TEST - MATRICES

1. What is a matrix?
a. an equation of over 5 numbers or symbols
b. a set of numbers in rows and columns
c. a method of finding the nth value of a series
d. a complicated number system
Solution: Correct answer: b)

If the answer is wrong: Let $M_1 = \{1, 2, ..., m\}, M_2 = \{1, 2, ..., n\}, \mathbb{C}$ set of complex numbers. A function $A : M_1 x M_2 \to \mathbb{C}$, $A(i, j) = a_{ij}$ is called a matrix of type (m, n) with elements complex numbers. A matrix has the general form

$$A = \begin{pmatrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{21} & a_{22} & ... & a_{2n} \\ ... & ... & ... & ... \\ a_{m1} & a_{m2} & ... & a_{mn} \end{pmatrix}.$$

A matrix is a rectangular grid with m rows and n columns or $A = \|a_{ij}\|_{\substack{i=\overline{1,m} \\ j=\overline{i,n}}}$

2. What is the name of each entry of a matrix?
a. row
b. element
c. dimension
d. numbers
Solution: Correct answer: b)

If the answer is wrong: The element is the entry of a matrix.

3. How many columns are in a $5X4$ matrix?
a. 5
b. 4
c. 20
d. 9
Solution: Correct answer: b)

If the answer is wrong: The mXn matrix has m row and n column.

4. How many rows are in a $7X3$ matrix?
a. 7
b. 3
c. 21
d. 10
Solution: Correct answer: a)

If the answer is wrong: The mXn matrix has m row and n column.

5. The transpose of a $5X6$ matrix has six columns and five rows.
a. true
b. false

Solution: Correct answer: b)

If the answer is wrong: The transpose of an mXn matrix A, written A is the nXm matrix whose i, j entry is the j, i entry of A.

6. Dimension/size:

$$\begin{pmatrix} 0 & -1 & 3 & 12 & 5 \\ 8 & 0 & 8 & 4 & 2 \\ 4 & 6 & 0 & 9 & 1 \end{pmatrix}$$

a. $3X5$
b. $4X5$
c. $5X3$
d. $5X4$
Solution: Correct answer: a)

If the answer is wrong: The dimensions of a matrix tells its size: the number of rows and columns of the matrix, in that order.

If the answer is still wrong: The matrix A has 3 rows and 5 columns, so it is a $3X5$ matrix.

7. What must be true in order to add two matrices?
a. they must be square
b. the dimensions/size must be equal
c. the determinant can't equal 0
d. the column of the 1^{st} must equal the row of the 2^{nd}
Solution: Correct answer: b)

If the answer is wrong: A matrix can only be added to another matrix if the two matrices have the same dimensions.

8. Add the matrices:

$$A = \begin{pmatrix} 5 & 5 \\ 1 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} 7 & 3 \\ -5 & 4 \end{pmatrix}$$

a. $\begin{pmatrix} 12 & 8 \\ -4 & 2 \end{pmatrix}$
b. $\begin{pmatrix} 3 & 2 \\ -6 & -6 \end{pmatrix}$
c. $\begin{pmatrix} -3 & 2 \\ -4 & -6 \end{pmatrix}$
d. $\begin{pmatrix} 10 & 8 \\ -6 & 2 \end{pmatrix}$
Solution: Correct answer: a)

If the answer is wrong: To add two matrices, just add the corresponding entries, and place this sum in the corresponding position in the matrix which results.

If the answer is still wrong:

$$A+B=\begin{pmatrix} 5 & 5 \\ 1 & -2 \end{pmatrix}+\begin{pmatrix} 7 & 3 \\ -5 & 4 \end{pmatrix}=\begin{pmatrix} 5+7 & 5+3 \\ 1-5 & -2+4 \end{pmatrix}=\begin{pmatrix} 12 & 8 \\ -4 & 2 \end{pmatrix}$$

9. Substract the matrices:

$$A=\begin{pmatrix} 3 & -6 \\ 4 & -1 \end{pmatrix}, \quad B=\begin{pmatrix} 0 & -2 \\ -4 & 6 \end{pmatrix}$$

a. $\begin{pmatrix} 3 & -8 \\ 0 & -1 \end{pmatrix}$

b. $\begin{pmatrix} 3 & -4 \\ 8 & -7 \end{pmatrix}$

c. $\begin{pmatrix} -3 & 8 \\ 0 & 1 \end{pmatrix}$

d. $\begin{pmatrix} 3 & -8 \\ -8 & 1 \end{pmatrix}$

Solution: Correct answer: b)

If the answer is wrong: To substract two matrices, just substract the corresponding entries, and place this difference in the corresponding position in the matrix which results.

If the answer is still wrong:

$$A-B=\begin{pmatrix} 3 & -6 \\ 4 & -1 \end{pmatrix}-\begin{pmatrix} 0 & -2 \\ -4 & 6 \end{pmatrix}=\begin{pmatrix} 3-0 & -6-(-2) \\ 4-(-4) & -1-6 \end{pmatrix}=\begin{pmatrix} 3 & -4 \\ 8 & -7 \end{pmatrix}$$

10. Multiply as necessary:

$$5\cdot\begin{pmatrix} -4 & 3 & -2 \\ 6 & -1 & 0 \end{pmatrix}$$

a. $\begin{pmatrix} 20 & 15 & -10 \\ 30 & -5 & 0 \end{pmatrix}$

b. $\begin{pmatrix} -20 & 15 & -10 \\ 30 & -5 & 0 \end{pmatrix}$

c. $\begin{pmatrix} 1 & 8 & 3 \\ 11 & 4 & 5 \end{pmatrix}$

d. $\begin{pmatrix} 20 & -15 & 10 \\ -30 & 5 & 0 \end{pmatrix}$

Solution: Correct answer: b)

If the answer is wrong: In scalar multiplication (refers to the product of a real number and a matrix), each entry in the matrix is multiplied by the given scalar.

If the answer is still wrong:

$$5 \cdot \begin{pmatrix} -4 & 3 & -2 \\ 6 & -1 & 0 \end{pmatrix} = \begin{pmatrix} -20 & 15 & -10 \\ 30 & -5 & 0 \end{pmatrix}$$

11. Compute $A \cdot B$ where:

$$A = \begin{pmatrix} 3 & 4 & 0 \\ 2 & 7 & 1 \\ 6 & 5 & 7 \end{pmatrix}, \ B = \begin{pmatrix} 2 & 2 & 8 \\ 5 & 7 & 0 \\ 6 & 4 & 3 \end{pmatrix}$$

a. not possible

b. $\begin{pmatrix} 26 & 34 & 24 \\ 45 & 57 & 19 \\ 79 & 75 & 69 \end{pmatrix}$

c. $\begin{pmatrix} 45 & 24 & 10 \\ 45 & 57 & 19 \\ 14 & 80 & 70 \end{pmatrix}$

d. $\begin{pmatrix} 79 & 75 & 69 \\ 45 & 57 & 19 \\ 26 & 34 & 24 \end{pmatrix}$

e. $\begin{pmatrix} 26 & 24 & 34 \\ 45 & 57 & 19 \\ 69 & 75 & 71 \end{pmatrix}$

Solution: Correct answer: b)

If the answer is wrong: Let $A = \|a_{ij}\|_{\substack{i=\overline{1,m} \\ j=\overline{i,n}}}$ and $B = \|b_{ij}\|_{\substack{i=\overline{1,n} \\ j=\overline{i,p}}}$. The matrix product AB is the mXp matrix whose i, j entry is

$$\sum_{k=1}^{n} a_{ik} b_{kj}$$

Remark: We can define the product $A \cdot B$ when the number of columns of A is the same as the number of rows of B.

If the answer is still wrong:

$$\begin{aligned} A \cdot B &= \begin{pmatrix} 3 & 4 & 0 \\ 2 & 7 & 1 \\ 6 & 5 & 7 \end{pmatrix} \cdot \begin{pmatrix} 2 & 2 & 8 \\ 5 & 7 & 0 \\ 6 & 4 & 3 \end{pmatrix} = \\ &= \begin{pmatrix} 3 \cdot 2 + 4 \cdot 5 + 0 \cdot 6 & 3 \cdot 2 + 4 \cdot 7 + 0 \cdot 4 & 3 \cdot 8 + 4 \cdot 0 + 0 \cdot 3 \\ 2 \cdot 2 + 7 \cdot 5 + 1 \cdot 6 & 2 \cdot 2 + 7 \cdot 7 + 1 \cdot 4 & 2 \cdot 8 + 7 \cdot 0 + 1 \cdot 3 \\ 6 \cdot 2 + 5 \cdot 5 + 7 \cdot 6 & 6 \cdot 2 + 5 \cdot 7 + 7 \cdot 4 & 6 \cdot 8 + 5 \cdot 0 + 7 \cdot 3 \end{pmatrix} = \\ &= \begin{pmatrix} 26 & 34 & 24 \\ 45 & 57 & 19 \\ 79 & 75 & 69 \end{pmatrix} \end{aligned}$$

12. Compute $A \cdot B$ where

$$A = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}, \ B = \begin{pmatrix} 1 & 3 & 5 \end{pmatrix}$$

a. not possible

b. $\begin{pmatrix} 2 & 9 & 10 \\ 3 & 6 & 15 \\ 4 & 20 & 12 \end{pmatrix}$

c. $\begin{pmatrix} 2 & 6 & 10 \\ 3 & 9 & 15 \\ 4 & 12 & 20 \end{pmatrix}$

d. $\begin{pmatrix} 2 & 1 & 1 \\ 3 & 3 & 4 \\ 4 & 12 & 20 \end{pmatrix}$

Solution: Correct answer: c)

If the answer is wrong: A is a $3X1$ matrix, B is a $1X3$ matrix$\Rightarrow$ $A \cdot B$ is a $3X3$ matrix.

$$A \cdot B = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 & 5 \end{pmatrix} = \begin{pmatrix} 2 & 6 & 10 \\ 3 & 9 & 15 \\ 4 & 12 & 20 \end{pmatrix}$$

13. Compute $A \cdot B$ where

$$A = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}, \ B = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$$

a. $\begin{pmatrix} 2 & 25 \\ 7 & 3 \end{pmatrix}$

b. not possible

c. $\begin{pmatrix} 2 & 25 \end{pmatrix}$

d. $\begin{pmatrix} 2 \\ 7 \\ 25 \end{pmatrix}$

e. $\begin{pmatrix} 1 \\ 25 \end{pmatrix}$

Solution: Correct answer: b)

If the answer is wrong: A is a $3X1$ matrix, B is a $2X1$ matrix$\Rightarrow$ is not possible to compute $A \cdot B$, because the number of columns of A is not the same as the number of rows of B.

14. If A and B are $2X2$ matrices such that $AB = 0$, then $BA = 0$.

a. true
b. false
Solution: Correct answer: b)

If the answer is wrong: For example, let $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 0 \\ 2 & 3 \end{pmatrix}$. The product of the matices $A \cdot B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = O_2$, $B \cdot A = \begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix} \Rightarrow A \cdot B \neq B \cdot A$.

15. If $AB = 0$, then either A or B is a zero matrix.
a. true
b. false
Solution: Correct answer: b)

If the answer is wrong: $A \cdot B = O_n \nRightarrow A = O_n$ or $B = O_n$
For example, let $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 0 \\ 1 & 2 \end{pmatrix}$. The product of the matices is $A \cdot B = O_n$, but $A \neq O_n$ and also $B \neq O_n$

16. Can you multiply a $3X4$ matrix with a $4X2$ matrix?
a. yes
b. no
Solution: Correct answer: a)

If the answer is wrong: We can define the product $A \cdot B$ when the number of columns of A is the same as the number of rows of B.

If the answer is still wrong: The number of columns of A is 4 and is the same number of rows of B.

17. You can multiply a $2X3$ matrix by which matrix below?
a. $2X2$
b. $2X12$
c. $3X12$
d. $2X3$
Solution: Correct answer: c)

If the answer is wrong: The $(2, 3)$ matrix can be multiply by any $(3, p)$ matrix, $p \in \mathbb{N}^*$, the product is a matrix of the type $(2, p)$.

18. These matrices are being multiplied. Determine the dimension/size of the new matrix

$$A = \begin{pmatrix} 6 & 4 & -3 \\ 2 & 1 & -5 \\ -1 & 6 & -7 \\ 3 & 1 & 7 \end{pmatrix}, \ B = \begin{pmatrix} 1 & 3 & 2 \\ 4 & -1 & 6 \end{pmatrix}$$

a. can't multiply them
b. $4X2$

c. $3X3$
d. $4X3$
Solution: Correct answer: a)

If the answer is wrong: A is a $4X3$ matrix, B is a $2X3$ matrix$\Rightarrow$ is not possible to compute $A \cdot B$, because the number of columns of A is not the same as the number of rows of B.

19. "Not invertible" is the same thing as "singular".
a. true
b. false
Solution: Correct answer: a)

If the answer is wrong: A non-invertible matrix is referred to as singular matrix.

20. If a sistem of linear equations is represented by $AX = B$ and A is invertible, then the system has a unique solution.
a. true
b. false
Solution: Correct answer: a)

If the answer is wrong:

$$\begin{aligned} A \cdot X &= B \ \mid \cdot A^{-1} \\ A^{-1} \cdot A \cdot X &= A^{-1} \cdot B \\ X &= A^{-1} \cdot B \Rightarrow \text{unique solution} \end{aligned}$$

21. What is the determinant of the matrix

$$\begin{pmatrix} -3 & 5 \\ -2 & -7 \end{pmatrix}?$$

a. -31
b. -29
c. -20
d. 31
e. -24
Solution: Correct answer: d)

If the answer is wrong: The value of a second-order determinant is equal to the product of the elements on the principal diagonal, minus the product of the elements on the secondary diagonal.
If $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ then $\det A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} \cdot a_{22} - a_{12} \cdot a_{21}$
If the answer is still wrong:

$$\begin{vmatrix} -3 & 5 \\ -2 & -7 \end{vmatrix} = -3 \cdot (-7) - 5 \cdot (-2) = 21 + 10 = 31$$

22. Calculate the determinant of the matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$$

a. 0
b. 3
c. 4
d. 6
Solution: Correct answer: a)

If the answer is wrong: If $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$, then the triangle rule is $\det A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \cdot a_{22} \cdot a_{33} + a_{21} \cdot a_{32} \cdot a_{13} + a_{12} \cdot a_{23} \cdot a_{31} - a_{13} \cdot a_{22} \cdot a_{31} - a_{23} \cdot a_{32} \cdot a_{11} - a_{12} \cdot a_{21} \cdot a_{33}$
or we can apply the rule of Sarrus
$\det A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{vmatrix} = a_{11} \cdot a_{22} \cdot a_{33} + a_{21} \cdot a_{32} \cdot a_{13} + a_{12} \cdot a_{23} \cdot a_{31} - a_{13} \cdot a_{22} \cdot a_{31} - a_{23} \cdot a_{32} \cdot a_{11} - a_{12} \cdot a_{21} \cdot a_{33}$

Feedback 2: If two rows/colums of a matrix are identical then $\det A = 0$.

23. Calculate the trace of the matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$$

a. 0
b. 3
c. 4
d. 6
Solution: Correct answer: d)

If the answer is wrong: Trace of a matrix is defined as the sum of the principal diagonal elements of a square matrix. It is usually represented as $tr(A)$.
Let $A = \begin{pmatrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{21} & a_{22} & ... & a_{2n} \\ ... & ... & ... & ... \\ a_{n1} & a_{n2} & ... & a_{nn} \end{pmatrix}$, then $tr(A) = a_{11} + a_{22} + ... + a_{nn}$
If the answer is still wrong: $tr(A) = 1 + 2 + 3 = 6$

24. Calculate the rank of the matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$$

a. 4
b. 1
c. 0
d. 2
Solution: Correct answer: b)

If the answer is wrong: If $A = \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{pmatrix}, a_{ij} \in \mathbb{C}, i = \overline{1,m}, j = \overline{1,n}$, then by minor of the order $r \leq \min(m,n), r \in \mathbb{N}^*$ is understood the determinant whose elements are the points of intersection of lines $i_1, i_2, ..., i_r$ with columns $j_1, j_2, ..., j_r$,

$$\Delta = \begin{vmatrix} a_{i_1j_1} & a_{i_1j_2} & \ldots & a_{i_1j_r} \\ a_{i_2j_1} & a_{i_2j_2} & \ldots & a_{i_2j_r} \\ \ldots & \ldots & \ldots & \ldots \\ a_{i_rj_1} & a_{i_rj_2} & \ldots & a_{i_rj_r} \end{vmatrix}$$

Matrix A has the rank r if A contains a non zero minor of order r, and all minors of order higher than r (if exists) are zero. Notation: $rankA = r$.
For $A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$, we can observed that all the minors of second order and the minor of third order are zero (the determinate that has 2 identical rows/columns is 0). So, $rankA = 1$.

25. Find the eigenvalues of the matrix

$$A = \begin{pmatrix} 5 & -3 \\ 1 & -2 \end{pmatrix}$$

a. $\begin{pmatrix} -4.54 & -1.54 \end{pmatrix}$
b. $\begin{pmatrix} -4.54 & 1.54 \end{pmatrix}$
c. $\begin{pmatrix} 4.54 & 1.54 \end{pmatrix}$
d. $\begin{pmatrix} 4.54 & -1.54 \end{pmatrix}$
Solution: Correct answer: d)

If the answer is wrong: Eigenvalues of the real type (m,n) matrix are the real solutions of the equation $\det(A - \lambda I_n) = 0$, where I_n is identity matrix (square matrix with ones on the main diagonal and zeros elsewhere).
If $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ then $\det A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} \cdot a_{22} - a_{12} \cdot a_{21}$

If $A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$, then the triangle rule is $\det A=\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}=$ $a_{11}\cdot a_{22}\cdot a_{33}+a_{21}\cdot a_{32}\cdot a_{13}+a_{12}\cdot a_{23}\cdot a_{31}-a_{13}\cdot a_{22}\cdot a_{31}-a_{23}\cdot a_{32}\cdot a_{11}-a_{12}\cdot a_{21}\cdot a_{33}$

or we can apply the rule of Sarrus $\det A=\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{vmatrix}=a_{11}\cdot a_{22}\cdot a_{33}+$ $a_{21}\cdot a_{32}\cdot a_{13}+a_{12}\cdot a_{23}\cdot a_{31}-a_{13}\cdot a_{22}\cdot a_{31}-a_{23}\cdot a_{32}\cdot a_{11}-a_{12}\cdot a_{21}\cdot a_{33}$

For $A=\begin{pmatrix} 5 & -3 \\ 1 & -2 \end{pmatrix}$, we have

$$I_2=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

$$\det(A-\lambda I_n)=\begin{vmatrix} 5-\lambda & -3 \\ 1 & -2-\lambda \end{vmatrix}=(5-\lambda)\cdot(-2-\lambda)+3.$$

$$\det(A-\lambda I_n)=0\Leftrightarrow(5-\lambda)\cdot(-2-\lambda)+3=0\Leftrightarrow\lambda^2-3\lambda-7=0\Rightarrow$$

$$\lambda_1=\frac{3+\sqrt{37}}{2}\simeq 4.54,\ \ \lambda_2=\frac{3-\sqrt{37}}{2}\simeq -1,54$$

26. Calculate the inverse of

$$A=\begin{pmatrix} 1 & 5 \\ 2 & 4 \end{pmatrix}$$

a. 2
b. $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$
c. $\begin{pmatrix} 2 & 3 \\ 5 & 6 \end{pmatrix}$
d. $\begin{pmatrix} -\frac{2}{3} & \frac{5}{6} \\ \frac{1}{3} & -\frac{1}{6} \end{pmatrix}$
Solution: Correct answer: d)

If the answer is wrong: Aquadratic matrix A of order n is invertible if there exist a quadratic matrix B of order n, such that :

$$AB=BA=I_n$$

B represent the inverse matrix of A and it is denoted by $B=A^{-1}$.
From the relation

$$AA^{-1}=A^{-1}A=I_n$$

we have $\det A \neq 0$. If $\det A = 0$, then A is not invertible.
To construct A^{-1}, we follow the steps:
$i)$ calculate $\det A$
$ii)$ write the transposed matrix A^T (the transpose of a matrix is obtained by changing its rows into columns (or equivalently, its columns into rows)).
$iii)$ calculate

$$A^* = \begin{pmatrix} A_{11} & A_{12} & ... & A_{n1} \\ A_{21} & A_{22} & ... & A_{2n} \\ ... & ... & ... & ... \\ A_{n1} & A_{n2} & ... & A_{nn} \end{pmatrix}$$

where $A_{ij} = (-1)^{i+j} \cdot \Delta_{i,j}, i, j \in \{1, 2, ..., n\}$
$\Delta_{i,j}$ represents the determinant that is obtained from the determinant of matrix A by removing the row of rank i and the column of rank j.
$iv)$

$$A^{-1} = \frac{1}{\det A} \cdot A^*$$

In our case,

$$\det A = \begin{vmatrix} 1 & 5 \\ 2 & 4 \end{vmatrix} = 4 - 10 = -6,$$

$$A^T = \begin{pmatrix} 1 & 2 \\ 5 & 4 \end{pmatrix},$$

$$A^* = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, \text{ where}$$

$$\begin{aligned} A_{11} &= (-1)^{1+1} \cdot 4 = 4 \\ A_{12} &= (-1)^{1+2} \cdot 5 = -5 \\ A_{21} &= (-1)^{2+1} \cdot 2 = -2 \\ A_{22} &= (-1)^{2+2} \cdot 1 = 1 \end{aligned}$$

In conclusion,

$$A^* = \begin{pmatrix} 4 & -5 \\ -2 & 1 \end{pmatrix}$$

and

$$A^{-1} = \frac{1}{\det A} \cdot A^* = -\frac{1}{6} \begin{pmatrix} 4 & -5 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} -\frac{2}{3} & \frac{5}{6} \\ \frac{1}{3} & -\frac{1}{6} \end{pmatrix}$$

27. Determine if the matrix

$$A = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

is orthogonal or not.

a. orthogonal
b. not orthogonal
Solution: Correct answer: a)

If the answer is wrong: A square matrix A of order n is an orthogonal matrix if

$$A^T = A^{-1} \text{ or } AA^T = A^T A = I_n$$

28. Find the trace of the following matrix

$$\begin{pmatrix} 14 & 15 \\ 2 & -5 \\ 5 & 10 \end{pmatrix}$$

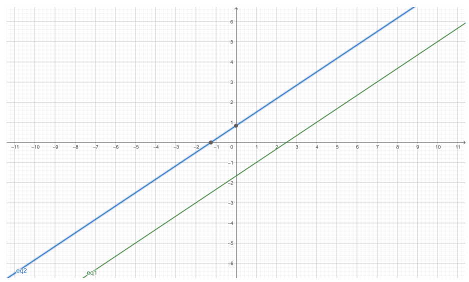
a. 16
b. 10
c. 9
d. 17
e. not possible to calculate
Solution: Correct answer: e)

If the answer is wrong: The matrix A is of type $(3, 2) \Rightarrow A$ is not a square matrix$\Rightarrow \nexists \ tr(A)$.

29. Find the number of solutions of the following pair of linear equations: $4x - 6y = 10$ and $-8x + 12y = 10$.

a. 0
b. 1
c. 2
d. infinite
Solution: Correct answer: a)

Feekback1: The equations represent the equations of two straight lines. The intersection of these straight lines represents the solution to the problem.
In our case, we can observe that the straight lines are parallel $\Rightarrow$ the problem has no solution.



Feekback2: We solve the system:

$$\begin{cases} 4x - 6y = 10 \\ -8x + 12y = 10 \mid : (-2) \end{cases} \Leftrightarrow \begin{cases} 4x - 6y = 10 \\ 4x - 6y = -5 \end{cases} \Rightarrow 10 = -5, \text{ false}$$

Feekback3: We write the equations in matrix form:

$$\begin{pmatrix} 4 & -6 \\ -8 & 12 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ 10 \end{pmatrix}$$

Matrix A has proportional lines.
The

$$\overline{A} = (A|b) = \begin{pmatrix} 4 & -6 & 10 \\ -8 & 12 & 10 \end{pmatrix}$$

has no proportional lines$\Rightarrow A \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ 10 \end{pmatrix}$ has no solution

30. The pair of equations $3x - 5y = 7$ and $-6x + 10y = 7$ have
a. a unique solution
b. infinitely many solutions
c. two solutions
d. no solution
Solution: Correct answer: d)

If the answer is wrong: We have:

$$A = \begin{pmatrix} 3 & -5 \\ -6 & 10 \end{pmatrix}, \ \overline{A} = \begin{pmatrix} 3 & -5 & 7 \\ -6 & 10 & 7 \end{pmatrix}$$

Matrix A has proportional lines: $\frac{3}{-6} = \frac{-5}{10}$
Matrix $\overline{A}$ has no proportional lines: $\frac{3}{-6} = \frac{-5}{10} = \frac{7}{7} \Rightarrow$the system has no solution