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TEACHING MATHEMATICS TO ENGINEERING STUDENTS CHALLENGE AND ADVENTURE

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INTRODUCTION

- Information about experiment with introduction of activating learning methods
- eduScrum team work versus individual work
- analysis of the students' opinions

MATHS IS THE ONLY SUBJECT THAT COUNTS!

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ERAZMUS+ Strategic Partnership

European Project No. 2017-1-PT01-KA203-035866

DRIVE MATH – Development of Innovative Mathematical Teaching Strategies in European Engineering Degrees

Coordinator – **ISEP University in Porto, Carla Pinto**

Project partners

- Technical University in Chemnitz, Germany
- University Lyon 1, Claude Bernard, France
- Slovak University of Technology in Bratislava, Slovakia

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EXPERIMENT IMPLEMENTATION

academic year 2018/19 and 2019/2020

- basic courses - Mathematics I, Mathematics II

  1st year mechanical engineering students,

  bachelor study programs

- randomly chosen group of 99 students

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TEAM WORK

- 99 students attended together lectures in lecture theater
- students were distributed to 5 working groups for tutorials, 19 - 20 students in each
- team work in small teams of 4 - 5 students
- 1 team leader - scrum master
- eduScrum method was introduced in both semesters
- 5 topics in winter semester
- 3 topics in summer semester

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TOPICS

WINTER SEMESTER

- Linear algebra – matrices, determinants, linear systems of equations
- Functions with one real variable – domains, basic properties
- Differential calculus
- Integral calculus I – indefinite integrals
- Integral calculus II – definite integrals

SUMMER SEMESTER

- Differential equations
- Functions with 2 variables – graphs and extremes
- Multiple integrals with applications

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TEAM WORK

- teams worked on prepared sprints
- each sprint consisted of 5 problems – discussed together
- each student in team, but the team leader – scrum master, was responsible for solution of one from the problems
- 1 problem was formulated as applied problem from the mechanical engineering field – solved together
- problems were followed by a short question from related mathematical theory
- open-books approach
- team leader organised the work distribution and was responsible for the final presentation of team results
- all students in the team received the same point score achieved together by the team

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INDIVIDUAL WORK

- applied as an alternative method
- realised in both semesters
- organised after the team work
- individual tests on prepared worksheets
- 4 topics in winter semester
- 3 topics in summer semester

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INDIVIDUAL WORK

- students answered questions and solved simple problems in prepared worksheets
- about 8 tasks in each topic
- problems were of more technical character
- closed-books approach
- students received the individually achieved point score

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QUESTIONNAIRE

Experiment was completed by diagnostic questionnaire, posed questions were aimed

- to discover attitudes and opinions of students on this different teaching scenario
- to receive feedback from students about inclusion of application problems in the subject of mathematics
- to open free informal interviews with students that followed.

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BASIC INFORMATION ABOUT STUDENTS

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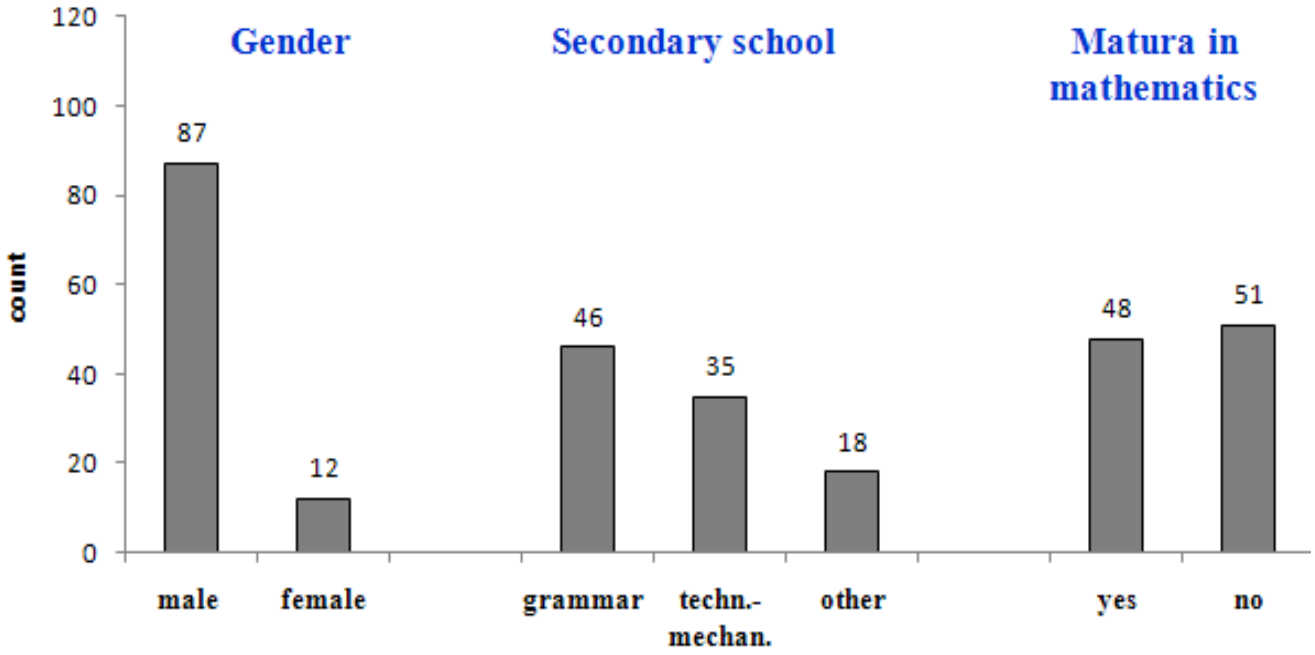

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BASIC INFORMATION ABOUT STUDENTS

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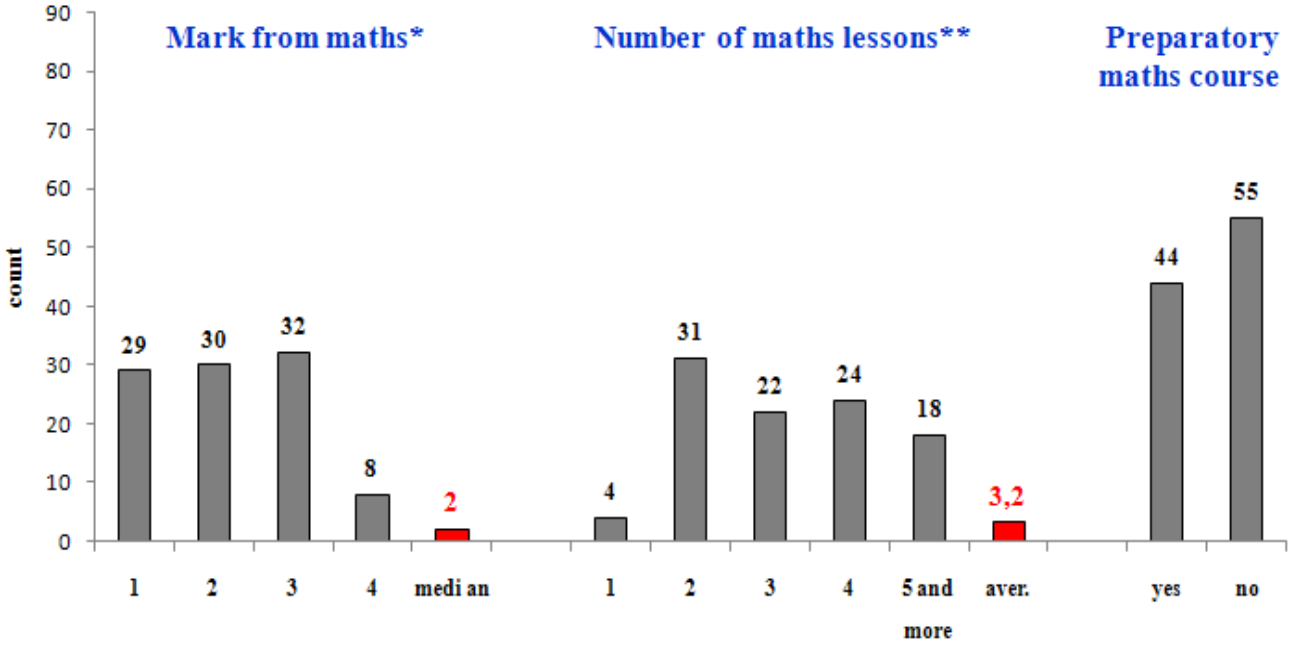

- very good average mark from mathematics at secondary school – 2,2 (median 2)
- almost one half of students attended Preparatory Course of Mathematics at STU

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TEAM WORK

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INDIVIDUAL WORK

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RESULTS OF TEAM AND INDIVIDUAL WORK

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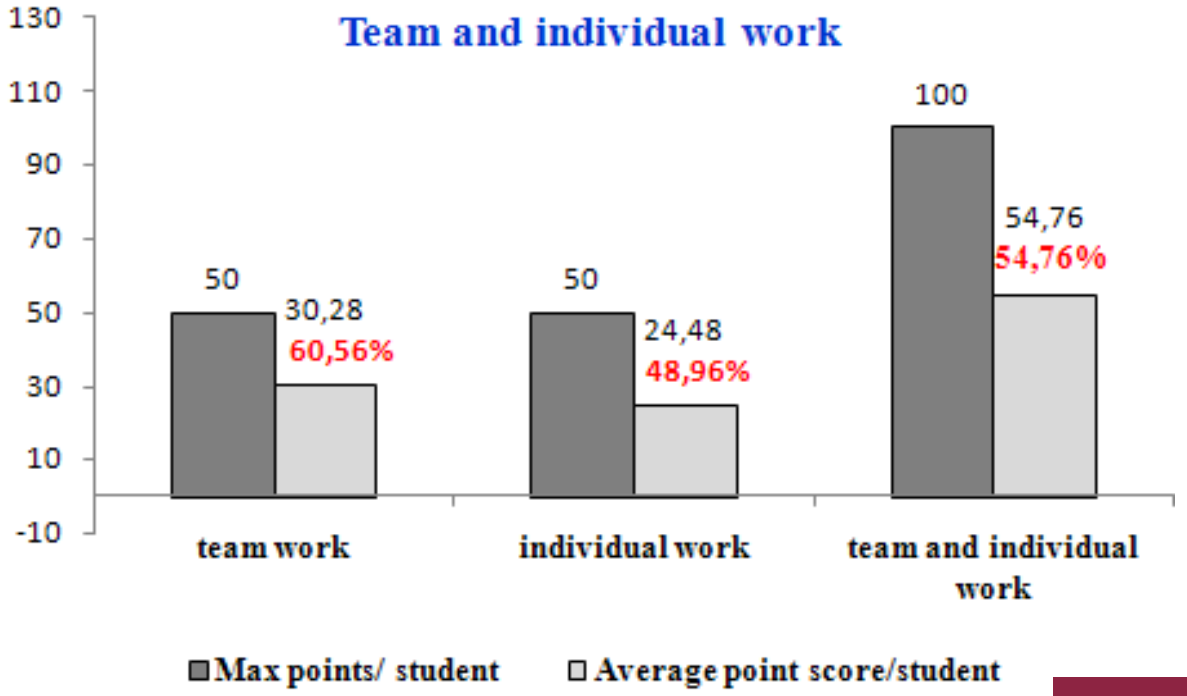

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RESULTS OF TEAM WORK

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MATHEMATICS I						
TEAM	Linear algebra	Calculus I	Calculus II	Indefinite Integrals	Definite Integrals	Results
	10 points	10 points	10 points	10 points	10 points	50 points
1.	9,5	9	5	9,5	8,5	41,5
2.	9,5	6,5	6	7,5	8	37,5
3.	6	4,5	2,5	6	3	22
4.	9,5	5,5	6,25	8,125	5,625	35

MATHEMATICS II				
TEAM	Coordinate Geometry	Extrema of $f(x, y)$	Multiple Integrals	Results
	12 points	20 points	16 points	48 points
1.	11	17	17	41,5
2.	10,5	12	14,5	37,5
3.	5	12	11	22
4.	7	14,5	11,5	35

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MATHEMATICS
is not about
numbers, equations,
computations, or
algorithms:
it is about
UNDERSTANDING.

William Paul Thurston

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QUESTIONNAIRE

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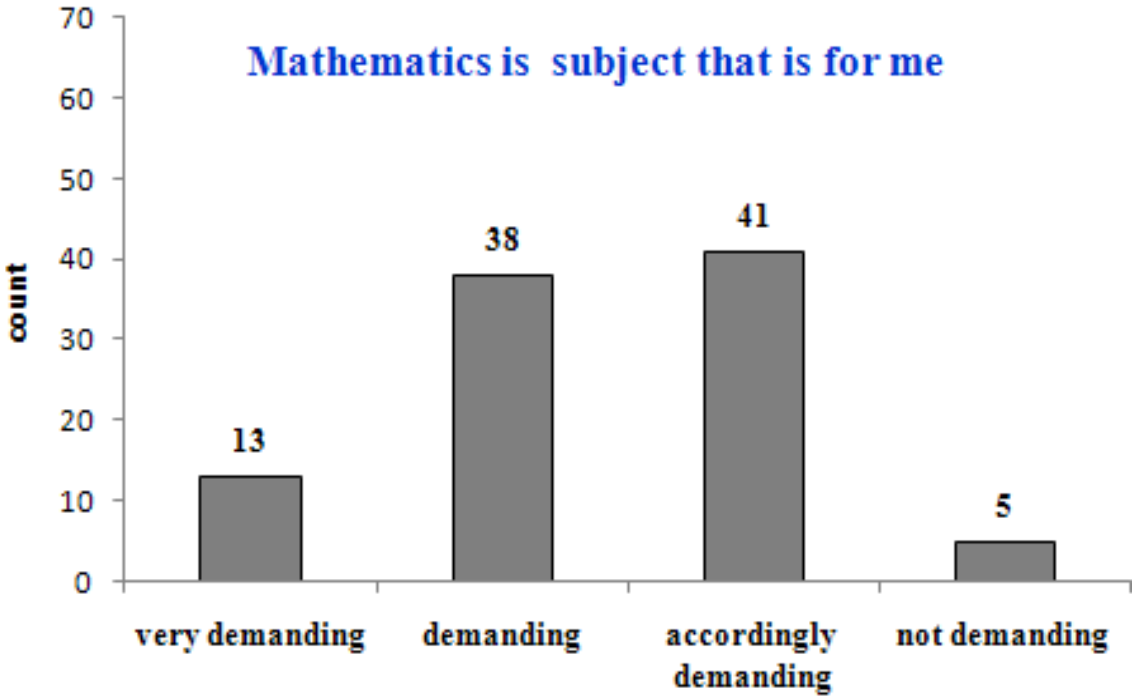

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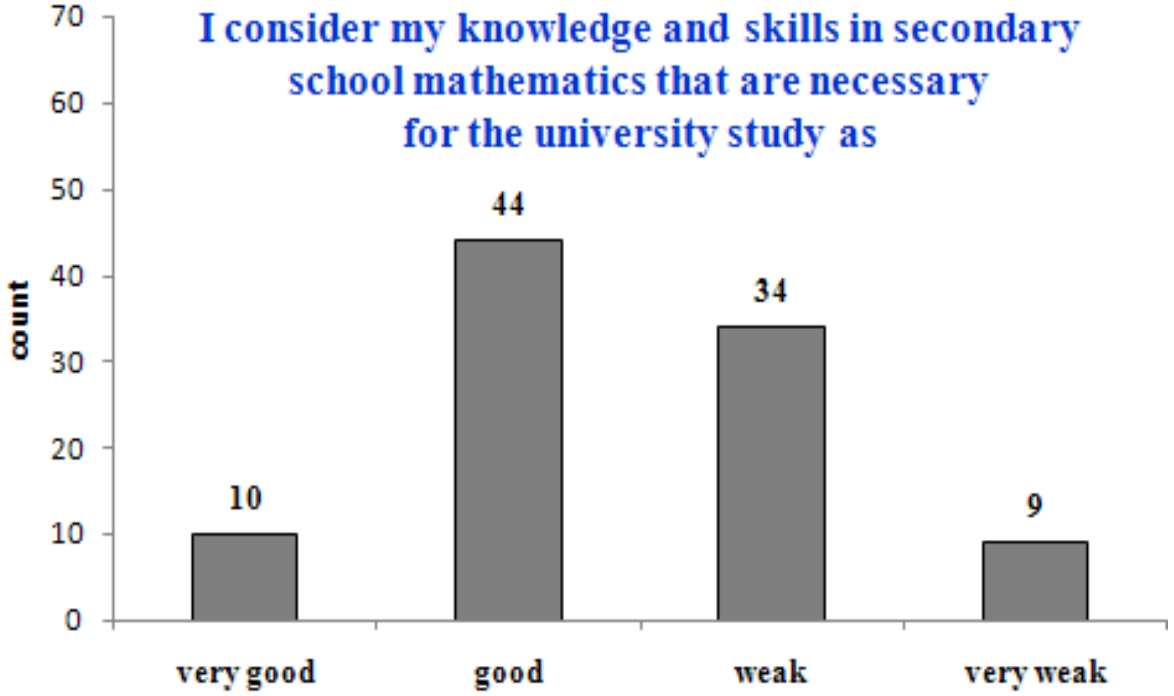

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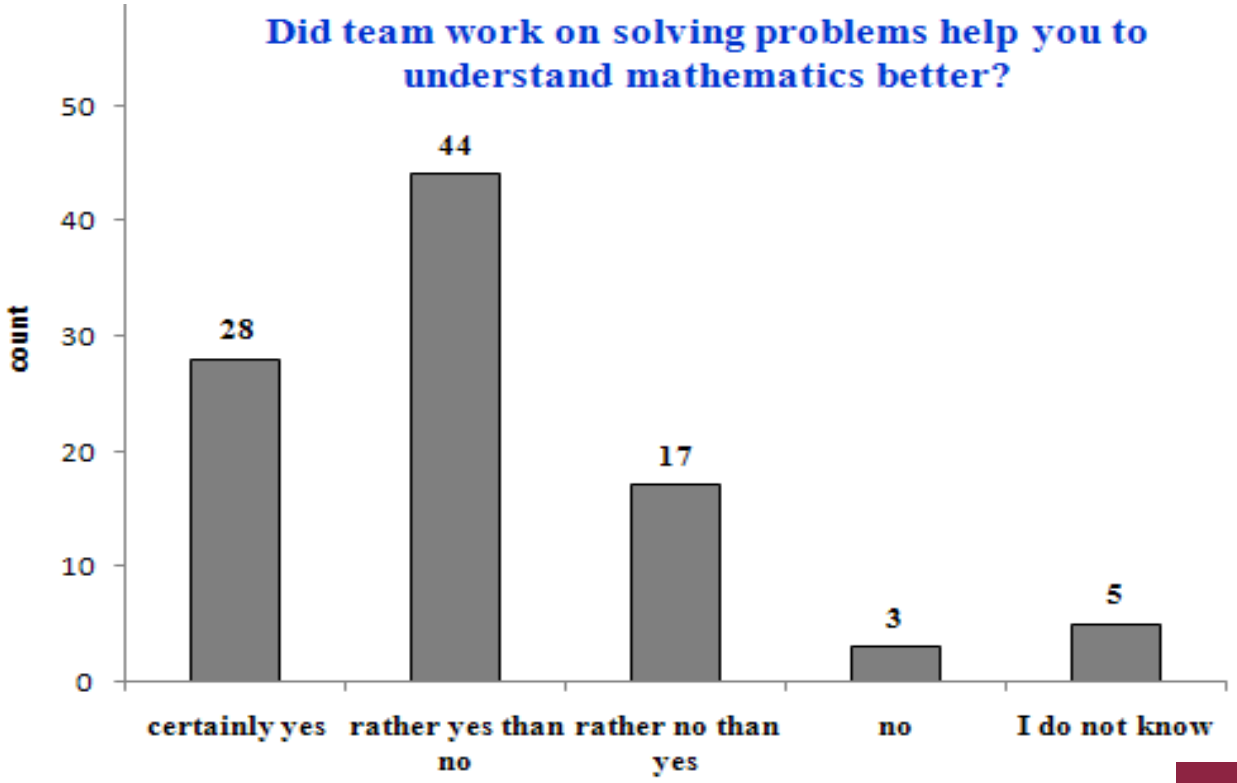

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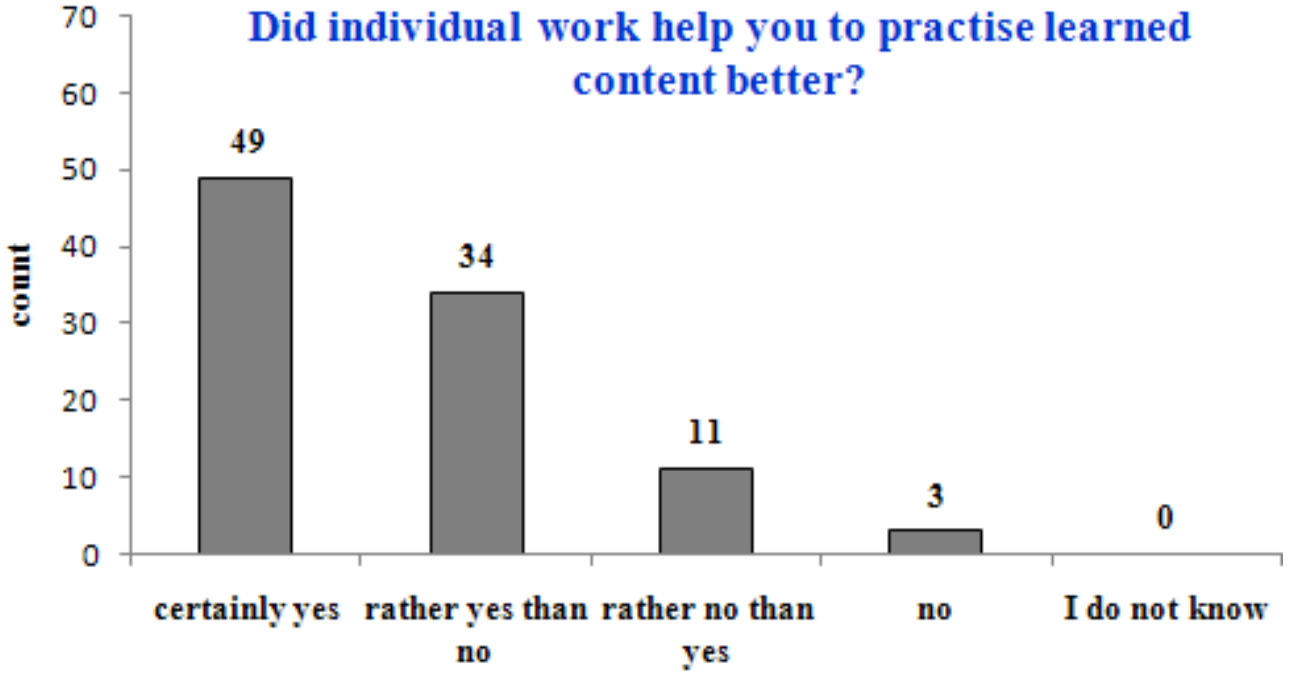

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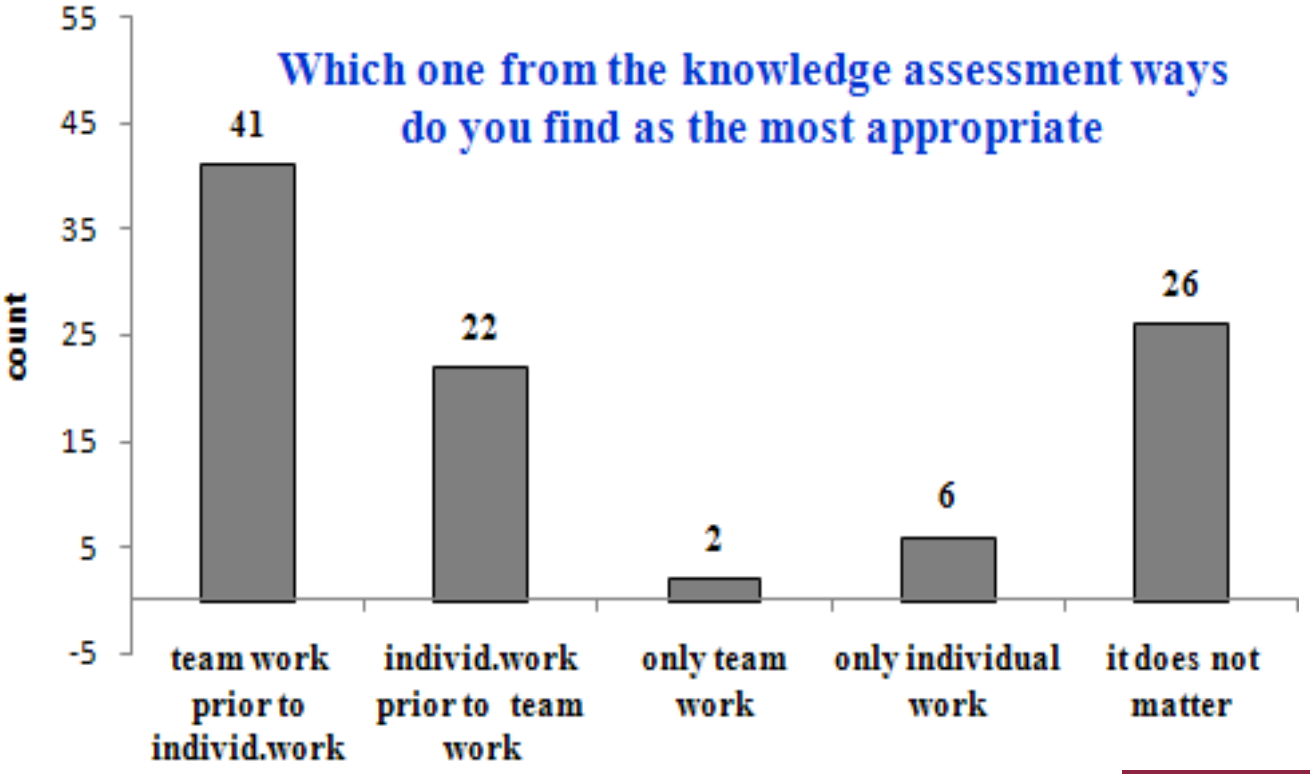

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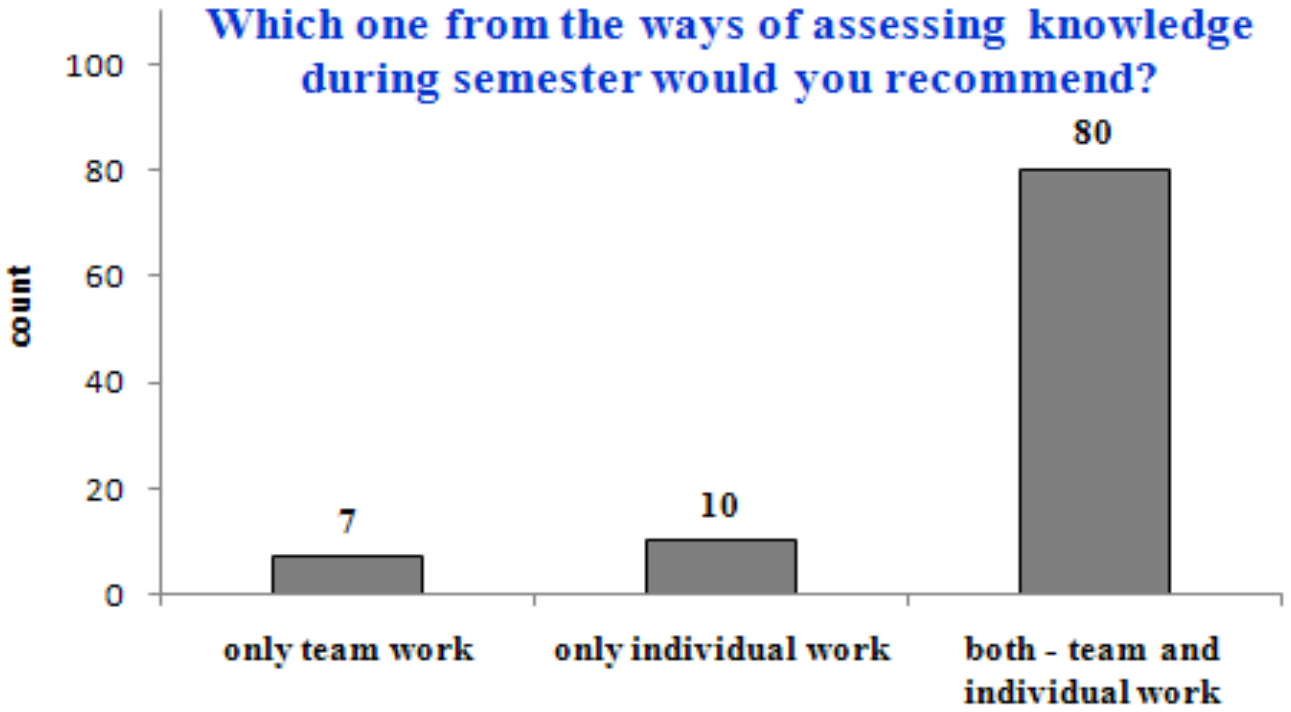

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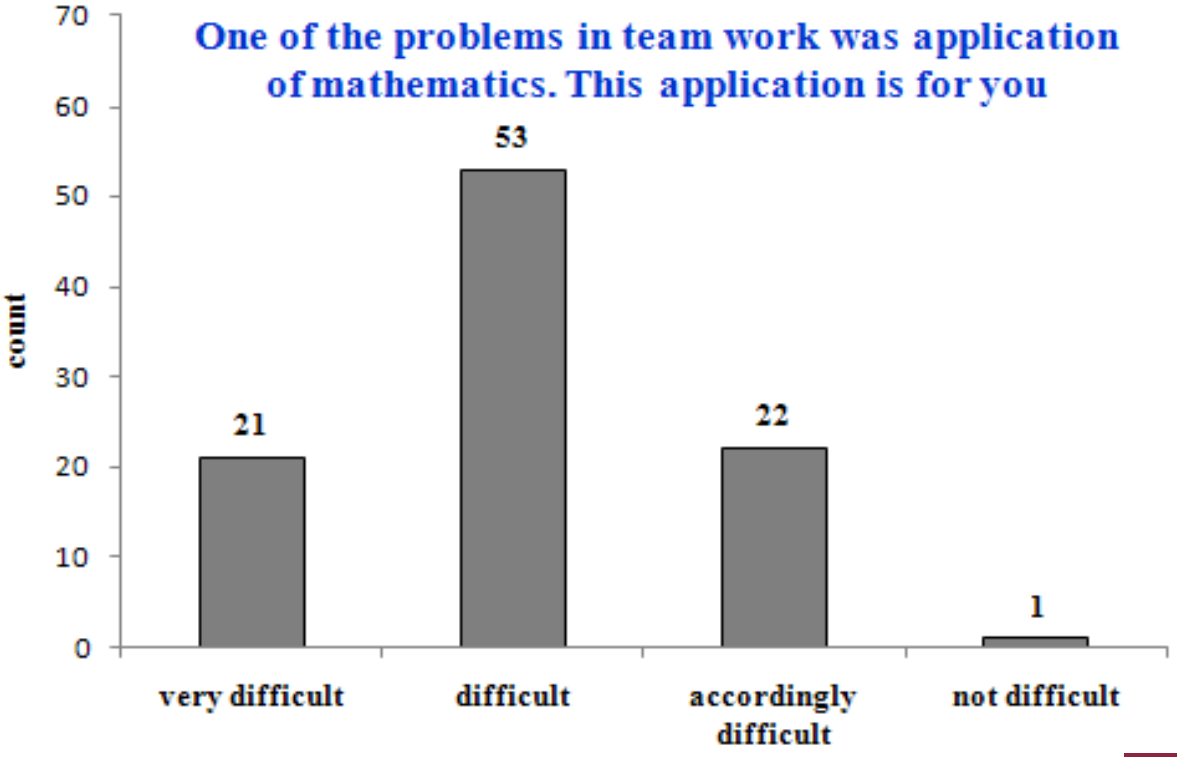

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APPLIED PROBLEM

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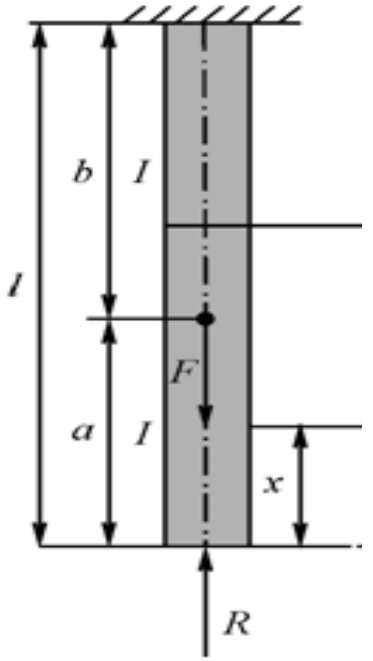
Prismatic rod of length *l*, which is firmly clamped at one end and free at the other one, is loaded by axial forces *F* and *R*. (Force of the rod own weight is neglected.) Dependence of the strain energy of the accumulation *A* in this rod on the force *R* , under assumption that force *F* is constant, is

$$A = 12\left[\left(\frac{R}{F}\right)^2 - \frac{4}{3}\left(\frac{R}{F}\right) + \frac{2}{3}\right]$$



Draw graph of the strain energy of the accumulation *A* for $R \in [-F, 2F]$.

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APPLIED PROBLEM

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Car is moving on a horizontal surface. Roadway before the junction and through the junction has the form of profiles, which are modeled by function $s(t) = 3\sin 2t$, while velocity $v(t)$ of the moving car is determined by function $v(t) = 6\cos 2t$.

- Sketch graph of the roadway function $s(t)$ and velocity function $v(t)$ into one diagram for time interval $\langle 0,2\pi \rangle$, while $t = 0$ is the time when the car will start its motion on profiles.
- At the time $t = \frac{7\pi}{6}$ calculate value of $s(t)$, length of the car trajectory, and velocity $v(t)$ of the car movement, and mark these values in the graph.

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APPLIED PROBLEM

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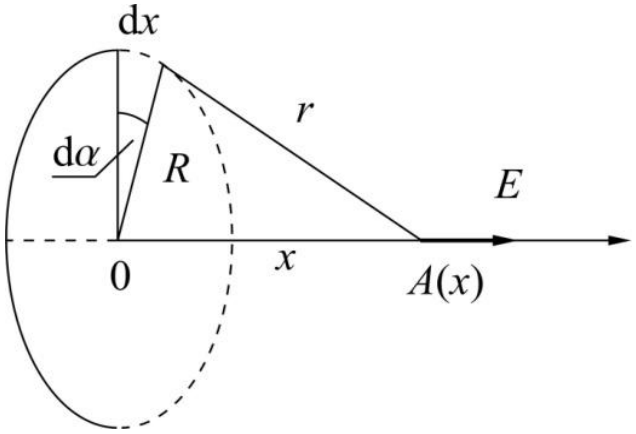
The electric charge Q is distributed with length density λ on a circle with radius R (see figure). Calculate electric potential $A(x)$ of field generated by a continuously distributed charge on the circle axis in distance x from the centre of circle.

$$A(x) = -\int E(x)\,dx$$



$$E(x) = \frac{\lambda R x}{2\varepsilon_0 (R^2 + x^2)^{3/2}}$$

intensity of electric field in direction of axis x

$\varepsilon_0 = 8{,}854 \cdot 10^{-12}\ F\,m^{-1}$ vacuum permitivity (electrical constant)

λ, R given constants

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APPLIED PROBLEM

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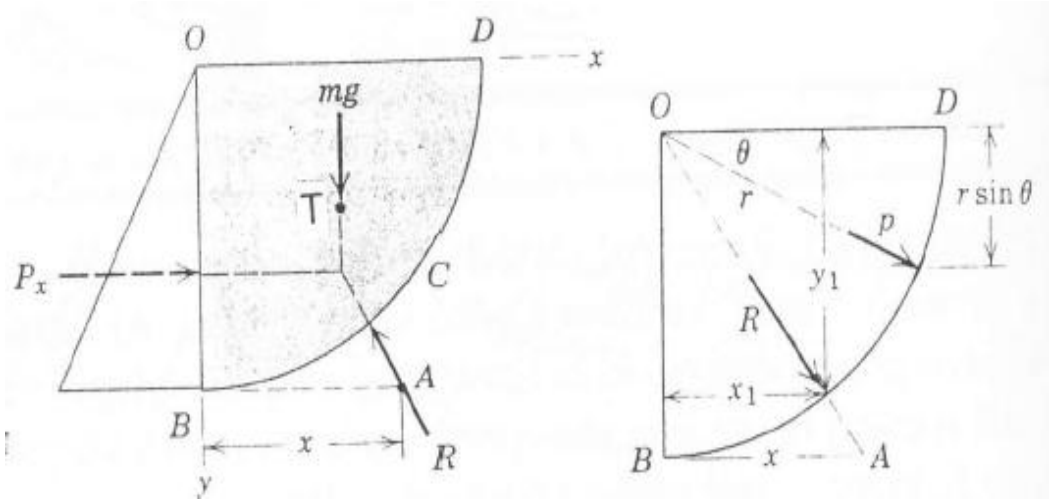
Calculate the resultant force R of the pressure of water with density ρ, acting on the surface of cylindrical dam with radius r and length b, (see in figure). Resultant force R of water pressure can be calculated as:

$$R=\sqrt{R_x^2+R_y^2} \qquad R_x=\int_0^{\pi/2}\rho g r^2 b\sin\theta\cos\theta\ d\theta \qquad R_y=\int_0^{\pi/2}\rho g r^2 b\sin^2\theta\ d\theta$$



ρ, g, b, r are constants.

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APPLIED PROBLEM

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Consider two projects. We will invest *x* million Euro to the first project, and *y* million Euro to the second one. Estimated income from both projects is determined by function $R(x, y) = x^2 y$, while we have 9 million Euro at disposal. How to estimate values of investments *x* and *y* in order to receive the maximal profit from both projects?

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APPLIED PROBLEM

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Let $y(t) \geq 0$ be the temperature of an object at time $t \in \boldsymbol{R}$, $t \geq 0$, while $y'(t)$ is the rate of its temperature drop, and k is a real constant determining the specific cooling process in the environment with the constant temperature T. Then the physical law of the cooling process can be described by the differential equation with separable variables

$$y'(t) = -k(y(t) - T)$$

Problems: Freshly baked bread has temperature 180°C, which drops after half an hour to 100°C. To cool it more rapidly, the bread will be stored in an air-conditioned chamber with a constant temperature 15°C.
How long we should wait in order to enjoy eating it still warm at 20°C?

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FEEDBACK ANALYSIS

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- EduScrum method introduced to students a new way of team work and knowledge assessment, which they rated quite positively.
- Mathematical, technical and social skills working in a team with colleagues were trained hand-in-hand.
- Up to 82,5% of students would prefer to be assessed in mathematics during semester in combination of both – individual and eduScrum team method.

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CONCLUSIONS

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Applied problems should be introduced into teaching mathematics from the very first year of study, because students need to:

- work with differently marked variables
  in Mathematics we usually use only *x*, *y* while
  in Physics and technical subjects they use mostly *t*
- use general physical constants and units,
  e.g. gravitational acceleration constant g , various other constants as ε, ρ, λ, μ, etc., which are obviously used in applied problems

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CONCLUSIONS

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Students need to:

- become aware of the parallels between concepts in mathematics and technical subjects
  - mathematical concept of a stationary point of function is related to the concept of the equilibrium position of the state in technical disciplines
  - value of function derivative at the particular point might represent the instantaneous velocity of a moving particle
  - eigen vectors of a matrix determine invariant directions of linear transformation defined by this matrix – used in photogrammetry for recognition of object real dimensions from photos

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CONCLUSIONS

Students need to:

- understand the role of mathematics
  - in science
  - in technology
  - in simple ordinary life

Our civilization is born on mathematical literacy, as we:

- measure
  - compare
    - analyse
      - estimate
        - optimize
          - evaluate
            - distribute
              - **solve problems ...**

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CONCLUSIONS

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- Students need a variety of opportunities to apply and use knowledge and skills in different learning situations and modes of instructional practise.
- Hands-on activities and real-world problems are more interesting and much more motivational than theoretical calculations without application.
- Social skills are not less important than any other.
- Students must be treated with respect, sincere interest, attention and appreciation.

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CONCLUSIONS

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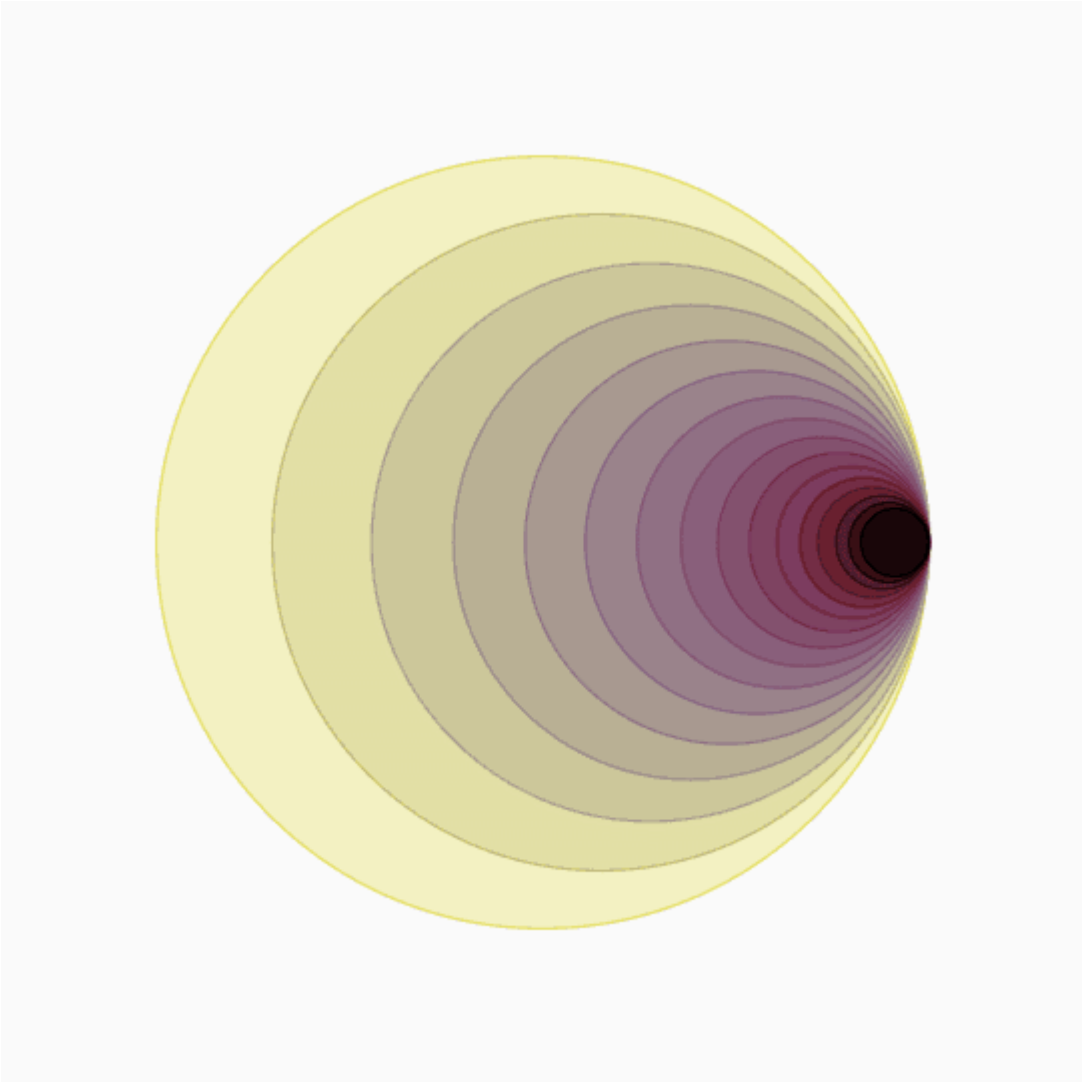
Excellent students pointed also to some negative aspects of team work that should be overcome:

- weaker students could achieve the same score for less work in the team as the better ones who worked more in order to reach the high score
- team work slightly hindered the best students´ study success and development, and some of them considered individual work as more challenging
- team leader responsible for the work distribution should play an important role in assessing individual students´ involvement and score distribution within the team according to the real work done by each of the students

Our students enjoyed the eduScrum method

- After completing Mathematics I in the first semester they asked to apply it also in the second semester subject Mathematics II.
- Some of these students who attended Basics of Statistics in the second year asked for the same approach.
- There was recorded a remarkable increase of motivation.
- It was not possible to analyse the method impact on the students´ success at exams, due to the random choice of the experiment respondents.



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WOULD YOU LIKE TO EXPERIENCE THIS EXPERIMENT LIVE?

WHY NOT !?

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