic quality, art pieces, sound effects, and soundtrack. Thoughts, identified as the most important element, shape the game's narrative, defining the experiences that lead to the desired learning outcomes.

These models can give an impression of what needs to be incorporated into the developing stages of a GLE.

Motivational theories for gamification

The relevance of gamification is grounded in its capability to increase students' motivation, and consequently performance. The most important motivational theories that support gamification, argued by Huang, Hew & Kapp, are the self-determination theory, the flow theory, goal-setting theory, social comparison theory, behavior reinforcement theory and expectancy theory (Huang & Hew, 2018; Kapp, 2012). We will now explain the different motivational theories, and their relevance to gamification (see table 3 for summary if theories).

Self-determination theory

Rewards-based gamification targets extrinsic motivation based on the self-determination Theory (SDT). SDT is concerned with the psychological needs behind motivation and the social conditions that foster these processes

(Ryan & Deci, 2000). SDT identifies two distinct types of motivation: intrinsic and extrinsic. Intrinsic motivation is the "manifestation of the human tendency toward learning and creativity" (Ryan & Deci, 2000). SDT is less concerned about the causes of intrinsic motivation, but rather the "conditions which elicit and sustain, versus subdue and diminish" this type of motivation (Ryan & Deci, 2000). Extrinsic motivation on the other hand is a socially generated motivation, often occurring due to social expectations, or possible rewards for certain actions or behavior. SDT argues that extrinsic motivation can be variably autonomous, as the reward can be something of self-determined value, or it could be something completely external. Rewards-based gamification can be used to incite extrinsic motivation or increase its autonomy among students. In many instances, extrinsic rewards help foster intrinsic rewards. A good extrinsic motivation is a good map to intrinsic motivation. The better a designer (or teacher) knows their players, the better game design (Schaaf & Mohan, 2014). When gamifying mathematics it is therefore important to provide the learner with a feeling of competence, opportunities for autonomy, and relatedness with others to illicit the intrinsic motivation (Kapp, 2012, p. 74). It is also important to appeal both to the extrinsic and the intrinsic motivation and use extrinsic game reward systems as a gateway to unlocking the intrinsic motivation in students.

Flow theory

Flow theory, as defined by Csikszentmihalyi, encapsulates a positive psychological state wherein individuals experience total involvement in an activity, resulting from a perceived balance between challenges and one's ability to meet those challenges (Csikszcntmihalyi, 1990). The eight elements of flow as revised by Kapp include *achievable tasks, concentration, clear goals, feedback, effortless involvement, control over actions, concern for self disappears, and loss of sense of time,* and they all contribute to the holistic sensation of engagement (Kapp, 2012, p. 71-74). This theory holds particular significance in educational settings, as Seligman and Csikszentmihalyi advocate for fostering strengths and creating communities that facilitate flow experiences (Seligman & Csikszentmihalyi, 2000). In the context of gamification design for mathematics education, the integration of flow theory becomes imperative. Kapp emphasizes the need for adaptive systems that maintain a constant state of learner interest by continually adjusting to the appropriate challenge level to be not too difficult and not too easy, aligning with the principles of flow theory (Kapp, 2012, p. 71-74). Wherever there is a need for educational improvement, the application of flow theory, with its emphasis on engagement and challenge, can guide the design and implementation of effective educational practices. By applying this theory in the gamification of mathematics, one can make sure that student's heterogeneous skill levels are met, and accordingly challenged in game-based learning activities so that the become completely immersed.

Goal-setting theory

Goal-setting theory, as articulated by Locke and Latham, commences with the identification or creation of specific objectives, acting as a mechanism to guide individual behavior (E. Locke & Latham, 2006). The significance of this theory lies in its assertion that establishing clear goals serves as a foundation for self-directed or accepted actions, fostering heightened motivation and performance. Notably, research by Locke and Latham emphasizes the efficacy of well-defined and challenging goals in enhancing performance across diverse domains (E. A. Locke & Latham, 2002). Schunk emphasizes the importance of creating a motivating learning environment by setting achievable long- and short-term goals, providing performance feedback, and assisting students in self-evaluation (Schunk, 1991). Huang and Hew reviewed empirical studies on the use of goal-setting theory in gamification and found that that game design elements such as badges and leaderboards can direct learners' attention to targeted learning activities (Huang & Hew, 2018). By incorporating this motivational framework into the gamification of mathematics, educators can leverage the power of clear objectives to stimulate intrinsic motivation and self-regulation among students. By incorporating goal-setting principles into gamification, educators can provide students with a roadmap for their academic journey, making the learning process more structured and purposeful. Achieving milestones in the gamified system becomes a source of motivation, as students receive immediate feedback and rewards for their accomplishments. The sense of accomplishment derived from reaching these goals can fuel intrinsic motivation and a desire to tackle more challenging tasks.

Social comparison theory

Social Comparison Theory, developed by Leon Festinger, posits that individuals evaluate their own abilities and opinions by comparing them to those of others in their social environment (Festinger, 1954). In the context of higher education and gamification, this theory can be relevant in understanding how students perceive their academic achievements relative to their peers. Gamification, the integration of game elements into non-game contexts like education, often introduces competitive or collaborative elements. By incorporating features that allow students to compare their progress, achievements, or learning outcomes with their peers, gamification aligns with the principles of Social Comparison Theory (Kapp, 2012, p. 74). This comparison can potentially motivate students by fostering a sense of competition, encouraging them to outperform others or collaborate for mutual success, thus enhancing engagement and overall learning experiences in higher education settings.

Behavior reinforcement theory

In Behavioral Reinforcement Theory, B.F. Skinner introduced the concept of operant conditioning, which involves shaping behavior through the use of positive and negative reinforcement or punishment (Skinner, 1965, 1989). Positive reinforcement refers to the addition of a reward to encourage a behavior, while negative reinforcement involves the removal of an aversive stimulus to achieve the same goal. In gamification within higher education, positive reinforcement can be seen through rewards like points, badges, or other virtual incentives awarded to students for completing assignments, achieving academic goals, or participating in collaborative activities. This approach taps into the fundamental idea that individuals are more likely to repeat behaviors for which they have been positively reinforced.

Expectancy theory

Victor Vroom's Expectancy Theory is built on three key components: expectancy, instrumentality, and valence (Vroom, 1964). Expectancy refers to an individual's belief that their efforts will lead to successful performance. Instrumentality is the belief that successful performance will be followed by a specific outcome or reward. Valence is the perceived value or desirability of that outcome. In the context of gamification in higher education, students' engagement is influenced by their expectations regarding the correlation between their efforts, academic performance, and the rewards embedded in the gamified system. If students believe that their efforts will result in successful learning outcomes (expectancy), that these outcomes will lead to desirable rewards or recognition (instrumentality), and they value those rewards (valence), they are more likely to be motivated and engaged in the learning process. Gamification strategies in higher education, such as incorporating meaningful rewards or recognition for academic achievements, aim to enhance students' motivation by aligning with the principles of Expectancy Theory.

The GAFCC model – a gamification design model based on motivational theories

Huang & Hew explores how to design and implement a gamification model, known as the GAFCC design model (Goal-Access-Feedback-Challenge-Collaboration), based on aspects derived from five of the six introduced motivation theories in this paper (Huang & Hew, 2018). The motivation theories include flow theory, goal-setting theory, social comparison theory, self-determination theory, and behavior reinforcement theory. The GAFCC model is proposed as a framework for integrating gamification into educational settings, specifically in the context of flipped learning. Flipped learning is pedagogical approach where students are introduced to the learning material before class with classroom time then being used to deepen understanding through discussion with peers and problem-solving activities facilitated. Flipped learning and online education can therefore be combined.

The researchers advocate a five-stage gamification design procedure, namely examine, decide, match, launch, and evaluate. This procedure is essential for the practical implementation of the GAFCC model. The goal is to enhance student engagement and motivation in out-of-class activities, addressing challenges encountered in flipped classroom scenarios where some students may fail to access learning materials outside of class.

The theoretical components of the GAFCC model are outlined, emphasizing the reification of goals, access, feedback, challenge, and collaboration in a gamified educational environment. The study also incorporates insights from empirical studies, presenting findings from two quasi-experimental studies involving postgraduate students. Results indicate that the GAFCC class demonstrated higher engagement in pre- and post-class activities compared to the control class, producing work of superior quality. The study underscores the potential of gamification, specifically the GAFCC model, as a strategy to motivate students and enhance their participation in out-of-class activities within the context of flipped learning.

Furthermore, the study provides a practical flowchart for designing a gamified course, aligning motivation theories, gamification strategies, and instructional objectives. The five-stage design procedure is detailed, encompassing the examination of instructional objectives and learner context, decision-making regarding motivating elements, alignment of motivating elements with game design elements, implementation of the design, and evaluation of its effectiveness. The proposed model and design procedure offer valuable insights for educators seeking to integrate gamification effectively into their teaching practices.

Table 3: Motivational theories, definitions, and motivating elements.

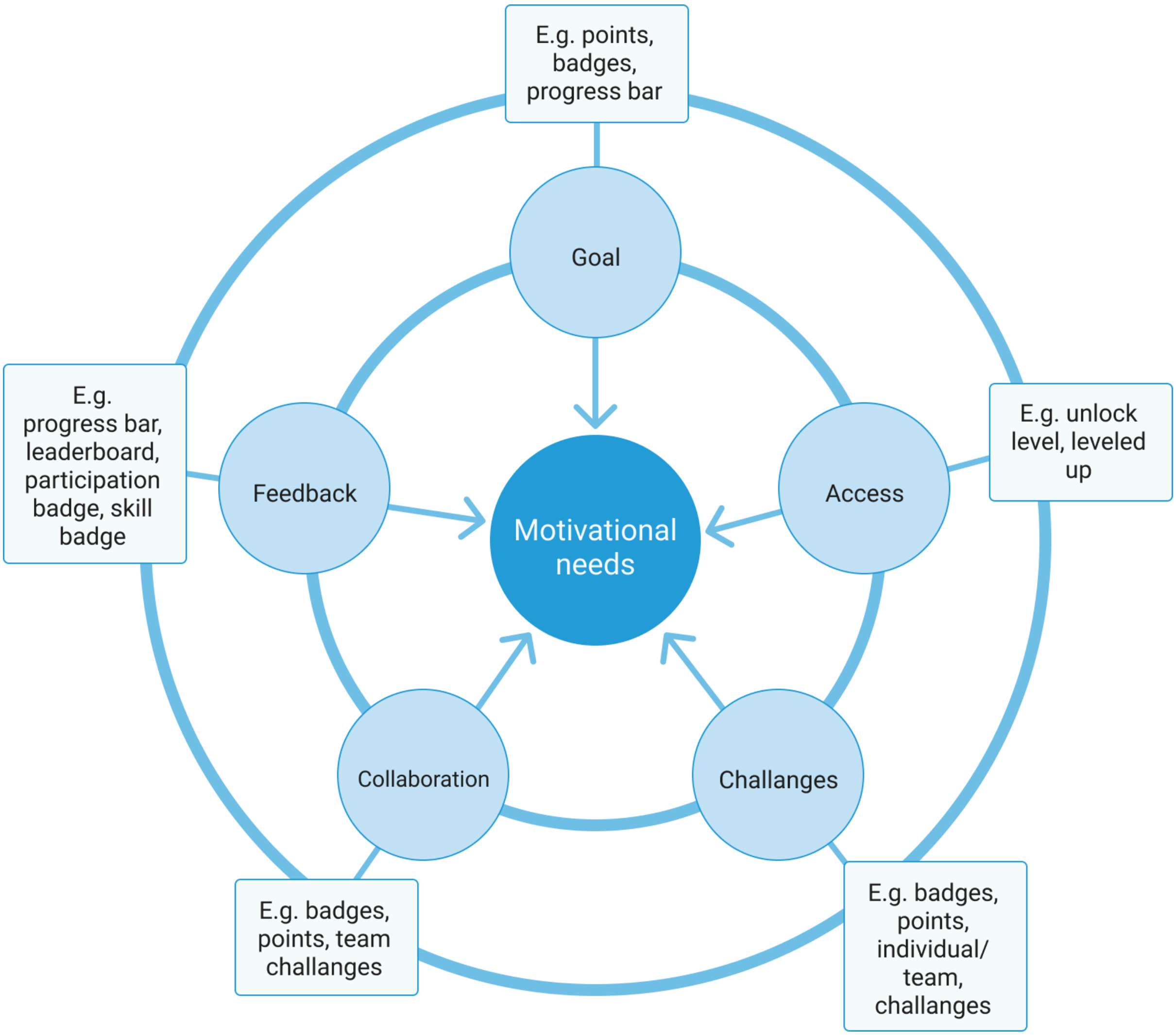
Motivation theory	Definition	Motivating elements
Self-determination theory (Ryan & Deci, 2000)	Explores why people do the things they do and how their motivation is influenced. It suggests that individuals are more likely to be motivated and satisfied when their basic psychological needs for autonomy, competence, and relatedness are fulfilled. Also introduces the concept of intrinsic and extrinsic motivation.	1. Allow learners to choose between several courses of action (access) 2. Offer opportunities for learners to compete with their own selves or with peers (access, competence, feedback) 3. Offer opportunities for learners to work together to achieve a shared goal, or to interact with each other (collaboration)

Flow theory (Csikszcntmihalyi, 1990)	The idea that people are happiest when they are completely absorbed in an activity, experiencing a state of "flow.". Flow happens when the challenge of the task matches your skill level, creating a harmonious and immersive experience.	1. Clear goals (goal) 2. immediate feedback on performance and progress (feedback) 3. suitable level of challenges (challenge, competence)
Goal-setting theory (E. A. Locke & Latham, 2002; E. Locke & Latham, 2006; Schunk & Swartz, 1993)	Suggests that setting specific and challenging goals can significantly enhance motivation and performance. When people have clear objectives that are a bit of a stretch but still achievable, it can inspire them to work harder and smarter to reach those goals. This theory emphasizes the importance of having well-defined targets to drive motivation and improve overall performance.	1. set up long-term and short-term goals (goal) 2. provide feedback on their performance (feedback, competence)
Social comparison theory (Festinger, 1954)	Suggests that people determine their own social and personal worth based on how they stack up against others. In simple terms, we tend to evaluate ourselves by comparing our abilities, opinions, and success to those of others around us. This comparison helps us understand where we stand in various aspects of life and can influence our self-esteem and motivation.	1. self-evaluation (feedback) 2. offer opportunities for learners to compete with peers (access, challenge)
Behavior reinforcement theory (Skinner, 1965, 1989)	Is based on the idea that behaviors can be strengthened or weakened through reinforcement. In simple terms, if you reward a behavior, it's more likely to be repeated; if you punish it, it's less	1. reinforcement (feedback, goal)

	likely. Reinforcement can be positive (adding something desirable) or negative (removing something undesirable).	
Expectancy theory (Vroom, 1964)	Expectancy theory posits that individuals are motivated when they believe their efforts will lead to good performance and, consequently, valuable rewards. It involves three elements: expectancy, instrumentality, and valence, emphasizing the link between effort, performance, and desired outcomes.	1. clear expectations and requirements (goal) 2. consistent rewards and recognition (goal, feedback)

Inspired by and partly adopted from "Table 1" in Huang, B., & Hew, K. F. (2018). Implementing a theory-driven gamification model in higher education flipped courses: Effects on out-of-class activity completion and quality of artifacts. Computers & Education, 125, 254–272. https://doi.org/10.1016/j.compedu.2018.06.018, With some adjustments and expectancy theory as an additional theory..

Figure 1: Model of motivation needs, motivating elements and game design element enablers.



Adopted from "Fig. 1" in Huang, B., & Hew, K. F. (2018). Implementing a theory-driven gamification model in higher education flipped courses: Effects on out-of-class activity completion and quality of artifacts. Computers & Education, 125, 254–272. https://doi.org/10.1016/j.compedu.2018.06.018

Motivation, gamification user styles, and online educational activities

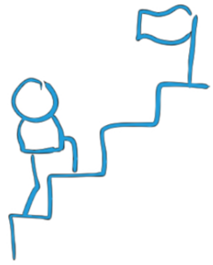
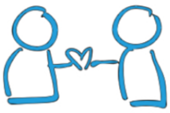
Bartle introduced a classification of player types in video games, encompassing Achievers, Explorers, Socializers, and Killers (Bartle, 1996). Kim extends this taxonomy by incorporating social actions (Compete, Express, Collaborate, Explore) (Kim, 2014). Based on these earlier theories, Marczewski identifies user types for gamification grounded in psychological needs for intrinsic motivation for suggested game mechanics adopted for online learning (Marczewski, 2015). The user types are archetypes and explain the different characteristics and mind-sets game participant usually have. The types consists of the Achievers (motivated by mastery, wants to be challenged, improve themselves and learn new things), Philanthropists (motivated by purpose and meaning, wants to enrich the life of others without any rewards), Free Spirits (motivated by autonomy and self-expression, wants to explore and create), Socializers (motivated by relatedness and wants to interact with others and create social connections), and Players (motivated by rewards, wants to do what it takes to collect rewards from the system) (Marczewski, 2015).

The study of Bovermann & Bastiaens build upon these concepts to explore the relationships between user types and motivational concepts as introduced earlier (Bovermann & Bastiaens, 2020). They tested how the online educational activities forum (posting comments, sharing pictures and videos and ideas in a forum), peer assessment (submitting their work to be peer reviewed), test (quiz with different question types), task (working on a problem alone or in collaboration with others), tutorial (a quick explanation of a concept through sets of short videos) and wiki (an online collaborative tool to make authored web documents) have different agreement levels with the game user styles of Marczewski in a gamified educational environment. The results of the study show that, firstly, Forum engagement is linked to students preferring interaction, teamwork, and knowledge sharing. Subsequently, Peer Assessment connects with those motivated by interacting with others, working in teams, and gaining motivation from being observed. Quiz activity is indicative of students inclined to invest time in experimentation. Task engagement is associated with learners desiring interaction while carefully weighing their actions to understand their investments and goals. Tutorial activity attracts students who seek interaction, steady progress, and alignment with specific learning objectives, displaying a willingness to assist others and find deeper meanings in tasks. Lastly, Wiki engagement corresponds to learners valuing interaction, teamwork, continuous practice, and knowledge sharing.

Students are heterogeneous and are motivated by different things. This is why user styles can be used to identify and categorize the different students with regards to which game mechanic elements and learning activities should be included and targeted towards which user style in the gamification process of mathematics. It is important to note that in reality, people can be a combination of the different player styles.

Table 4: Marczewski player types, their motivation and corresponding activity

Player types	**Motivated by**	**Online gamified educational activity related to increased motivation**

Achievers 	Mastery Challenges Self improvement Learning new things	Forum activity Peer assessment Quiz activity Task engagement Tutorial activity Wiki activity
Philanthropist 	Purpose Meaning Helping others No extrinsic rewards	Peer assessment Task engagement Tutorial activity Wiki activity
Free spirits 	Autonomy Self-expression Exploration Creating	Forum activity Quiz activity Wiki activity
Socializers 	Relatedness Socialization Social connection Teamwork	Forum activity Peer assessment Task engagement Tutorial activity Wiki activity
Players 	Rewards	Quiz activity Tutorial activity

Based on the content of Marczewskis' article: Marczewski, A. (2015). *User Types. In Even Ninja Monkeys Like to Play: Gamification, Game Thinking and Motivational Design* ((1st ed., s. 65-80)). CreateSpace Independent Publishing Platform. https://www.gamified.uk/even-ninja-monkeys-like-to-play/

What gamification is not

After going deep into what gamification is, we also want to include what gamification isn't. Gamification, as cautioned by Werbach and Hunter (Werbach & Hunter, 2012, p. 62), should not involve mindlessly attaching extrinsic motivators to activities that can be motivated intrinsically. To grasp the true essence of gamification, it is essential to debunk misconceptions. According to Bernardes et al., gamification is not merely about badges, points, and rewards; these elements, often associated with gamification, represent the least exciting and least useful aspects of games (Bernardes et al., 2022, p. 667). Much of the research presented earlier is overrepresented by examples mostly related to PBL elements (points, badges, leaderboards). Be aware of limiting educational activities to only be supported by these elements when making a gamified learning experience. Don't feel restricted by these examples and explore other gamification elements as well. The power of game-based thinking lies in engagement, storytelling, visualization of characters, and problem-solving.

That gamification with its often-playful elements is trivializing learning is another misconception, as gamification is a serious approach to accelerating the learning experience, teaching complex subjects, and fostering systems thinking. It is also not a new concept, with historical roots traced back to military war games and simulations (Kapp, 2012). Teachers and trainers have long employed game-like techniques, such as embedding stories and creating challenges, which are integral elements of gamification withing education. To combat educators and students sometimes skepticism towards gamification, the concept of the magic circle can be practiced and introduced as a way of actively encouraging playfulness (as mentioned under the section of playfulness).

Nine gamification heuristics for effective gamification and the challenges they address

Gamification is recognized as a potential remedy for declining learner motivation, yet the efficacy of gamification remains inconclusive in prior research (van Roy & Zaman, 2017). Van Roy and Zaman contend that the focus should be on understanding how gamification operates. Drawing from Self-Determination Theory (SDT), they propose 9 Gamification Heuristics to guide effective gamification implementation in educational contexts, emphasizing autonomous motivation and addressing fundamental psychological needs (van Roy & Zaman, 2017). These heuristics include not forcing users into gamified systems, offering suitable options, setting challenging yet manageable goals, providing positive feedback mechanisms, fostering user interaction and a sense of belonging, considering other psychological needs, facilitating goal-oriented motivation, and accommodating personal differences and preferences (see table 5). Holistically understanding gamification also involves considering user characteristics, system properties, and contextual demands. These heuristics can both be used when designing for a gamified learning environment in mathematics, and when evaluating it.

Table 5: nine gamification heuristics for effective gamification

Challenge	**Heuristic**
Support learner's autonomy	**#1 Avoid obligatory uses** Avoid forcing the user to use (a part of) the gamified system in order not to give them the feeling of being controlled.
	#2 Provide a moderate amount of meaningful options Find the sweet spot between supporting users' autonomy by providing them with at least one option that is meaningful and complies with their values, while avoiding placing them in a dilemma by offering too many options.
Support learner's competence	**#3 Set challenging, but manageable goals** In order to support the user's feelings of competence, create tasks that pose a significant challenge while remaining perceived as feasible to fulfil.
	#4 Provide positive, competence-related feedback Support feelings of competence by integrating feedback mechanisms that positively inform learners about their progress in gaining competences, and avoid negative feedback.
Support learner's relatedness	**#5 Facilitate social interaction** Eliminate factors that hinder social interactions between users, and facilitate them to interact and support their feelings of relatedness instead.
Support interplay between needs	**#6 When supporting a particular psychological need, wary to not thwart the other needs** When designing a specific element in order to support users in

	one of their basic psychological needs, wary to not thwart one of the other needs.
Integration of gamification into the activity	**#7 Align gamification with the goal of the activity in question** Alight the motivational pull of gamification with the goal of the activity, as such tuning gamification to both facilitate motivation and goal achievement.
Contextual characteristics	**#8 Create a need-supporting context** In order to support the user's basic psychological needs, the gamified system should be implemented in a setting that is perceived as open and supporting as opposed to controlling.
Individual characteristics	**#9 Make the system flexible** To account for personal differences, the gamified system should be flexible and adaptable in order to comply with the users' personal needs and preferences.

Adopted from "Table 1" in van Roy, R., & Zaman, B. (2017). Why Gamification Fails in Education and How to Make It Successful: Introducing Nine Gamification Heuristics Based on Self-Determination Theory (s. 485–509). https://doi.org/10.1007/978-3-319-51645-5_22

Research on applying gamification on mathematics in higher education

Introduction

Most research on gamification in education is done on the higher educational level (Torres-Toukoumidis et al., 2021). However, Yiğ & SezgiN found that the greater part of research within gamification of mathematics more specifically is still done on primary school level (Yiğ & SezgiN, 2021). Several literature reviews suggest that there is a general lack of studies on gamification within certain STEM areas and higher education (Khaldi et al., 2023; Ortiz Rojas et al., 2016), and Rojas et al. points more specifically to a lack of studies that identify the particular game element associated with the positive differential impact on student performance; a lack of validated psychometric measurements, and lack of focus on student variables that could/should be taken into account as mediating/moderating variables clarifying the impact of gamification in the HE focus on STEM learning and teaching (Ortiz Rojas et al., 2016). This means that even though education is modernizing and digitalizing, good examples of gamification in HE mathematics relevant to the Pythagoras project are scarce. However, we will still try to present some empirical studies and case studies that explain how they applied gamification in a mathematics context within HE successfully. Each study will be presented with its main applications of gamification, its corresponding results, and what could be learned from this.

Teaching business mathematics using Kahoot!, Socrative, Quizlet, Quizziz and Showbie

The article "A Case Study On Applying Gamification Tools In Business Mathematics For Higher Education Students" by Lawrence et al. presents a case study on the application of gamification tools in the context of Business Mathematics for higher education students (Lawrance et al., 2021). The study involved undergraduate students in the Bachelor of Commerce program at Grace International Academy, with 16 out of 17 students providing feedback. The gamification tools used include Kahoot!, Socrative, Quizlet, Quizizz, and Showbie.

These gamification tools were implemented to enhance students' motivation, engagement, critical thinking, and problem-solving skills. The study used questionnaires, observations, and interviews to collect data. Online feedback was conducted using Google Forms, as part of the semester completion for the Business Mathematics course.

The different gamification tools and how they were used to gamify is explained here:

Kahoot!:

- **Usage:** Kahoot! is a game-based learning platform that engages students through quizzes, discussions, and surveys.
- **Gamification Features:**
  - **Points System:** Students earn points for correct answers.
  - **Leaderboard:** Displays scores and names, fostering competition.
  - **Music:** Adds enthusiasm to classroom activities.

Socrative:

- **Usage:** Socrative is a formative assessment tool that supports collaborative activities.
- **Gamification Features:**
  - **Space Race Activity:** Creates a fun and engaging game event.
  - **Collaborative Activity:** Promotes teamwork in learning.

Quizlet:

- **Usage:** Quizlet is a web-based application for interactive study tools and games.
- **Gamification Features:**
    - **Live Collaborative Classroom Game:** Forms teams randomly to match terms with definitions.
    - **Interactive Study and Play Modes:** Supports student-paced activities.

Formative:

- **Usage:** Formative is an interactive tool to assess and monitor student performance.
- **Gamification Features:**
    - **Live Responses:** Provides real-time feedback to students.
    - **Tracks Student Work:** Maintains a record of students' performance.

Quizizz:

- **Usage:** Quizizz is a free tool for teaching and learning, allowing the creation of quizzes.
- **Gamification Features:**
    - **Self-Paced Learning:** Allows students to engage at their own pace.
    - **Points and Leaderboard:** Motivates students with points and displays leaderboard.

Showbie:

- **Usage:** Showbie is an application for assigning, collecting, and reviewing student work.
- **Gamification Features:**
    - **Organizes Student Work:** Categorizes work by classes and assignments.
    - **Feedback Options:** Provides grades and feedback in various formats (voice notes, annotations, videos).

Key findings from this study include:

Effectiveness of Gamification Tools:

- 79% of students found Kahoot! effective for learning Business Mathematics.
- 37% found Socrative effective for learning Business Mathematics.
- 52.2% found Quizlet effective for learning Business Mathematics.
- 47.3% found Formative effective for learning Business Mathematics.
- 66.4% found Quizizz effective for learning Business Mathematics.
- 37.06% found Showbie effective for learning Business Mathematics.

Motivation Using Gamification Tools:

- 81.4% of students were motivated by the gamification tools.

Understanding Mathematical Concepts Using Quizlet:

- 87% of students agreed that Quizlet was effective in understanding mathematical concepts.

Factors Boosting Engagement:

- 88% were motivated by game points.
- 44% preferred student-paced activities.
- 50% preferred teacher-paced activities.
- 50% reacted positively to team activities.
- 75% found leaderboards motivating.
- 56% preferred simple questions.
- 56% preferred difficult questions.
- 75% preferred individual activities.
- 50% preferred team activities.
- 69% were willing to participate again.
- 56% preferred music in activities.
- 56% preferred quick feedback.

Increase in Critical Thinking and Problem-Solving Skills:

- 87% of students agreed that gamification tools increased critical thinking and problem-solving skills.

In conclusion, the gamification tools significantly enhanced students' motivation, engagement, critical thinking, and problem-solving skills in the context of Business Mathematics. Despite challenges and impediments, both traditional classrooms and online gamification tools were found to be effective in teaching and learning Business Mathematics (Lawrance et al., 2021). The digital gamification tools Kahoot!, Socrative, Quizlet, Quizizz, Showbie and the game mechanics they support could be applied to more mathematics courses.

Big Data And Analytics Course With The Use Of "Jeopardy-Style Classroom Quiz"

This study explores the application of a 'Jeopardy-style classroom quiz' as a gamification tool in the context of a 'Big Data and Analytics' course (Mandke & Jahirabadkar, 2021). Jeopardy is a quiz show that has a unique answer-and-question format in which contestants are presented with clues in the form of answers and must phrase their responses in the form of a question. Written by Sakshi Mandke and Sunita Jahirabadkar, the study is conducted at Cummins College of Engineering for Women in Pune, India. The case study revolves around the 'Big Data and Analytics' course. The exact number of participants is not specified.

Gamification Tools Used:

- The primary gamification tool employed is a 'Jeopardy-style classroom quiz.'

Contribution of Tools to Gamification:

- The Jeopardy-style quiz is designed to be played individually or in teams, promoting active participation and teamwork.
- The quiz format allows instructors to assess students' knowledge during various tests, fostering engagement and participation.

Success Factors and Key Contributions to Gamification:

- **Improvement in Team Building and Leadership:** The gamification tool contributes to enhancing team-building and leadership qualities among students.
- **Technical and Interpersonal Skills:** Game-based learning positively influences both technical knowledge and interpersonal skills.
- **Engagement during Classroom Sessions:** Students find the gamification technique interesting, contributing to increased engagement during online teaching classes.
- **Identifying Difficult Topics:** Instructors can identify topics where students face difficulty through the game, enabling targeted revision sessions.

Students expressed interest in repeating the gamified learning activity for other topics, indicating a positive reception. In conclusion, the successful implementation of the Jeopardy-style quiz in the 'Big Data and Analytics' course is attributed to its ability to enhance teamwork, leadership skills, technical knowledge, and overall engagement among students. The positive feedback from students and their willingness to repeat the activity for other topics underline the effectiveness of gamification in the educational context.

Calculus And The Use of an RPG Educational Digital Game

Cultivating student engagement, particularly in challenging subjects like Calculus, remains a pressing concern in higher education. This study by Cezar et al. delves into the transformative potential of gamification by introducing an RPG-style educational digital game, "The Fellowship of the Calculus," in a Calculus course (Cezar et al., 2022). The objective is to explore the symbiotic relationship between gamification elements and academic standards to enhance student motivation, interaction, and learning outcomes.

Grounded in the principles of gamification, the game's development hinged on three core elements: Mechanics, Aesthetics, and Thoughts. Leveraging RPG (Role-Playing Game) format, students assumed roles inspired by eminent mathematicians, navigating a 3D scenario interwoven with Calculus concepts. A pivotal aspect was the integration of game aesthetics akin to commercial counterparts, ensuring familiarity and resonance with the digital-native cohort. The game unfolded as a narrative, seamlessly aligning with the theoretical content of Calculus.

The study's success lay in its ability to foster a positive learning environment through imersiveness and engagement. Notably, 73% of participants acknowledged the game's impact on the learning process, attesting to its effectiveness in transforming a traditionally challenging subject into an engaging endeavor. Continuous refinement, guided by student feedback, emerged as a key contributor to success. The iterative process of the game development addressed concerns related to movement dynamics, interface clarity, and overall game design, reaffirming the importance of adaptability in gamification endeavors.

Beyond its role as an educational tool, the gamified approach triggered a shift in students' study habits. A noteworthy 35% reported engaging with Calculus solely due to the game. The study sets the stage for future research, with implications for accessibility enhancements, such as a smartphone application, and expanded data collection. This gamification odyssey underscores its potential as a catalyst for positive change in mathematics education, offering insights that contribute to the evolving landscape of gamified learning in higher education.

Engineering - Gamified Activities and Online Platforms in Project-Based Learning

A study conducted by Leung & Pluskwik aimed to assess the effectiveness and student engagement with gamification tools in a project-based learning (PBL) engineering classroom setting (Leung & Pluskwik, 2018). The focus was on utilizing game-based elements, specifically online audience response systems with automated feedback, to enhance the existing active and collaborative learning environment of the Iron Range Engineering (IRE) program at Minnesota State University, Mankato. The IRE program, characterized by its project-based learning model, involves students in industry-focused design projects during their junior and senior years, emphasizing the integration of technical and professional knowledge.

Throughout the Fall 2017 semester, five gamification activities were implemented across six technical courses, including Signals & Systems, Engineering Economics, Statistics, Linear Control Systems, Lean Principles, and

Electric Machines. These activities incorporated three game-based online platforms and two hands-on activities, involving approximately 8 - 12 participants in each class, totaling around 300 student gaming interaction instances. The gamification tools provided real-time assessment, enabling instant feedback on students' understanding of technical concepts. The study collected feedback through student surveys, reflections from both students and faculty, and automated data from the game programs.

Preliminary analysis of the collected data suggested several positive outcomes. The gamification tools contributed to increased learner motivation, facilitated enhanced review of technical content, and fostered a positive classroom atmosphere. Faculty reflections highlighted the effectiveness of games in identifying students who successfully mastered concepts, enabling instructors to structure peer-to-peer active learning opportunities more effectively. The study indicated that the gamification tools served as a valuable addition to the PBL environment at Iron Range Engineering, offering significant value to the overall learning process.

Mathematics In Engineering and Robots

Hilario et al. did a pilot project where they incorporated robots and interdisciplinary collaboration to gamify a mathematics course (Hilario et al., 2022). The pilot project took place within a course in Industrial Design and Product Development within the framework of the EXPLORIA project at CEU University. The study involved a pilot project aimed at rethinking the learning processes of university students, focusing on STEAM (science, technology, engineering, arts, and mathematics) subjects. The project, specifically implemented in the context of a transportation challenge, engaged students in the application of mathematics and physics to solve real-world problems using LEGO EV3 robots.

The gamification tools employed in this study included educational robotics, particularly LEGO Mindstorms EV3, and the GeoGebra applet for Bézier curve design. The transportation challenge aimed to teach students Bézier curves and their applications in physics through an active methodology. The results of the pilot study indicated an increase in motivation among students due to the use of robots and the realistic context of the challenge.

The success of gamification in this context can be attributed to the integration of various STEAM disciplines, active learning methodologies, and the practical application of mathematical and physical concepts. The interdisciplinary nature of the project, combining mathematics, physics, and robotics, provided a holistic learning experience. The use of LEGO Mindstorms EV3, known for its effectiveness in educational robotics, contributed to promoting computational thinking and active learning.

Key elements contributing to the success of gamification included the creation of a learning community, where students worked collaboratively and discovered knowledge with the support of mentors. The study also emphasized the importance of connecting theoretical knowledge with real-world applications, enhancing students' understanding and positive attitudes toward mathematics.

The transportation challenge itself involved a series of sessions covering mathematical and physical concepts, robotics, and the practical application of Bézier curves. The GeoGebra applet facilitated the design of Bézier curves, and a MATLAB application controlled the LEGO EV3 robot to follow the trajectories. The challenge not only enhanced students' technical skills but also fostered critical thinking, creativity, and problem-solving abilities.

In the questionnaire administered to students, positive feedback was received regarding the impact of gamification tools on their learning experience. Students expressed increased interest in mathematics, particularly in the context of robotics, and acknowledged the relevance of the practical application of theoretical concepts. Some students highlighted the dynamic and engaging nature of the experiment, emphasizing its positive contribution to their understanding of real-world applications of mathematics.

As a pilot project within the larger EXPLORIA initiative, this study provides insights into the potential of gamification, particularly in STEAM subjects, to enhance students' motivation, understanding, and positive attitudes toward traditionally challenging disciplines like mathematics and physics. The interdisciplinary and integrative approach, along with the use of educational robotics, offers a promising avenue for future educational strategies (Hilario et al., 2022).

Other Interesting Cases

Because of the limitations in research specifically within HE gamification mathematics, we will also include some related studies that do not check all the inclusion criteria's but are still interesting for the Pythagoras project. These include successful implementations of Massive Open Online Courses, web-based platforms, and Intelligent Tutoring systems using AI in contexts related to mathematics in higher education.

MOOCs in A Mathematics Education Context

Research by Yiğ delves into the exploration of Massive Open Online Courses (MOOCs) as a medium for teaching mathematics for to-be-teachers (Yığ, 2022). While this study is not exploring the concept of gamification directly, MOOCs can often be the platform where gamified activities take place in distance learning and is therefore interesting to include. The research focuses on understanding the experiences of 30 teacher candidates enrolled in

MOOCs with a mathematics education context, with the aim of proposing reflections on design principles for MOOCs in mathematics education. The investigation explores the general course structure, pedagogical orientations, and mathematical connection elements embedded in these MOOCs.

The MOOCs in this study incorporated diverse pedagogical approaches, assessment methods, and motivational strategies. Notably, the study introduces the concept of "mathematical connection" as a crucial design component in MOOCs. Mathematical connection refers to a bridge that connects mathematical facts, procedures, representations, or meanings. Therefore, an alternative way to describe mathematical connections is to think of them as parts of a schema or as clusters of related schemata in a mental network (Eli et al., 2013). There exists different mathematical connection strategies that will not be elaborated upon here.

The analysis indicates that connections with real-life scenarios were the most intensively observed mathematical connection strategy. Additionally, connections between different mathematics subject areas and various representations were prevalent. The participants emphasized the importance of utilizing different representations, such as tables, graphics, and models, to enhance the learning experience. Interestingly, the research identified the use of MOOCs to foster mathematical connection skills, facilitating the interrelation of different forms of mathematical representation and promoting connections between formal mathematics and everyday life.

Moreover, the study provides valuable insights into the structural features of course videos, including visual and audiovisual elements, and video/course durations. Participants highlighted the significance of instructor engagement, appropriate tone of voice, and engaging visual materials. 4–5-minute videos were too short to properly learn the course subject, while 20-minute videos quickly reduced the attention of the viewer. The research therefore underscores the importance of flexible video durations to cater to diverse learning needs.

Gamit! Mathematics Gamification Resulting in A Web Platform

This study, as presented by Rincon-Flores et al., delves into the transformative effects of gamification on high school students' attitudes toward mathematics (Rincon-Flores et al., 2023). The collaboration between Tecnológico de Monterrey, Mexico, and the Universidad de Lima, Peru, led to the development and implementation of Gamit!, a web-based gamification platform. The research involved 454 high school students, utilizing a reward system managed through Gamit! The study explores the dynamics of gamification tools such as badges, leaderboards, and avatars in modifying the learning environment and student attitudes.

The participants, primarily aged between 15 and 16, showed improved attitudes toward mathematics, with reductions in anxiety and increased enjoyment. The results indicated a nuanced relationship between gamification dimensions, revealing positive correlations between anxiety and enjoyment. Professors noted benefits in managing gamification efficiently through Gamit! - fostering a dynamic learning environment. The study considered various factors, such as scholarship status, gender, program type, and semester, revealing nuanced effects on students' attitudes. Notably, non-scholarship students exhibited unfavorable results in motivation and procrastination dimensions, emphasizing the need for targeted interventions and further research on this topic.

Gamit! proved effective in enhancing students' attention, engagement, and resilience, as highlighted in focus group discussions. The platform's features, including leaderboards, badges, and avatars, contributed to a positive classroom atmosphere. Students reported emotions of pride, joy, and motivation, indicating the success of the reward system. Professors acknowledged increased participation, proposing a connection between gamification and improved classroom dynamics. Additionally, the study explored the platform's usability, revealing positive feedback on progress tracking but suggestions for interface improvements.

In conclusion, gamification, when implemented through the Gamit! platform, positively influenced high school students' attitudes toward mathematics. The study emphasizes the importance of collaboration between educational institutions in developing innovative strategies. While highlighting the success of gamification, the research underscores the need for continued exploration of its impact on learning outcomes across disciplines and educational levels.

AI-Assistant in an Intelligent Tutoring System for Physics

An interesting study done by Tan & Cheah explores the ongoing design efforts to create an AI-enabled gamified web-based online learning application for university introductory physics courses (Tan & Cheah, 2021). The application that was developed aimed to address the diverse learning needs of students, particularly those with a weak background in physics. The design incorporates gamified elements, including incremental difficulties, points and streaks, leaderboards, and a gamified user interface. The rationale behind these elements is to engage students emotionally, encourage hands-on practice, and strengthen their domain knowledge.

The incorporation of AI into this gamification platform is a key focus, introducing various AI models such as the “learner model”, “pedagogy model”, and “domain model”. These could be interactive with in different ways by the students. The AI aims to act as a personalized tutor, offering step-by-step instructions, feedback, and resources based on individual student progress. The platform also serves as an efficient feedback tool for teachers to optimize and redesign the curriculum according to individual learner needs. The potential benefits of AI as mentioned by Tan & Cheah include adapting assessments, providing tailored assistance, and offering real-time feedback to instructors.

The article highlights the challenges of implementing intelligent tutoring systems (ITS) and the limited widespread adoption of AI in education. It emphasizes the importance of addressing issues related to self-regulation, self-efficacy, and motivation among students (Tan & Cheah, 2021). The proposed solution to this involves combining gamification with ITS to enhance student engagement and address low self-efficacy in learning physics.

The study also describes the gamified application's design features in detail, such as short quizzes with incremental difficulties, points and streaks for motivation, leaderboards for competition, and a gamified user interface for visual appeal. The design aims to balance attractiveness for students (visuals) with the practical constraints of portability. The application's effectiveness is yet to be assessed, with potential research questions exploring its impact on self-directed learning, the role of portability, the interaction of gamification dynamics, and the effectiveness of a gamified ITS as an intervention for at-risk students.

Further, the study proposes future enhancements in the platform, including an early warning system for identifying at-risk students, automated question generation, and continuous development based on feedback and evaluation (Tan & Cheah, 2021). The ongoing research positions gamification and AI as complementary elements in shaping a positive and effective learning experience, particularly in challenging subjects like physics.

Challenges and Opportunities to Gamification

Even though there is a lack of research within gamification of mathematics within HE, research on gamification of HE in general shows predominantly positive results for the motivation, attendance and learning outcomes of students (Ortiz Rojas et al., 2016). The case studies introduced in this paper also show a favorability towards increased gamification of mathematics. So why are not more courses already gamified? To answer this question, we must explore some of the challenges associated with the process of gamification, and the research supporting it.

Similar barriers to gamification seem to be identified in multiple studies. The biggest barriers seems to be a lack of funding, lack of time to implement a gamification process, limited knowledge on the topic, stigma attached to students perceptions of gamification, lack of fit between gamification and the course content, a lack of guidance from gamification experts and professional training of the teachers, a lack of good and understandable examples of games, lack of flexible platform to implement the gamification on, and the fear of change by some teachers (An et al., 2021; Oscar Bernardes et al., 2022; Watson-Huggins & Trotman, 2019).

Sanchez & Lee elaborate on the challenges to research within the field of game-based learning and summarizes them as: (1) the blurred distinctions between games and similar training media, (2) the growing need to identify game characteristics for specific game based interventions, (3) the lack of evidence for the mechanisms through which game-based learning is experienced, (4) the continued ambiguity in the measurement of different training outcomes, and (5) the need to draw accurate implications from research-based findings (Oscar Bernardes et al., 2022, p. 543). They propose that to solve this, future research should (1) distinguish games from other media, (2) focus on game design and characteristics, (3) understand the learning process of games, (4) explain the effectiveness of games, and (5) draw accurate implications (Oscar Bernardes et al., 2022, p. 543). These are sound recommendations for future research, but in the case of the Pythagoras project, distinguishing games from other media would exclude important learnings and examples that could be applied in a real-life mathematics context.

Gamified Learning Environments (GLEs) in Higher Education present a nuanced landscape, offering advantages while grappling with inherent challenges. GLEs have seen a surge in research, often initiated by educators well-versed in research methodologies. Its strengths and weaknesses are studied by Giraldèz et al. (Oscar Bernardes et al., 2022, p. 63-). The strengths they identifies are improved academic performance and/or learning, improvement of psychological variables, creativity and cooperative work, improvement of students' commitment to the subject area, improved student motivation, and improvement in the evaluation processes of the subject area (Oscar Bernardes et al., 2022, p. 63-). The weaknesses they point out are a lack of standardized research protocols, little training in gamification by the teaching community, incorrect or inappropriate EAG designs, difficulty in improving the motivation of all students, mandatory participation of students in gamification to the detriment of voluntariness, low sample size of research, absence of a control group in the research and pre & post-test design, and a preferential use of self-reported evaluation over other data collection instruments (Oscar Bernardes et al., 2022, p. 63-).

The utilization of game-based approaches has been suggested to be a passing trend, as indicated by some scholars (Calleja & Callus, 2011). Critics argue that game elements might function more as attractive embellishments, diverting learners' attention away from relevant content (Fisch, 2005). Despite the growing body of research on game-based learning, there exists conflicting evidence regarding the positive impact of games on organizational outcomes such as learning (Akl et al., 2013; Breuer & Bente, 2010; Harris, 2008). This discrepancy is worrisome as ongoing disagreements might impede advancements in game-based learning (Oscar Bernardes et al., 2022, p. 542-543).

When designing serious games, Kara empathizes several key considerations and opportunities (Oscar Bernardes et al., 2022, p.464). These considerations can be applied to the whole process of gamification as a road to game-based learning. Firstly, an adequate budget should be allocated for designers, developers, and practitioners. Secondly, it is important to involve the target audience, including adult student play testers, which proves invaluable in shaping the game's design based on their pedagogical expectations and preferences. Lastly, the composition of the serious game design team, comprising educators, game designers, and possibly developers, significantly influences the game's effectiveness in both educational and entertainment aspects. This collaborative approach, supported by advancements in game technologies, enhances the creation of realistic gaming worlds, fostering a more responsible and engaging player experience.

Conclusion

In conclusion, the exploration of gamification in higher education, specifically in the context of mathematics, reveals a promising yet underutilized avenue to enhance student motivation and engagement. The literature review underscores the significance of incorporating gamification elements in online and hybrid education, citing potential benefits such as improved academic performance, enhanced psychological variables, and increased student commitment. Scholars like Alt emphasize the effectiveness of gamification in teaching mathematics, emphasizing key principles for successful implementation. The diverse definitions of gamification, including its application of game elements in non-game contexts, highlight its transformative nature.

However, many challenges exist in the gamification landscape, including a lack of funding, limited time for implementation, and a shortage of knowledge on the topic. Identifies barriers, such as the stigma associated with students' perceptions of gamification and the fear of change among teachers, underscore the need for careful consideration and strategic planning.

Case studies and initiatives on gamification in mathematics education are scarce, highlighting the need for more research and practical applications. The strengths and weaknesses of Gamified Learning Environments (GLEs) further emphasize the need for standardized research protocols and increased training within the teaching community. The potential passing trend of game-based approaches raises concerns, suggesting the importance of careful integration to avoid distractions from relevant content.

The mentioned literature, theories, and models can serve as valuable resources for gamifying courses in mathematics within higher education. However, it is crucial to navigate both opportunities and barriers with awareness.

To enable gamification to happen, Kara's considerations and opportunities for designing serious games emphasize the importance of budget allocation, involvement of the target audience, and a collaborative approach involving educators, game designers, and developers. The case studies together with the literature presented aim to inform, inspire, and help visualize successful gamification processes within online and hybrid education, offering a roadmap for implementing and realizing positive outcomes in the realm of mathematics education.

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