An ongoing project at Karlstad University

Designing for a combined use of a dynamic mathematics software environment and a computer-aided assessment system

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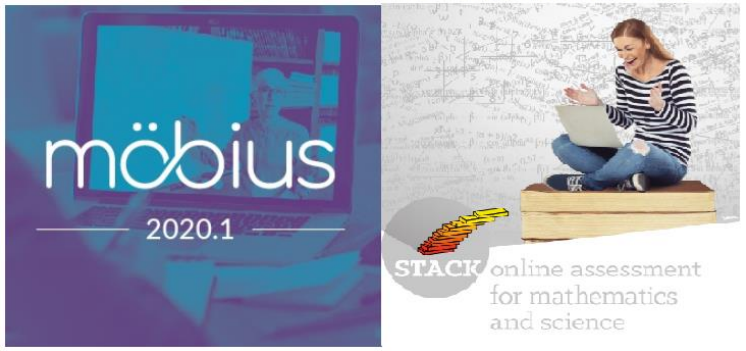
The aim of the project at Karlstad University

The aim of the project is to develop principles to guide the design of a technology-enhanced learning environment in which DMS tasks are embedded in a CAA system that (automatically) provides elaborated formative feedback based on students' responses

Dynamic Mathematics Software (DMS)

Computer Aided Assessment (CAA) Systems

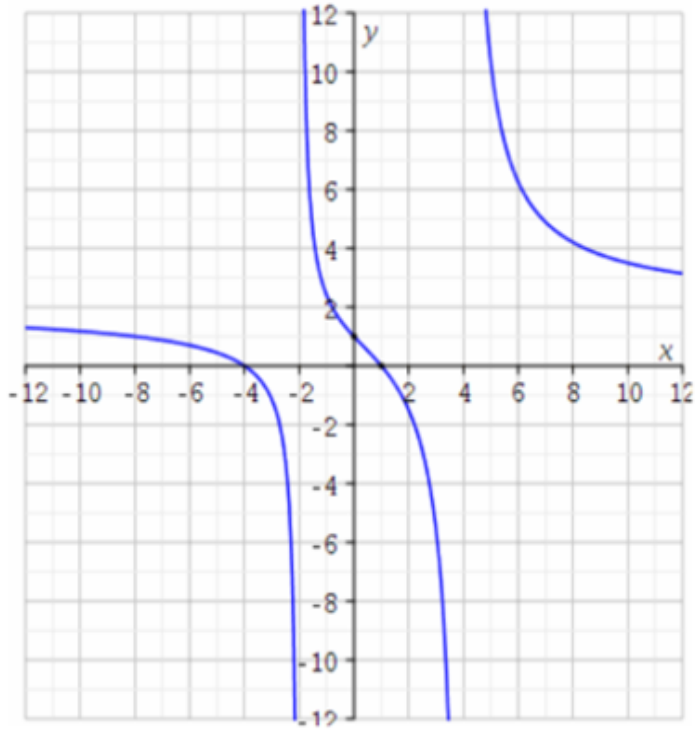


Some examples of tasks used in the project:

Translation task: From graph to formula

The task below (Task 5) is an example of a ‘translation task’ intended to be solved in groups.

Below is the graph of a function g.



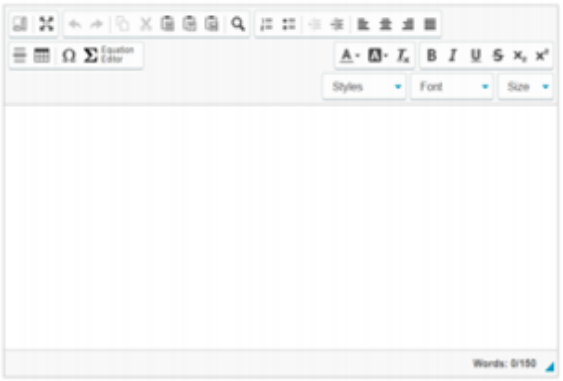
a) Use the graph to determine the function formula.
Check your suggestion in GeoGebra before submitting it a
an answer to the task.

Group agreed response

$g(x) =$

b) Give a brief account of how you used the graph to determine the function formula.

Group agreed response



How did the students use the graph to determine the function formula?

Some findings from a pilot study (256 students, 98 groups)

Code	Description of what students referred to	Total (number of groups)
VA	The vertical asymptotes, x = -2 and x = 4, or that the function is undefined for these values	84 (95,5%)
HA	The horizontal asymptote, y = 2	47 (53,4%)
Zs	The zeros, x = -4 and x = 1	44 (50,0%)
1P	One further point	25 (28,4%)
GG	GeoGebra	15 (17,0%)
ES	System of equations or two further points	20 (22,7%)

Possible feedback

Instead of asking for an explanation, ask students to declare what they used by choosing among various suggested options.

- The zeros
- The vertical asymptotes
- The horizontal asymptotes
- One more point that is not a zero
- Three points that are not zeros
- GeoGebra

Depending on their response, they will receive different elaborated feedback.

For example, if they not have used the horizontal asymptote, they will be asked to solve a new task in which they are (explicitly) asked to utilize the horizontal asymptote.

Example-generating task

The task below (Task 7) is an example of a task in which students are asked to generate examples that fulfil certain conditions. In this task students received different values of the asymptotes, and were supposed to provide individual answers.

Give examples of two different functions, f and g, both of which have

- two vertical asymptotes, $x = -6$ and $x = 3$, as well as
- a horizontal asymptote, $y = 2$.

Note:

- Group members may have received different asymptotes.
- Check in GeoGebra if your suggested functions really have the given asymptotes.

Individual response:

$f(x) =$

$g(x) =$

Most of the students managed to provide two correct examples. However, almost all students provided the same type of formula in both their examples.

Possible feedback

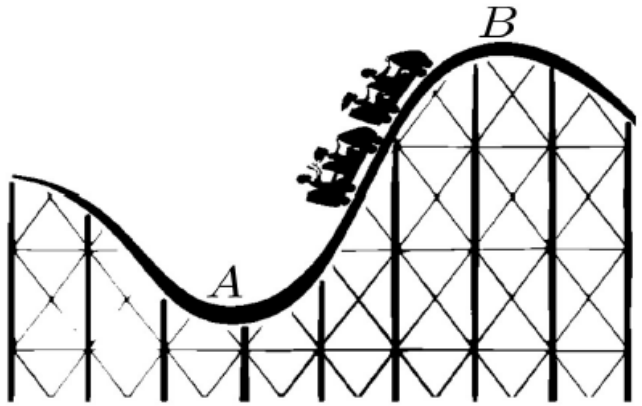
Since we think that it is instructive for students to realize that there are various ways of thinking that results in different types of formula, it would have been great if the CAA system could recognize the type of formula used by a student.

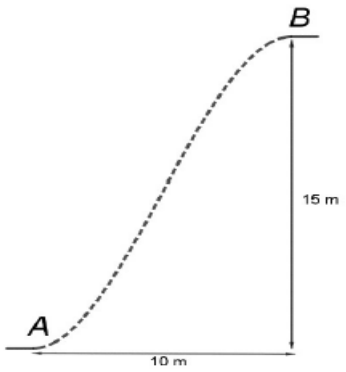
So, for example, if a student uses a formula of the following type: $f(x) = \frac{ax+b}{(x+6)(x-3)} + 2$ (in both examples), the elaborated feedback could be something like:

"Great, the answers are correct. However, another correct answer could be: $f(x) = \frac{2x^2}{(x+6)(x-3)}$.

How do you think a student who came up with this answer has been reasoning? Now, use this strategy to provide an example of a function with the following asymptotes…"

One more example-generating task





Parts of a structure in a roller coaster need to be replaced. It is the part between points *A* and *B* (see figures below) to be replaced. Your task is to determine two different function formulas (*f* and *g*) that can be used during construction so that there is a smooth transition in points *A* and *B*.

- Use the measurements in the sketch to the right below.
- Use GeoGebra to check if your suggested expressions are suitable before submitting them as answers to the task.
- There are a number of clues. However, it is important that you only use them when needed. So, try first to solve the problem without using any clue. If this does not work, then use as few clues as possible.

Group answer:

$f(x)$ = _____

$g(x)$ = _____

Hint	Penalty	
Clue 1:	0.0	View Hint
Clue 2:	0.0	View Hint
Clue 3:	0.0	View Hint

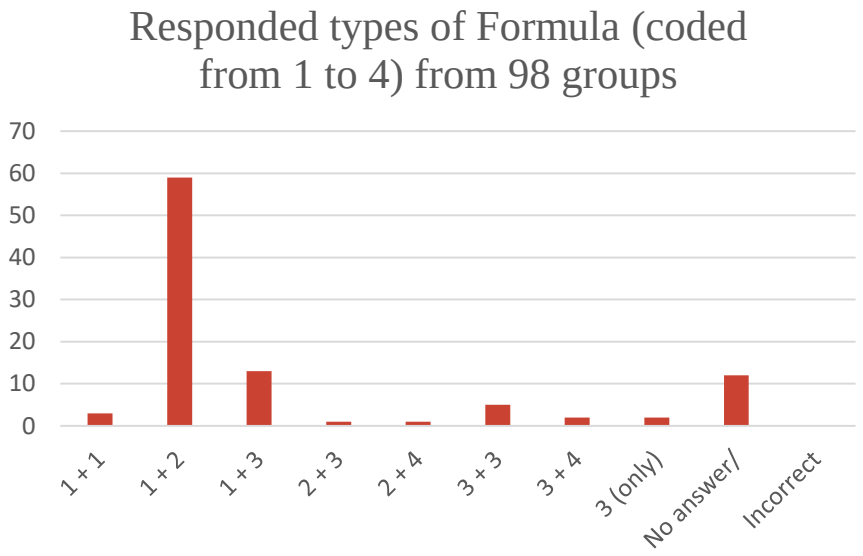
Hint 1: Start by placing points *A* and *B* in a coordinate system with the point *A* at the origin.

Hint 2: Use the conditions: $f(0)=0$, $f(10)=15$, $f'(0)=0$ och $f'(10)=0$

Hint 3: Suitable function types are polynomials of degree three and sine function

Student responses

1. A sine function
2. A cosine function
3. A Polynomial function, degree 3
4. A Polynomial function, degree 4 or 5



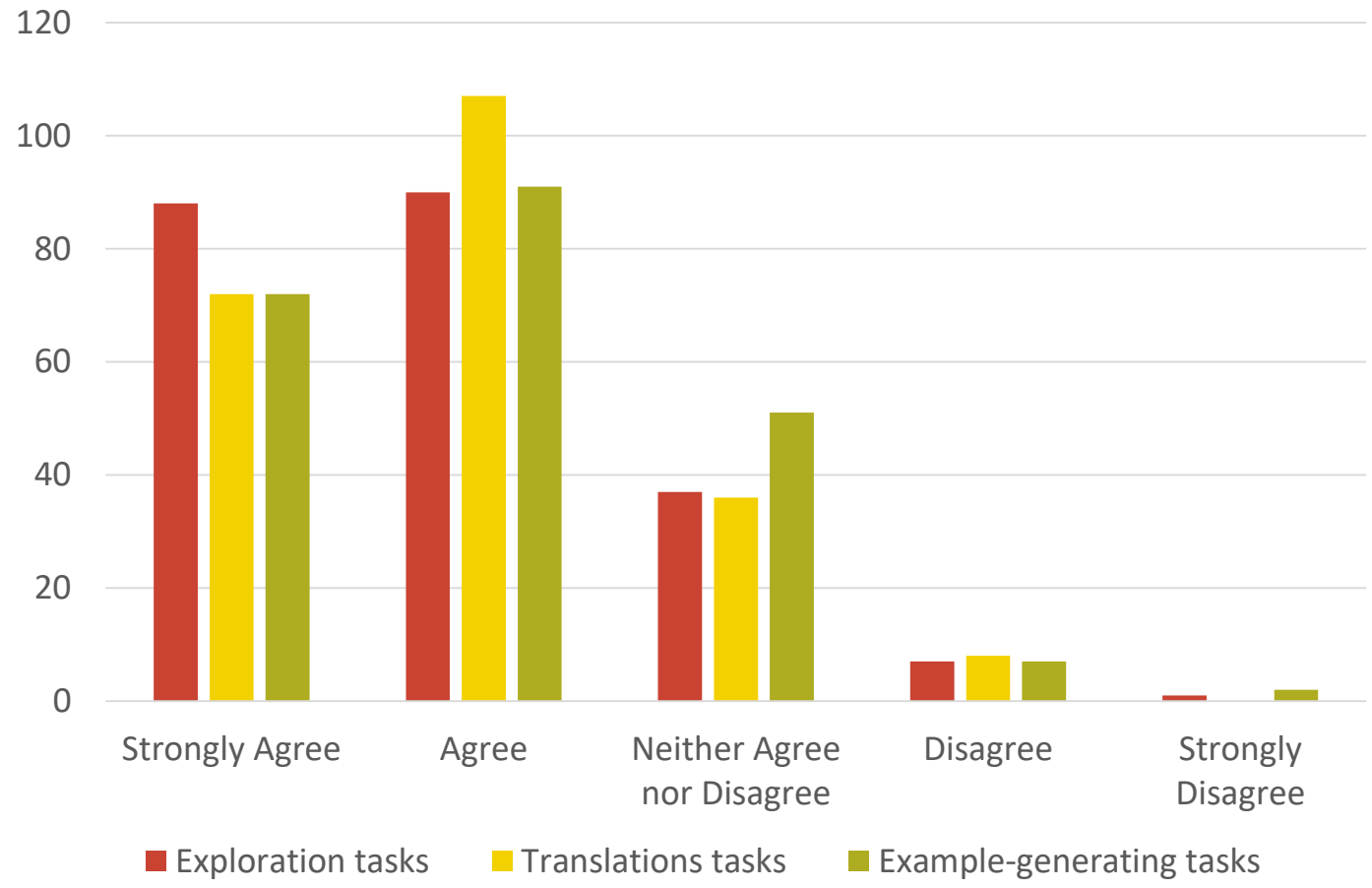
- 67/98 (68,4%) provided the same function formula (trigonometric)
- 14/98 (14,3%) provided one trigonometric function formula and one plynomial function formula, i.e. as we expected
- 24/98 (24,5%) provided at least one polynomial function formula

What do the students think about the activities?

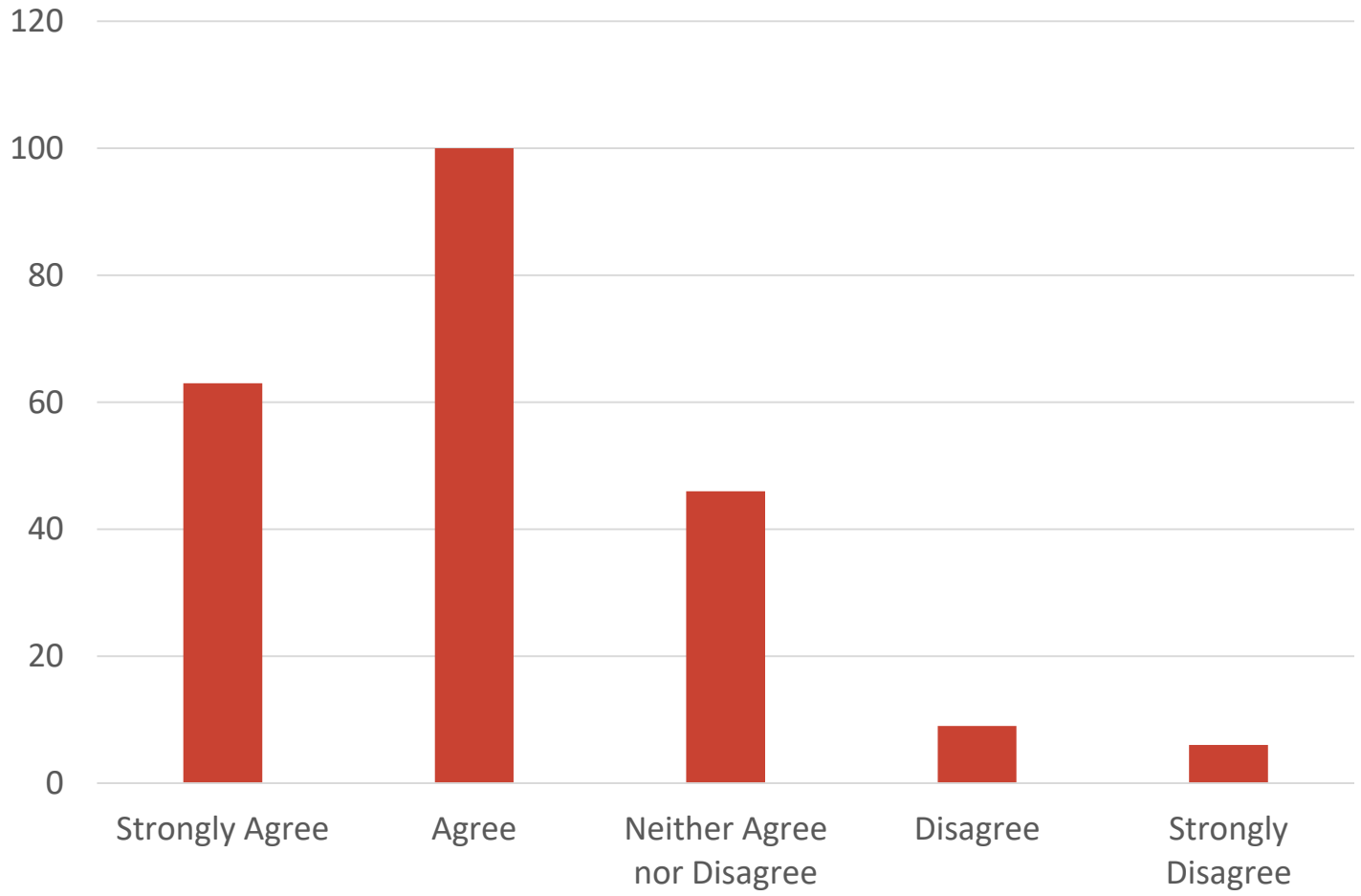
A voluntary questionnaire at the end of the course

- **2020: 84 out of 256 -> percentage of answers: 32,8%**
- **2021: 139 out of 235 -> percentage of answers: 59,1%**

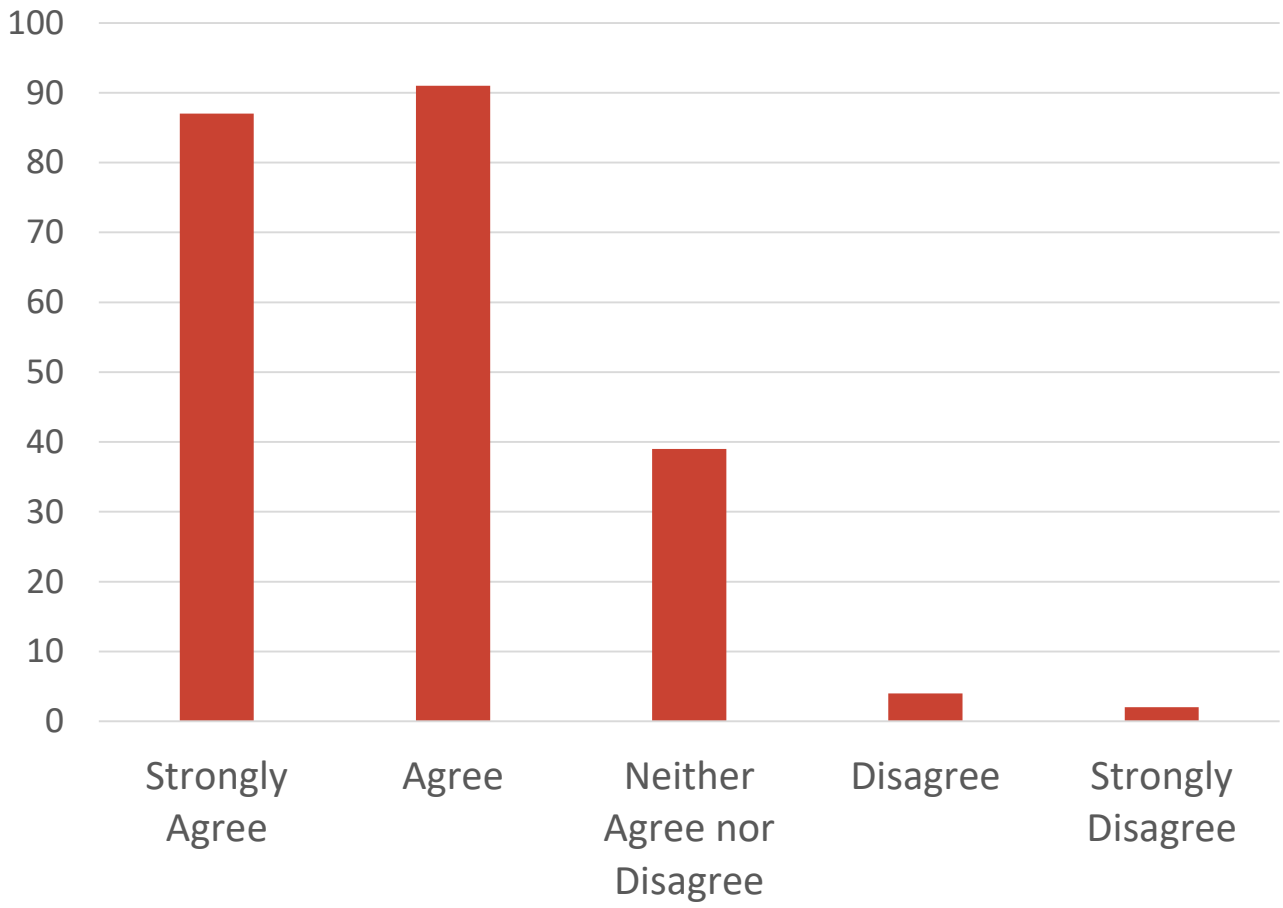
To what extent do you agree with the statement that the tasks provided increased understanding?



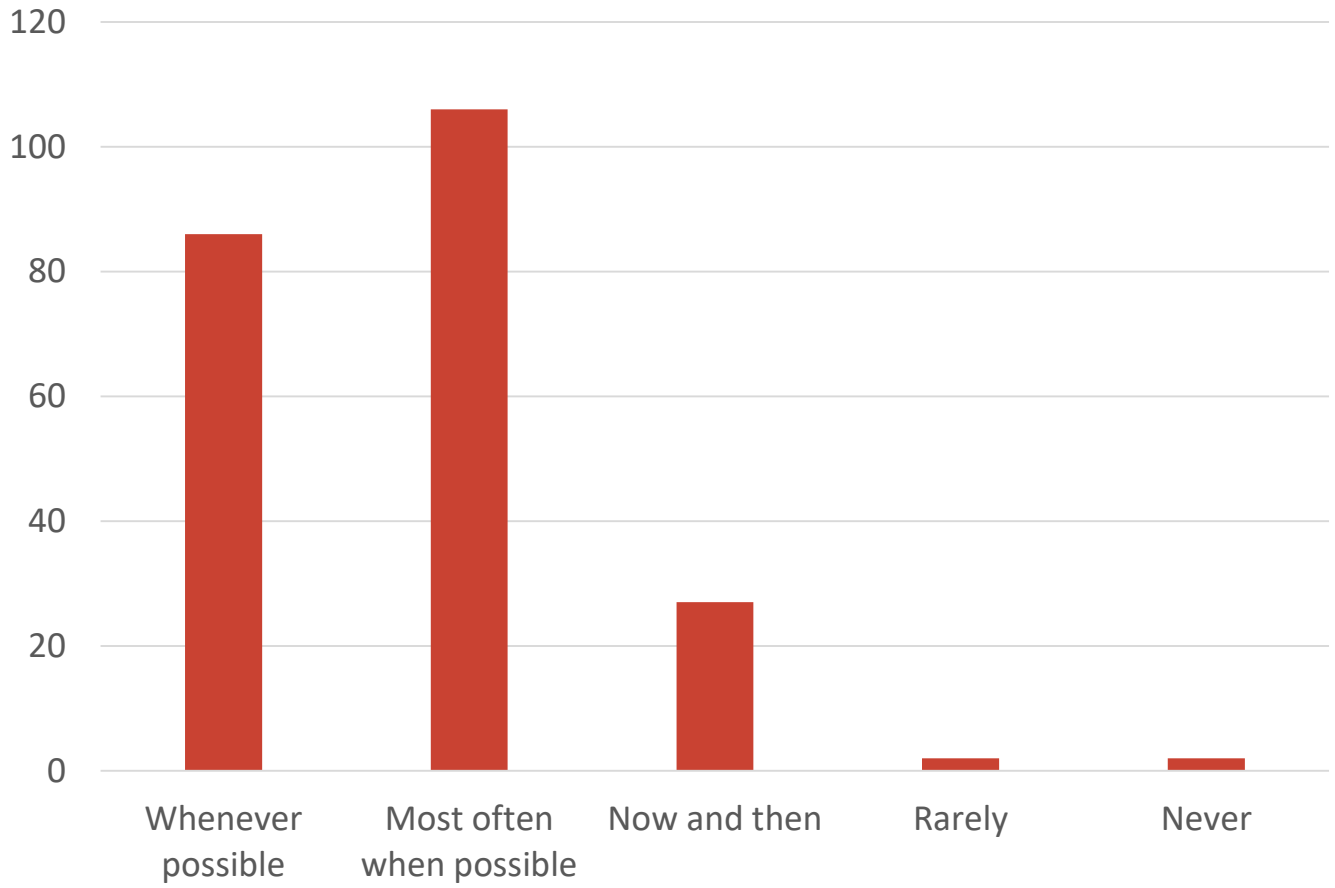
To what extent do you agree with the statement? The time spent on working with the activities was well worth



To what extent do you agree with the statement? It is good to have compulsory parts early in the course



To what extent did you use GeoGebra to check your answer before entering it in Möbius?



To what extent do you agree with the statement? Working in GeoGebra provides an increased understanding