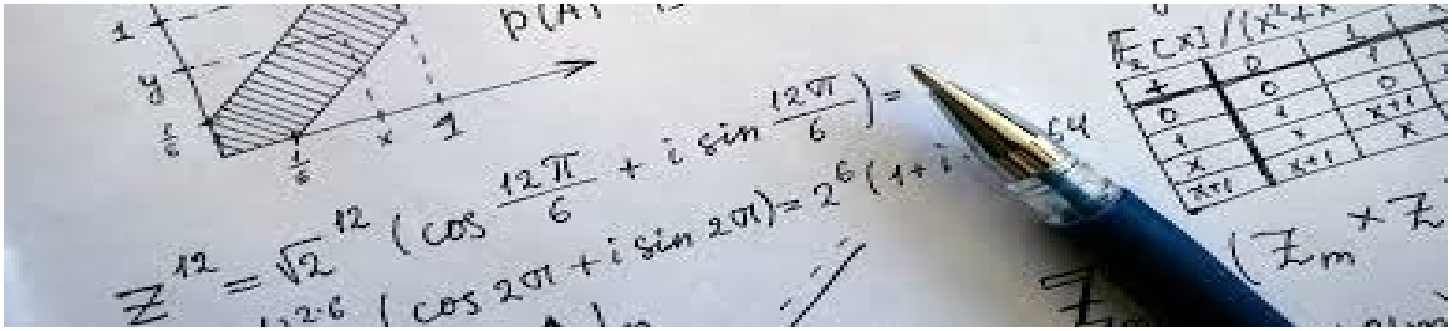

Innovating methods for teaching and learning: constructing and implementing examples in a computer assessment system (STACK)

INNOVATIVE STRATEGIES FOR TEACHING AND LEARNING MATHEMATICS

Workshop for teacher trainers

La Laguna University, Tenerife, Spain

March 11 – March 15 2024

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Background

Continuous assignments have become increasingly common thanks to an intensive development of digital solutions in the form of computer aided assessment systems, where students receive direct feedback

However, the feedback given is often limited to **right or wrong**, sometimes with the correct answer added. In this way, students are not offered feedback that can function **formatively**.

The purpose of the WP3 in Pythagoras is to design tasks with customized automatically generated **formative feedback.**

To make this possible, we combine a computer-aided assessment system (STACK) with a dynamic mathematics software (GeoGebra).

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In focus

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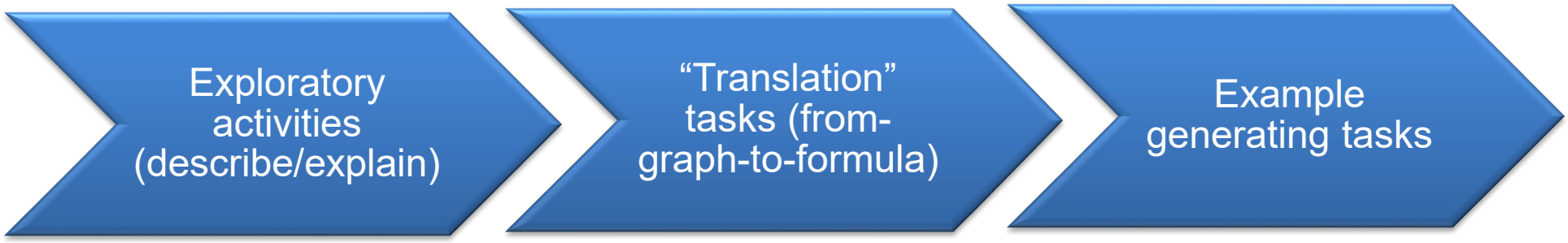
Examples of type of tasks



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Examples of formative feedback

Silent recorded demonstrations

GeoGebra visualization

Multiple choice question that help the student reflect

Depending on the type of answer, give a new easier question that can help.

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STACK is an assessment system for mathematics, science and related disciplines, designed to enable students to answer questions with a mathematical expression.

Math questions often have to be multiple choice questions, but with the STACK question type, students can enter direct math answers into Moodle.

STACK questions can have several parts and each part can be assessed separately. STACK questions can also include randomly generated components, which makes it much easier to create a series of practical questions and also prevent students from collaborating during a test.

Staff feedback options are now essential. Student responses can be assessed on the basis of a series of tests, with feedback and different grades returned to students based on test results.

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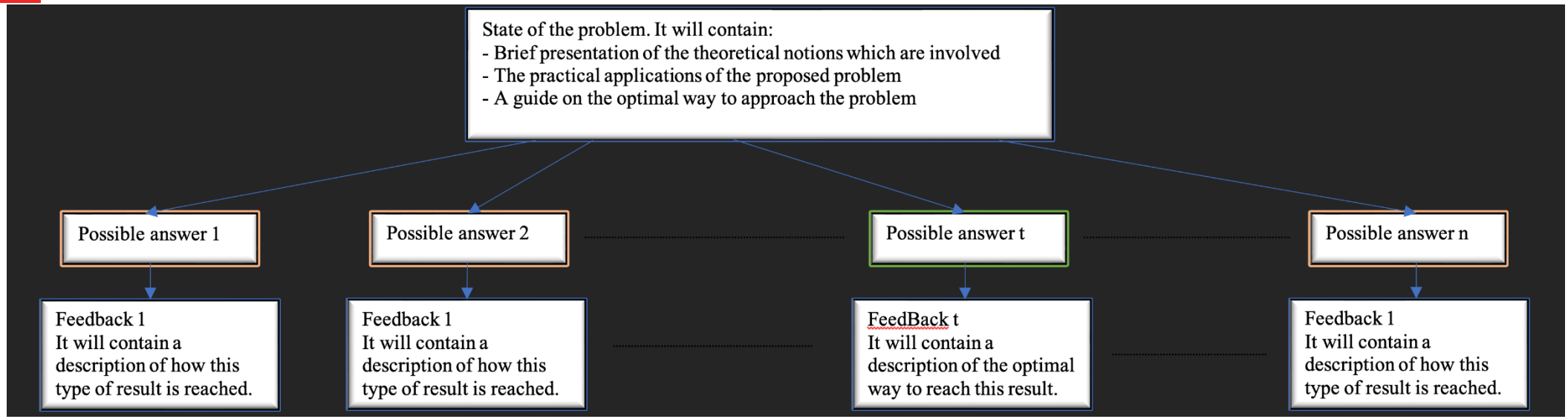

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STACK offers the possibility to introduce at the beginning of the evaluation a theoretical part that will give the student a short recapitulation that will help in obtaining a maximum score.

Information

Flag question

Definition: A sequence of real numbers is a function $f:\mathbb{N}\to\mathbb{R}$, $f(n)=a_n$ or $f:\mathbb{N}\setminus A\to\mathbb{R}$, where $A\subset\mathbb{N}$ finite, $f(n)=a_n$.

Notation: (a_n) is the sequence defined by the function f.

Definition: A sequence of real numbers (a_n) is increasing (decreasing) if $a_n\leq a_{n+1}$ $(a_n\geq a_{n+1})$, $\forall n\geq 0$. If the above inequalities are strictly, then the sequence is called strictly increasing (strictly decreasing).

To study the monotony of a sequence (a_n), the sign of the difference $\Delta a_n=a_{n+1}-a_n$ can be establish or to compare the ratio $\frac{a_{n+1}}{a_n}$ with 1, when $a_n>0, \forall n\geq 0$.

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After completing the theoretical parts, the student can complete the questions from the evaluation test.

There are several types of STACK questions that can be assigned to students to test the level of knowledge related to a subject.

Let $f : \mathbb{R} \to (0, \infty)$, $f(x) = e^{-2x}$, $g : (0, \infty) \to (1, \infty)$, $g(x) = \frac{1}{e^{-2x}}$ and $h : (0, \infty) \to \mathbb{R}$, $h(x) = -\frac{1}{2}lnx$. Which of the following statement(s) is/are true?

- a. g and h are inverses of each other.
- b. f and h are inverses of each other.
- c. f and g are inverses of each other.
- d. None of f, g or h are inverses of each other.

Let $f : (0, \infty) \to (0, 1)$, $f(x) = e^{-2x}$, $g : (0, \infty) \to (1, \infty)$, $g(x) = \frac{1}{e^{-2x}}$ and $h : (0, \infty) \to \mathbb{R}$, $h(x) = -\frac{1}{2}lnx$. Determine the following composite functions:

a) $(f \circ g)(x) = f(g(x)) =$

b) $(f \circ h)(x) = f(h(x)) =$

c) $(h \circ f)(x) = h(f(x)) =$

Investigate, by using GeoGebra, how the graph of the trigonometric function $f(x) = A\,sin(B(x + C)) + D$, depends on the values of the parameters A, B, C and D.

a) Describe in what way the various parameters alter the graph.

Compute the solutions for the equation: $3 \cdot y^2 - y - 1 = 0$

[y = ,y =]

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Students input equations directly into Moodle and can see a preview before they submit, and can receive feedback for each individual answer to the question

Tidy STACK

Let $(s_n) = \left\{ \frac{P_k(n)}{Q_i(n)}, n \in \mathbb{N} \right\}$ a sequence such that $P_k(n)$ and $Q_i(n)$ are two polinomyal of degrees $k \leq 3$ respectively.
Give an example of a sequence s_n such that sequence is
a) divergent;

-(2*n^3)+5*n+1

Your last answer was interpreted as follows:

$$-2 \cdot n^3 + 5 \cdot n + 1$$

The variables found in your answer were: [n]

b) convergent to zero;

{-(2*n^2)+5*n+1}/{n^3-27}

Your last answer was interpreted as follows:

$$\frac{\{-2 \cdot n^2 + 5 \cdot n + 1\}}{\{n^3 - 27\}}$$

The variables found in your answer were: [n]

c) convergent to $\frac{3}{5}$.

{6*n^3+5*n+1}/{10*n^3-1}

Your last answer was interpreted as follows:

$$\frac{\{6 \cdot n^3 + 5 \cdot n + 1\}}{\{10 \cdot n^3 - 1\}}$$

The variables found in your answer were: [n]

Tidy STACK question tool

Let $(s_n) = \left\{ \frac{P_k(n)}{Q_i(n)}, n \in \mathbb{N} \right\}$ a sequence such that $P_k(n)$ and $Q_i(n)$ are two polinomyal of degrees $k \leq 3$ respectively.
Give an example of a sequence s_n such that sequence is
a) divergent;

-(2*n^3)+5*n+1

Your last answer was interpreted as follows:

$$-2 \cdot n^3 + 5 \cdot n + 1$$

The variables found in your answer were: [n]

✖ **Incorrect answer.**

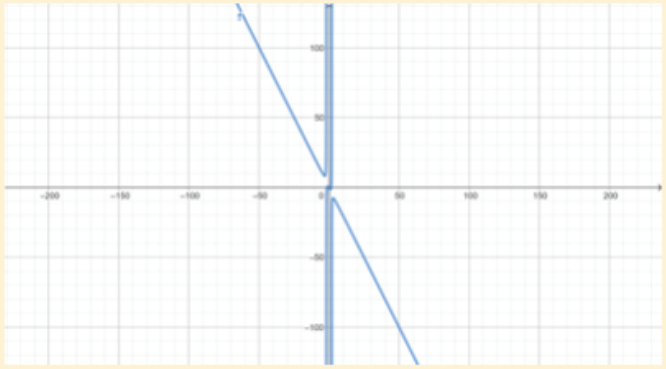
Consider the sequence (s_n), $s_n = \frac{a_k n^k + a_{k-1} n^{k-1} + \ldots + a_1 n + a_0}{b_i n^i + b_{i-1} n^{i-1} + \ldots + b_i n + b_0}$.

(s_n) is divergent if $k > i$

For example:

(s_n), $s_n = \frac{-2n^3+5n+1}{n^2-4}$, $n \in \mathbb{N}$ is divergent, $\lim_{n \to \infty} s_n = -\infty$

Using GeoGebra, we can consider the function $f(x) = \frac{-2x^3+5x+1}{x^2-4}$, $x \in \mathbb{R} \backslash \{-2, 2\}$



b) convergent to zero;

{-(2*n^2)+5*n+1}/{n^3-27}

Your last answer was interpreted as follows:

$$\frac{\{-2 \cdot n^2 + 5 \cdot n + 1\}}{\{n^3 - 27\}}$$

The variables found in your answer were: [n]

✔ **Correct answer, well done.**

Implementation of questions. Example

➢Question variables (optional) – In the following example, the variables ta1, ta2, ta3 represent the correct answers provided by the teacher, against which the student's responses will be checked.

Question variables

```
ta1:{-2*n^3+5*n+1}/{n^2-4};
ta2:{-2*n^2+5*n+1}/{n^3-27};
ta3:{6*n^3+5*n+1}/{10*n^3-1};
```

➢Question text

Let \((s_n)=\left\{\frac{P_k(n)}{Q_i(n)}, n\in\mathbb{N}\right\}\) a sequence such that \(P_k(n)\) and \(Q_i(n)\) are two polinomyal of degrees \(k\leq 3\) respectively.
Give an example of a sequence \(s_n\) such that sequence is
a) divergent; [[input:ans1]] [[validation:ans1]][[feedback:prt1]]

b) convergent to zero; [[input:ans2]] [[validation:ans2]][[feedback:prt2]]

c) convergent to \(\frac{3}{5}\). [[input:ans3]] [[validation:ans3]][[feedback:prt3]]

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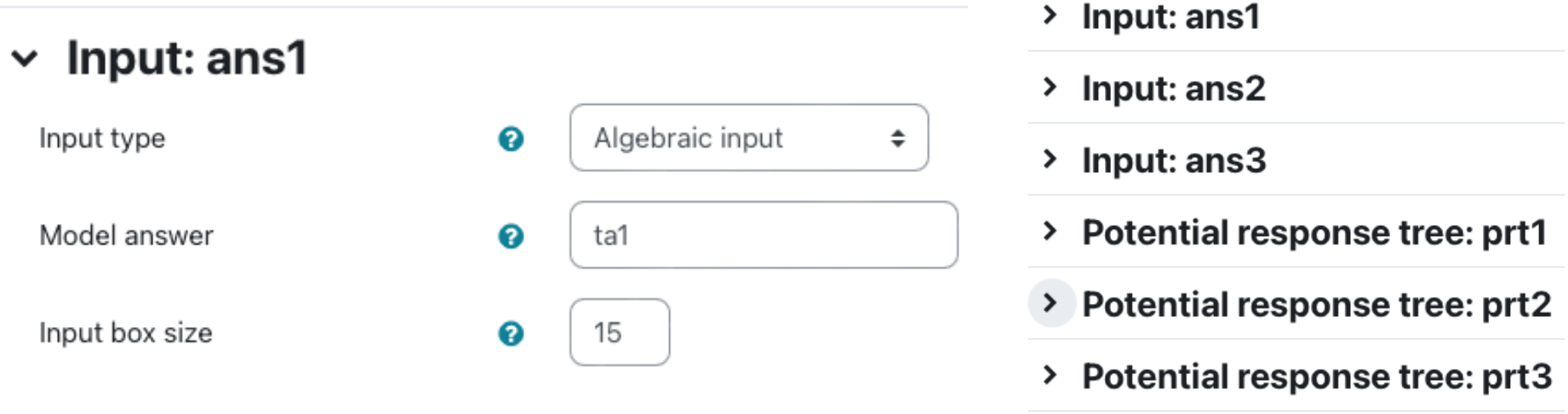
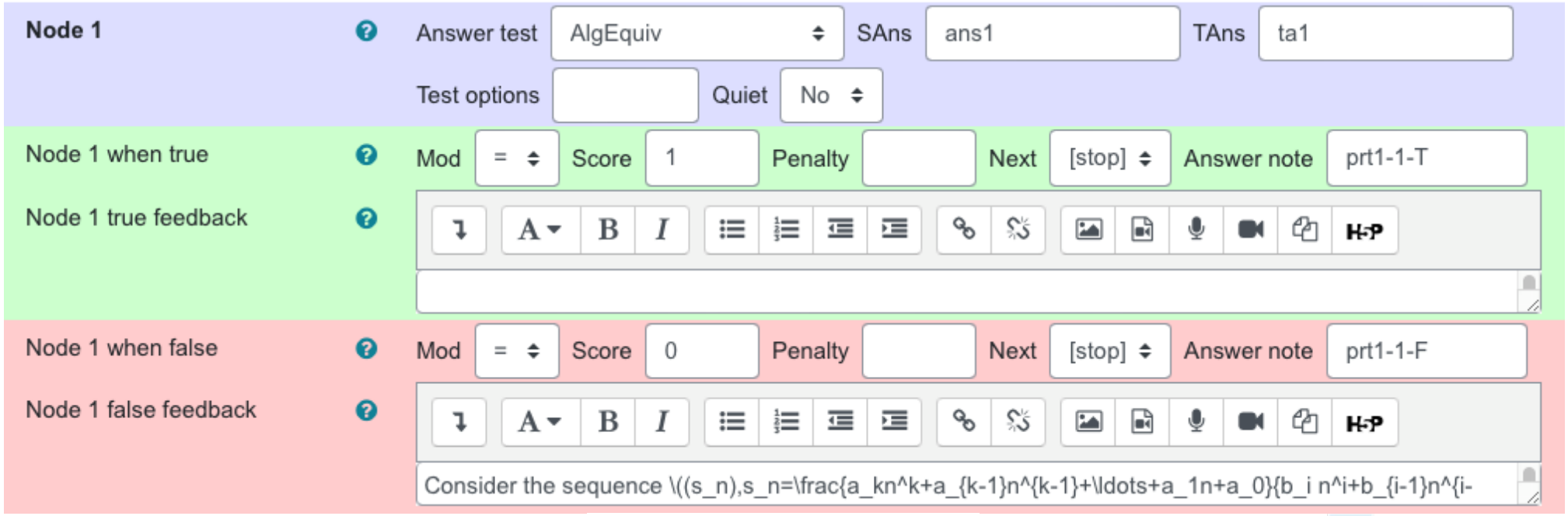
Implementation of questions. Example

- Input answers: ans1, ans2, ans3, etc



- Potential response tree for each answer



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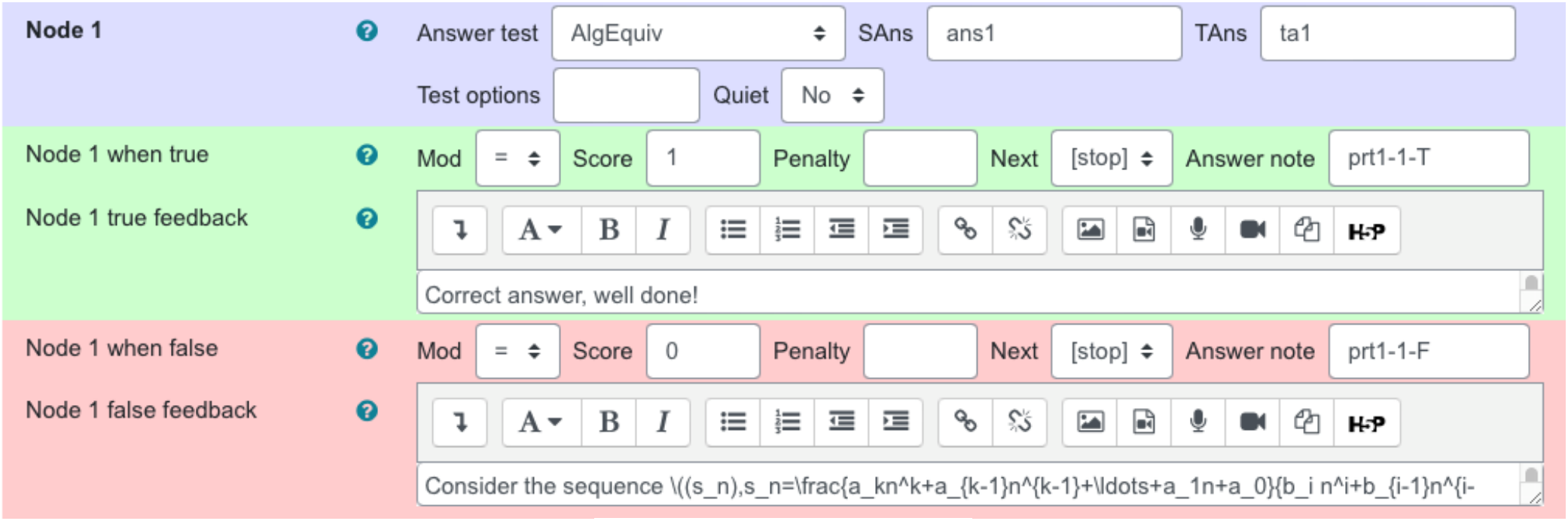
Implementation of questions. Example

➢Potential response tree for each answer - this is where formative feedback comes:

- ❖ for whatever the student's answer is, we can give him feedback, either for the correct answer, partially correct or for the wrong answer
- ❖ if the student answered correctly, we can send him a feedback with the solution proposed by the teacher, and in case of a wrong answer, he will receive the correct answer and the related solution.



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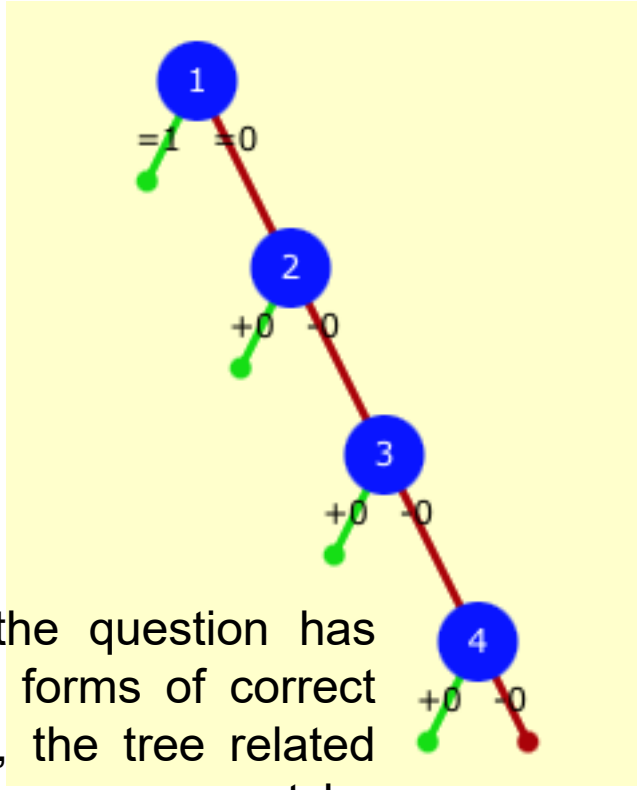
Implementation of questions. Example

There are questions where the answer can be written in several forms. At the time of implementation, the teacher must provide all forms of correct answers. For example, the following variables constitute all forms of correct answer:

Question variables

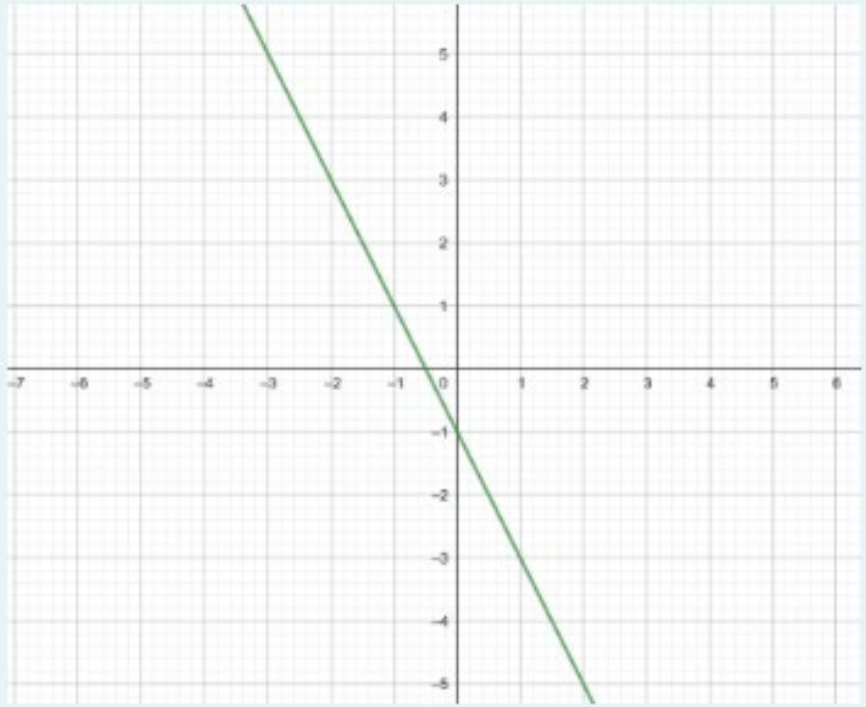
ta1: y=-2*x-1
ta2: y=-1-2*x
ta3: -2*x-y-1=0
ta4: 2*x+y+1=0



Since the question has several forms of correct answer, the tree related to the answers must be implemented

The question is:

Write an equation for line in the graph below:



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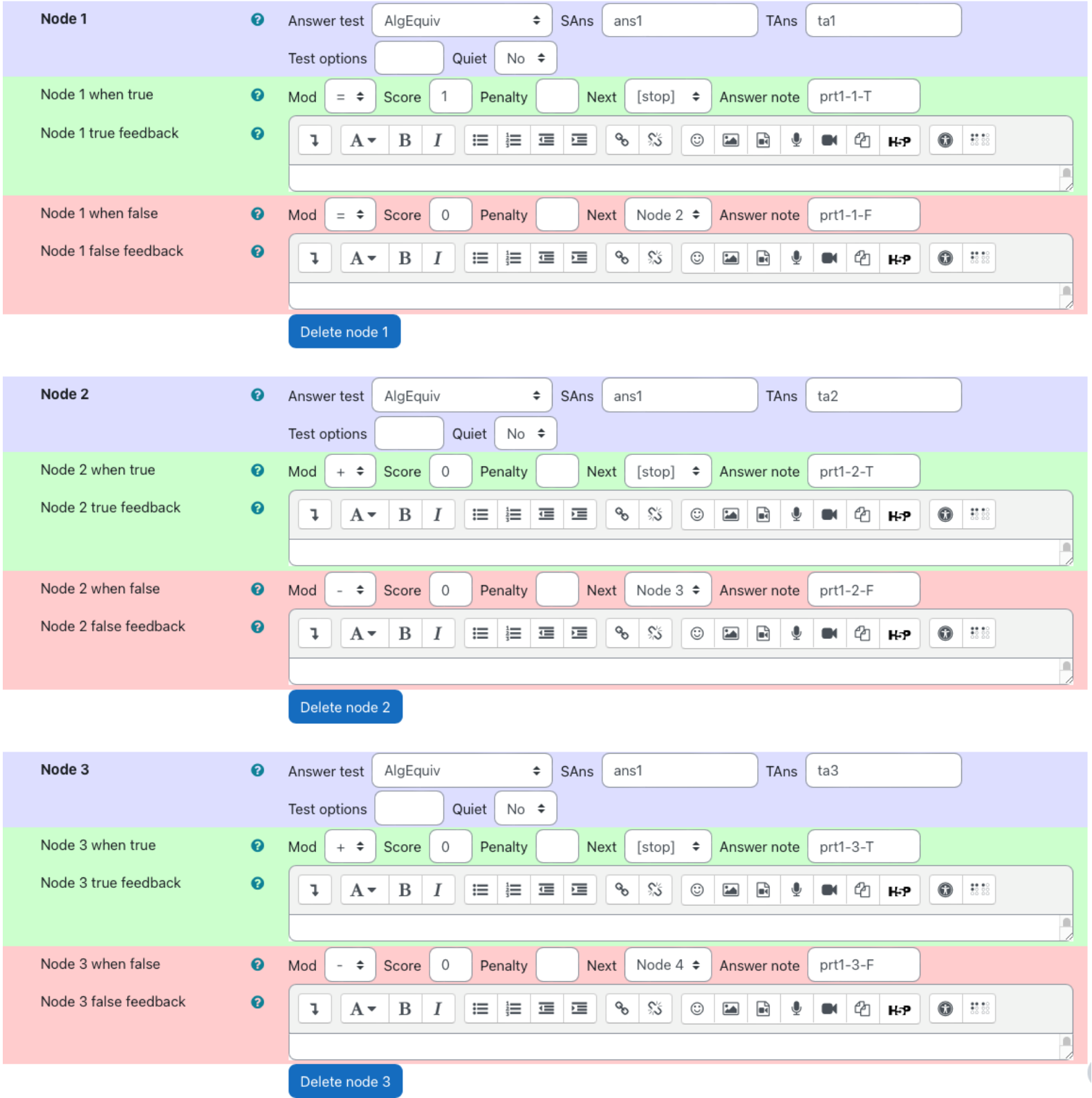
Implementing the tree related to the correct answers:



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Types of questions

In the stack there are several types of questions that can be implemented:

- *Multiple choice* – allows the selection of a singular o multiple responses from a pre-defined list
- *True/False* – a simple form of multiple choice question with just the two choices “True” and ”Fals”
- *Matching* – the answer to each of a number of subquestion must be selected from a list of possibilities
- *Essay* – allows a response of a file upload and/or online text. This must then be graded manually
- *Drag and drop into text* – STACK provides mathematical questions for the Moodle quiz. These use a computer algebra system to establish the mathematical properties of the student's responses.
- *Select missing words* – missing words in the question text are filled in using drop-down menus
- *STACK* - STACK provides mathematical questions for the Moodle quiz. These use a computer algebra system to establish the mathematical properties of the student's responses.

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Types of questions

True/False – a simple form of multiple choice question with just the two choices “True” and ”Fals”

Determine whether the following relation is a function $\{(2, 1), (3, 2), (-1, 1), (0, 2)\}$

Select one:

- True
- False ✖

We can consider the function $f(x) = ax^3 + bx^2 + cx + d$, where a, b, c, d can be determined from the conditions:

$$\begin{cases} f(2) = 1 \\ f(3) = 2 \\ f(-1) = 1 \\ f(0) = 2 \end{cases}$$

The correct answer is 'True'.

Determine whether the following relation is a function $\{(2, 1), (3, 2), (-1, 1), (0, 2)\}$

Select one:

- True ✔
- False

Well done!

The correct answer is 'True'.

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Types of questions

Implementation of the True/False question type:

Correct answer: True

Feedback for the response 'True'.

Well done!

Feedback for the response 'False'.

We can consider the function \(f(x)=ax^{3}+bx^{2}+cx+d\), where \(a\), \(b\), \(c\), \(d\) can be determined from the conditions:

\(\begin{cases} f(2) = 1 \\ f(3) = 2 \\ f(-1) = 1\\ f(0) = 2 \end{cases} \)

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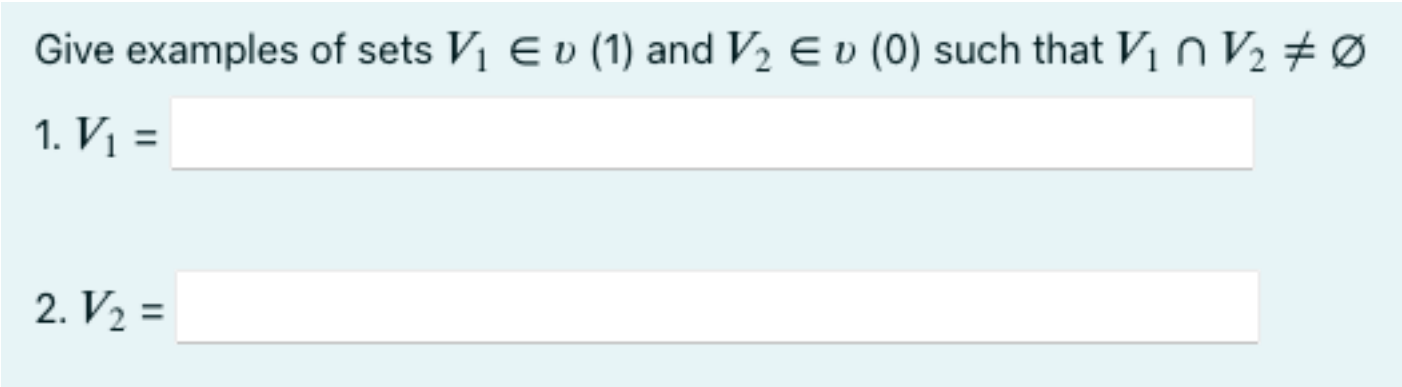
Types of questions

Implementation of the STACK question type:

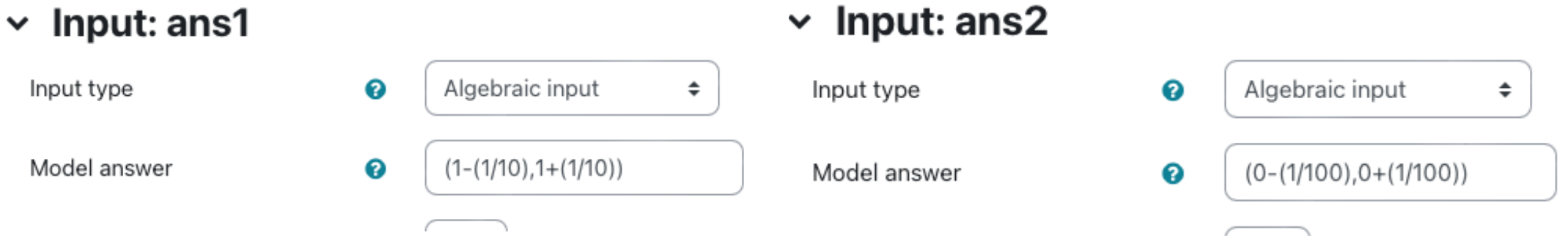


The stack code:

```
Give examples of sets \(V_{1}\in\) \(\upsilon\) (1) and \(V_2\in\) \(\upsilon\) (0) such that \(V_1\cap V_2\neq\) \
(\varnothing\)

1. \(V_1\) = [[input:ans1]] [[validation:ans1]] [[feedback:prt1]]

2. \(V_2\) = [[input:ans2]] [[validation:ans2]] [[feedback:prt2]]
```



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Types of questions

Implementation of the STACK question type

Potential response tree: prt1

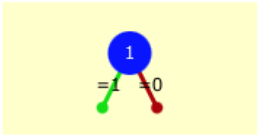
Question value: 1

Auto-simplify: Yes

PRT feedback style: Formative

Feedback variables:

This potential response tree will become active when the student has answered: **ans1**



Node 1 — Answer test: AlgEquiv | SAns: ans1 | TAns: ta1 | Test options: | Quiet: No

Node 1 when true — Mod: = | Score: 1 | Penalty: | Next: [stop] | Answer note: prt1-1-T

Node 1 true feedback: Well done!

Node 1 when false — Mod: = | Score: 0 | Penalty: | Next: [stop] | Answer note: prt1-1-F

Node 1 false feedback: - remember them the definitions of the neighborhood of a point and disjoint sets

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Types of questions

Implementation of the STACK question type

Potential response tree: prt2

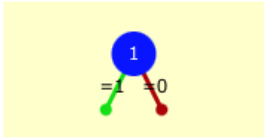
Question value: 1

Auto-simplify: Yes

PRT feedback style: Formative

Feedback variables:

This potential response tree will become active when the student has answered: **ans2**



Node 1 — Answer test: AlgEquiv | SAns: ans2 | TAns: ta2 | Test options: | Quiet: No

Node 1 when true — Mod: = | Score: 1 | Penalty: | Next: [stop] | Answer note: prt2-1-T

Node 1 true feedback: Well done!

Node 1 when false — Mod: = | Score: 0 | Penalty: | Next: [stop] | Answer note: prt2-1-F

Node 1 false feedback: - represents on the axis of real numbers the neighborhood of a point \(a\)

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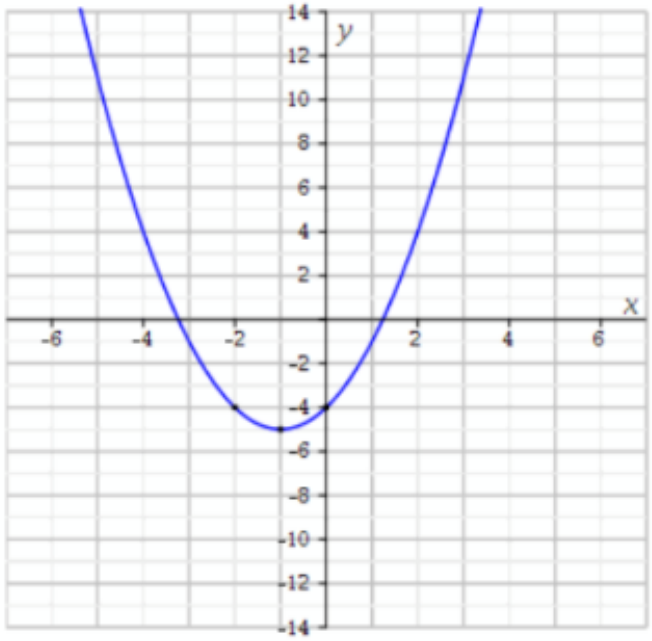
Types of questions

Another type of question(STACK question): within the statement, you can also insert the graph related to the problem, and the students input equations directly into Moodle, and can see a preview before they submit.

Let $f(x) = k(x + a)^2 + b$ be a quadratic function.

Investigate, how the graph of the function depends on the values of the parameters a, b and k.

d) The figure below shows the graph $y = f(x)$ of a quadratic function f. Use the method that you described in prompt c) to determine f. Use the method that you described in prompt c) to determine f.



$f(x)$ = 2*x^2+4*x-1

Your last answer was interpreted as follows:

$$2 \cdot x^2 + 4 \cdot x - 1$$

The variables found in your answer were: $[x]$

Types of questions: Multiple choice question

This type of question allows students to choose as an answer one or more of the items and has the possibility to access GeoGebra to calculate the correct answer.

For this type of question, for each student the items can be generated randomly, so that the items a, b, c, d are always in a different order.

Give an example of a sequence:

$(s_n)=\frac{an+b}{cn+d}, n \in \mathbb{N}$, where $a, b, c, d \in \mathbb{R}$ such that the sequence is:

a) increasing and convergent to 3;

b) decreasing and convergent to 3.

Use GeoGebra to check your sequences before you answer.

https://www.geogebra.org/calculator

Select the correct answer:

- [ ] a. sequence (s_n) is decreasing to 3 if $a = 3c, ad \leq bc$
- [ ] b. sequence (s_n) is increasing to 3 if $a = 3c, ad \geq bc, \frac{d}{c} \geq 0$
- [ ] c. sequence (s_n) is increasing to 3 if $a = 3c, ad \geq bc$
- [ ] d. sequence (s_n) is decreasing to 3 if $a = 3c, ad \leq bc, \frac{d}{c} \geq 0$

Check

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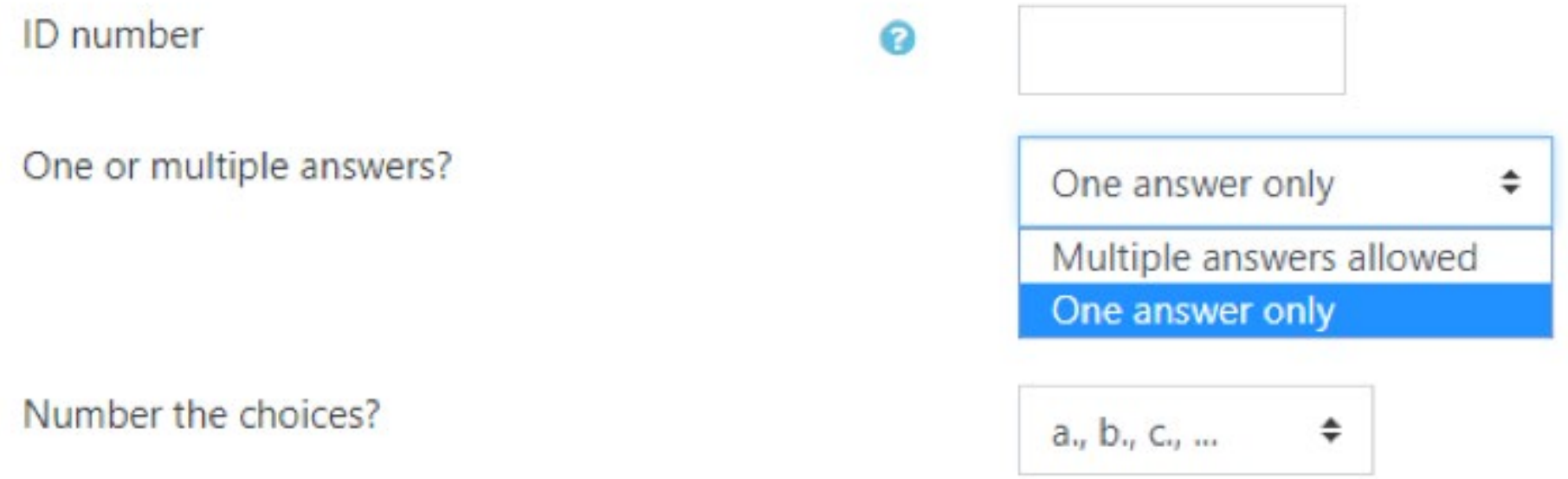
Multiple choice question

At the time of implementation, you can choose the subtype of the question:
1 single correct answer or multiple correct answers



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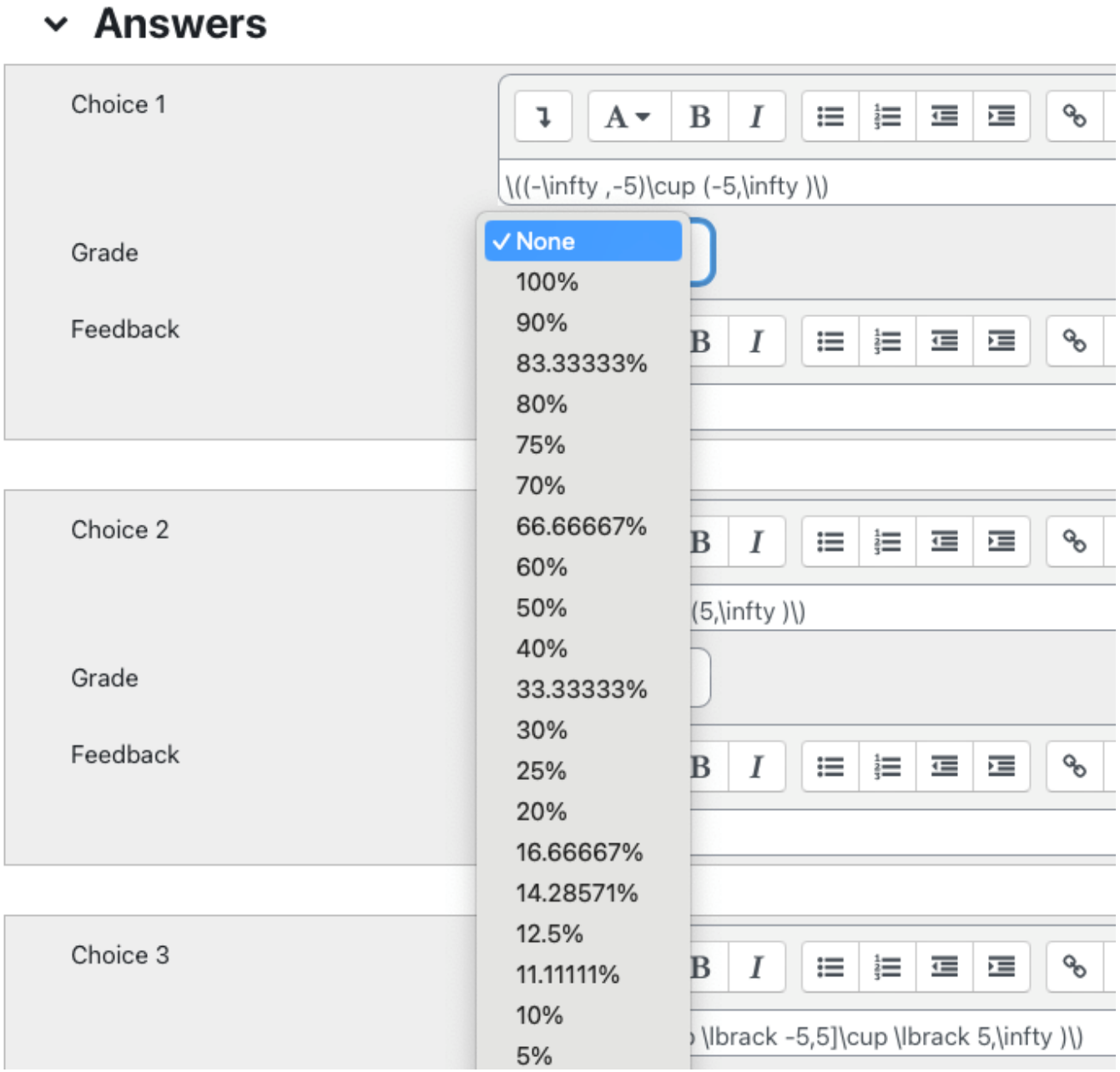
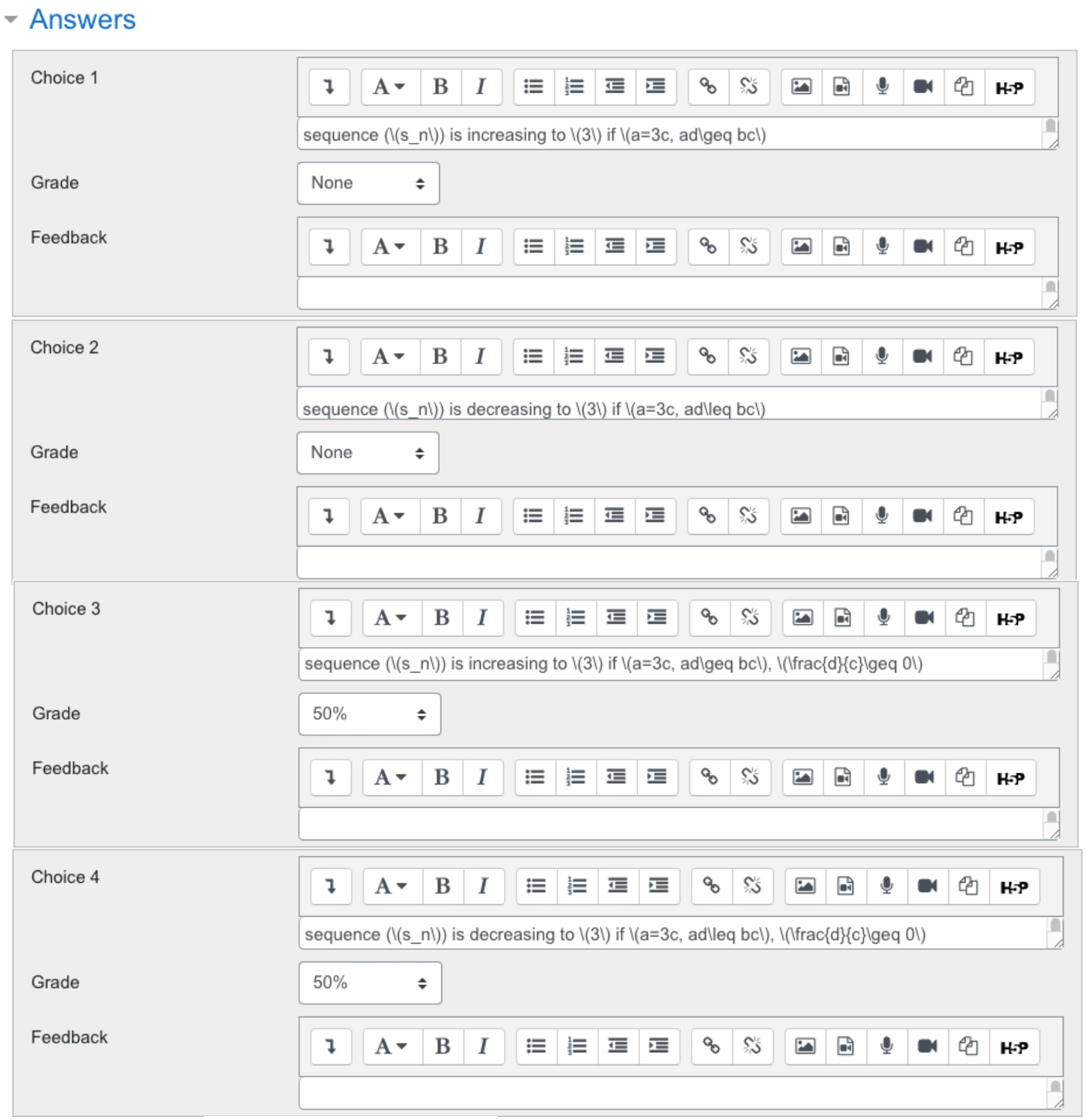
Multiple choice question

Depending on this attribute, the answers will be distributed and the total score of 100 points. Example: if there are two answers, 50 points will be distributed for each answer, if there is only one correct answer, all the points will be awarded



Multiple choice question

If the question has more than one answer, the score must be divided according to how many correct items it has, so that if the student chooses only part of the correct answer, feedback for the partial answer will be sent to him.



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Multiple choice question

For this question, depending on the student's answer, we implemented formative feedback for:

- correct answer
- partially correct answer
- incorrect answer

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Multiple choice question – formative feedback

Your answer is partially correct.

Theory: A sequence (s_n) is increasing (decreasing) if $s_{n+1} \geq s_n, \forall n \in \mathbb{N}, (s_{n+1} \leq s_n, \forall n \in \mathbb{N})$.

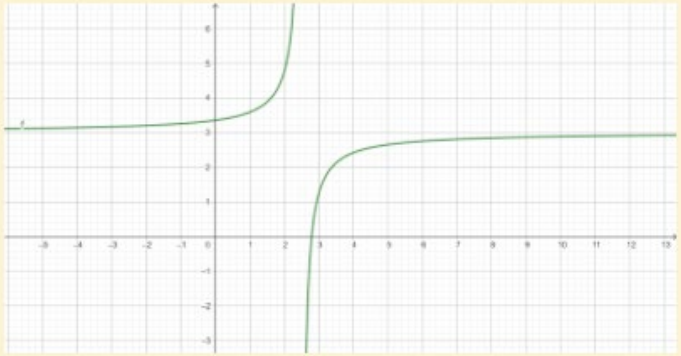
$\lim_{n\to\infty} s_n = \lim_{n\to\infty} \frac{a+\frac{b}{n}}{c+\frac{d}{n}} = \frac{a}{c} = 3, a = 3c, c \neq 0$

Approach: $s_{n+1} - s_n = \frac{ad-bc}{(cn+c+d)(cn+d)} = \frac{c(3d-b)}{c^2(n+1+\frac{d}{c})(n+\frac{d}{c})}$

We consider the function $f : \mathbb{R}\setminus\{-\frac{d}{c}\} \to \mathbb{R}, f(x) = \frac{ax+b}{cx+d}$ for study of monotony; $f'(x) = \frac{ad-bc}{(cx+d)^2}$.

For example $(s_n), s_n = \frac{24n-67}{8n-20}, n \in \mathbb{N}$ we have $ad - bc \geq 0$, but it is not increasing because $s_3 - s_2 < 0, s_3 - s_1 < 0, s_{n+1} - s_n > 0, \forall n \in \mathbb{N}\setminus 2$.

Using GeoGebra, we can represent the graphic of function $f(x) = \frac{24x-67}{8x-20}, x \in \mathbb{R}\setminus\{\frac{5}{2}\}$ and it si observed that $x = \frac{5}{2}$ is the vertical asymptote.

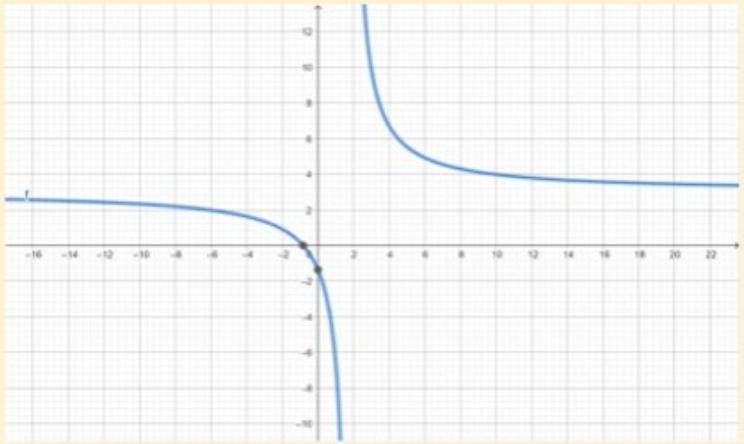


In conclusion, the function must have the vertical asymptote $x = -\frac{d}{c} \leq 0$.

In conclusion, the function must have the vertical asymptote $x = -\frac{d}{c} \leq 0$.

For example $(s_n), s_n = \frac{18n+15}{6n-11}, n \in \mathbb{N}$ we have $ad - bc < 0$, but it is not decreasing because $s_2 > s_1, s_{n+1} < s_n, \forall n \in \mathbb{N}\setminus\{1\}$.

Using GeoGebra, we can represent the graphic of function $f(x) = \frac{18x+15}{6x-11}, x \in \mathbb{R}\setminus\{\frac{11}{5}\}$.



In conclusion, the function must have the vertical asymptote $x = -\frac{d}{c} \leq 0$.

You have correctly selected 1.

The correct answers are:

sequence (s_n) is increasing to 3 if $a = 3c, ad \geq bc, \frac{d}{c} \geq 0$

sequence (s_n) is decreasing to 3 if $a = 3c, ad \leq bc, \frac{d}{c} \geq 0$

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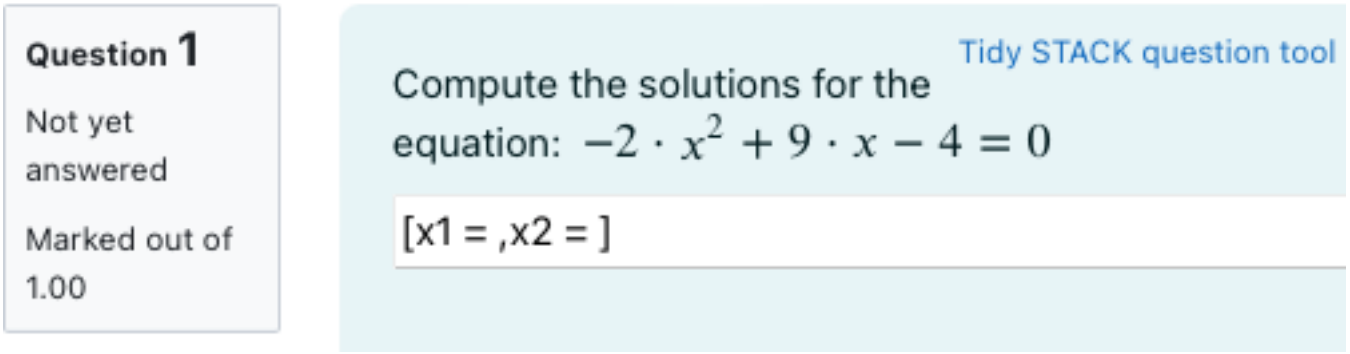
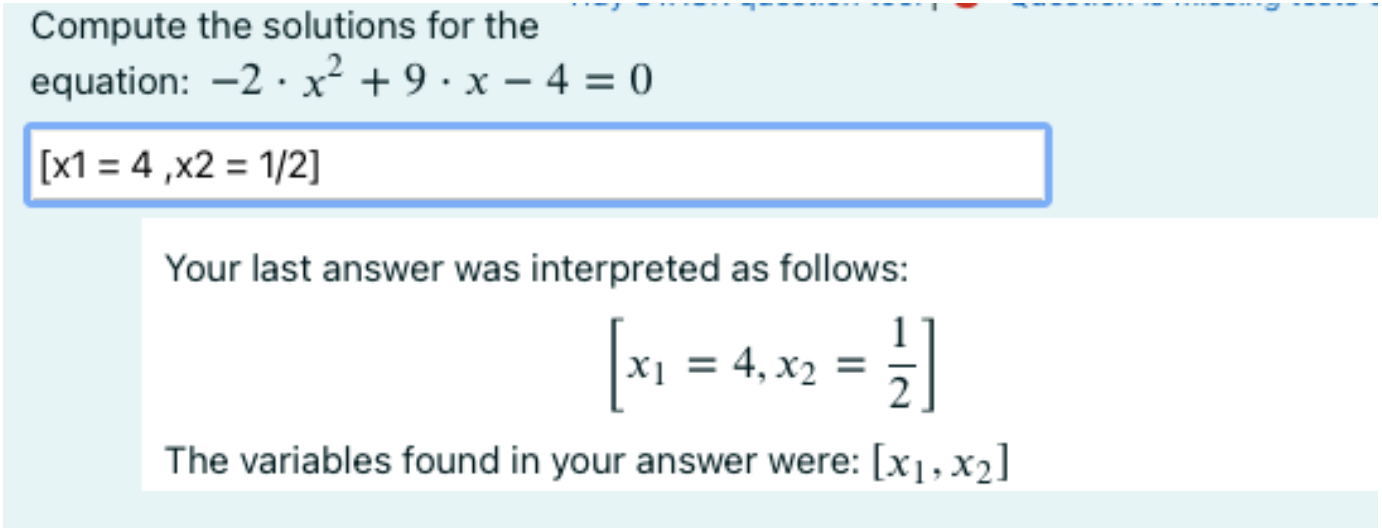
Questions with variable values

When creating the code related to the questions, in the field related to the answer, we can suggest the form of the answer to the student, so as to help him in obtaining the correct answer.





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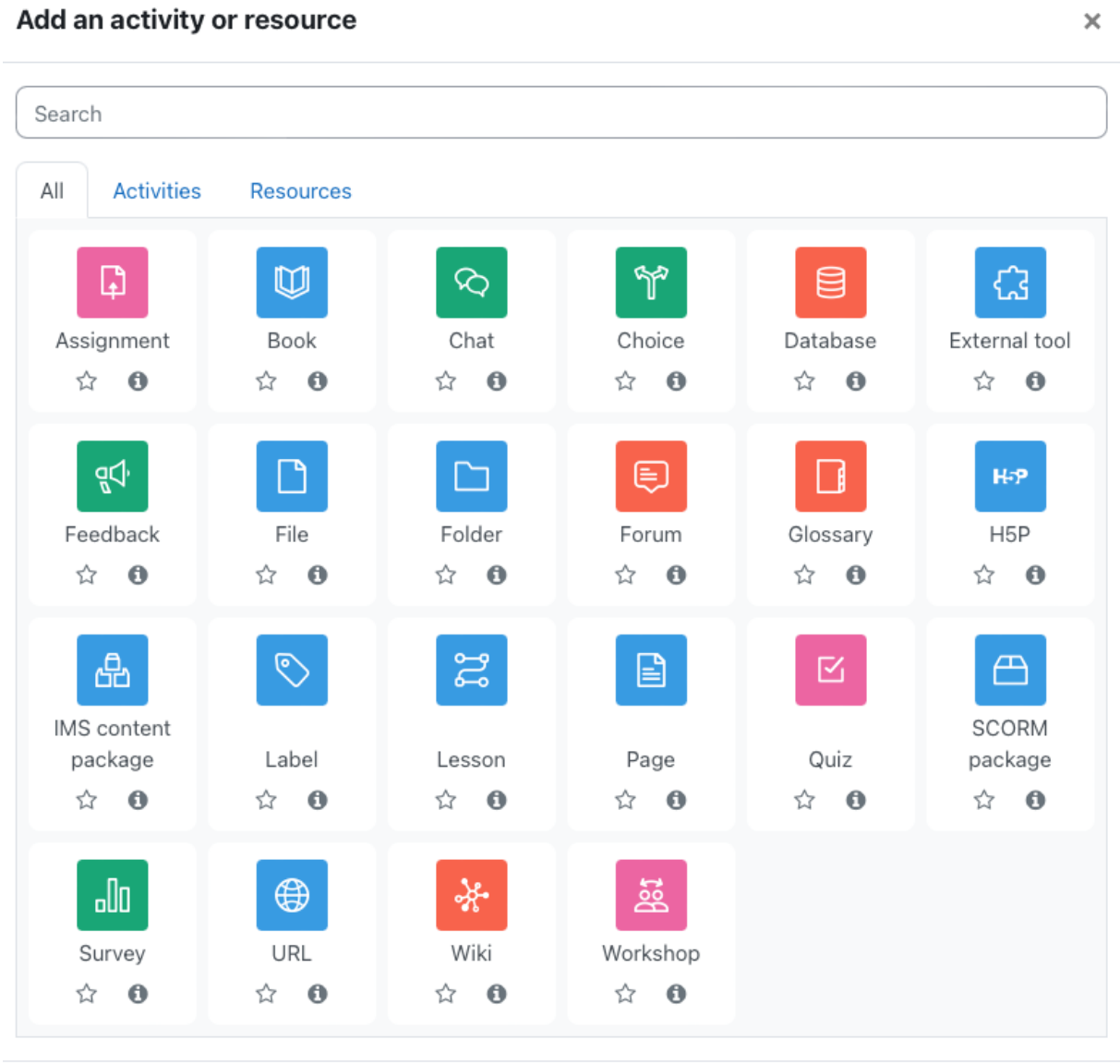
Create, configure the test and assigning it to students

Identify the section where you want to enter the test (for example, the current week) and add a new activity (Add an activity or resource) of the grid test type (Quiz)



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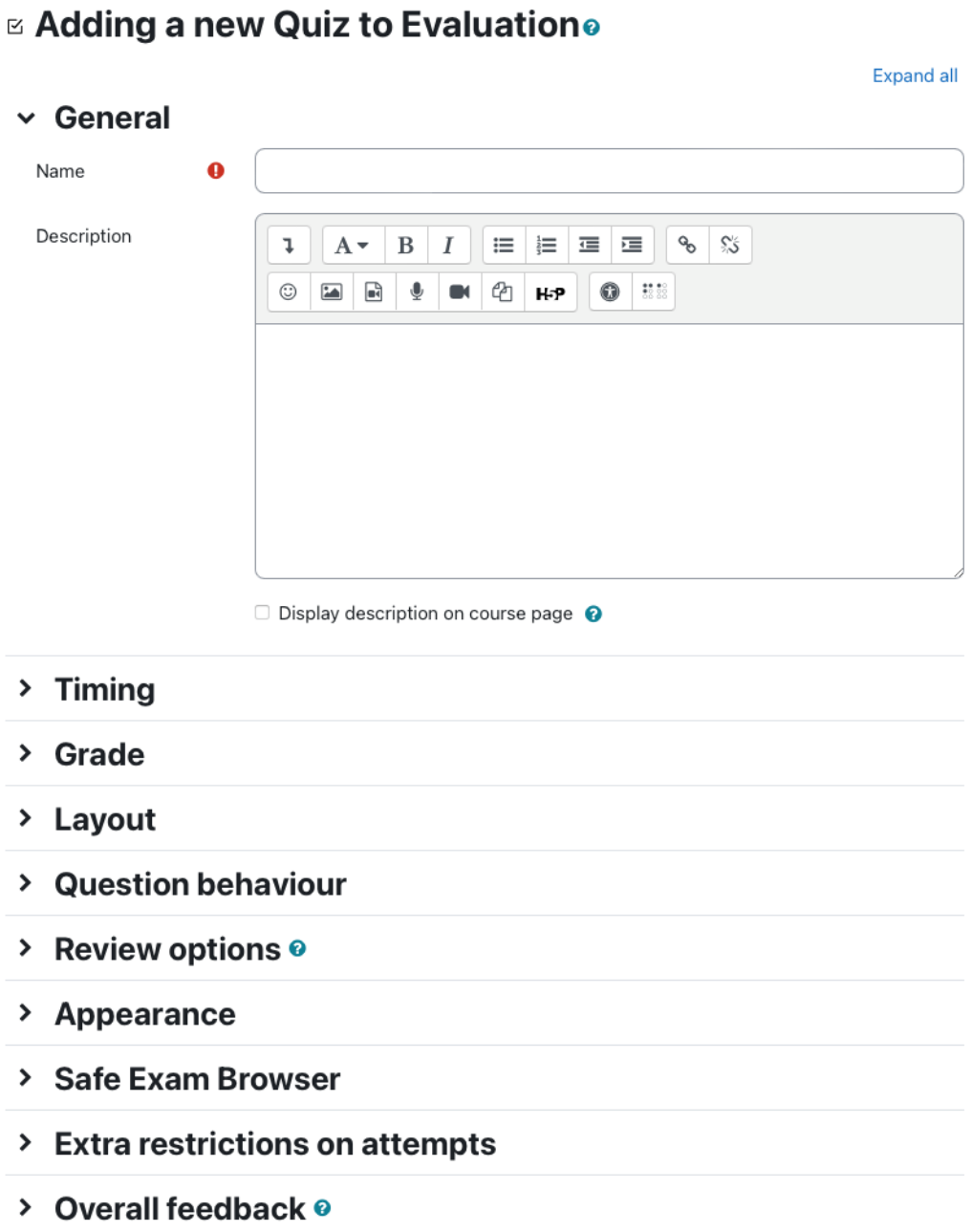
Create, configure the test and assigning it to students

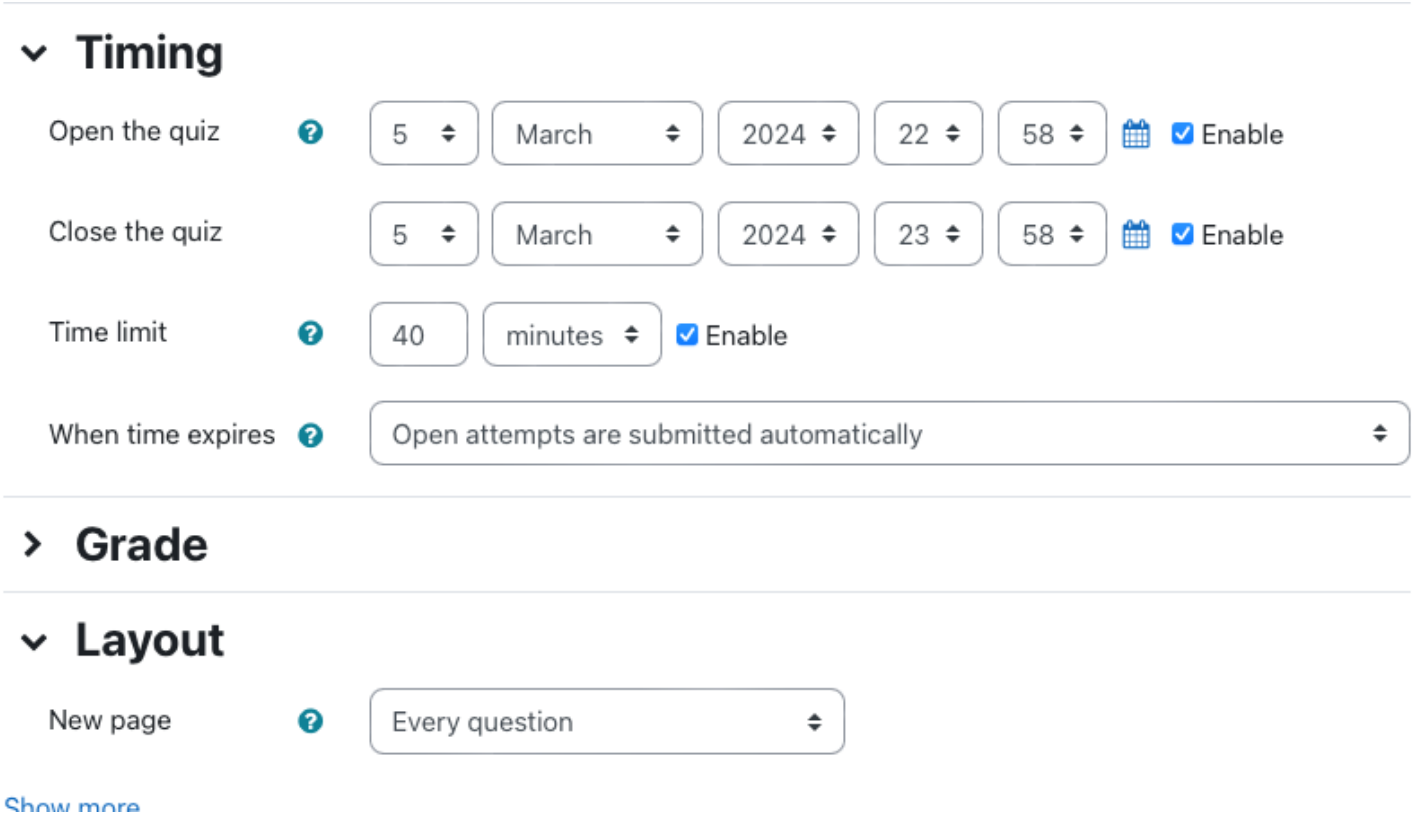
Complete the requested data about the new created test:

- In the General section:
  - The name of the test (Name), a field that will also appear on the main page of the discipline;
  - Description of the test (Description), a field that can appear on the main page of the discipline if you check the appropriate box below the description;



Create, configure the test and assigning it to students

- In the Timing section:
  - Date and time when the test becomes available to students (Open the quiz);
  - Date and time when the test becomes unavailable to students (Close the quiz). Attention: if students are still working at that time, the test will be closed automatically;
- In the Layout section:
  - How to display the questions (New page). We recommend using the way in which all questions are displayed on the same page (Never, all questions on one page) so that the number of interactions with the Moodle server is minimal;



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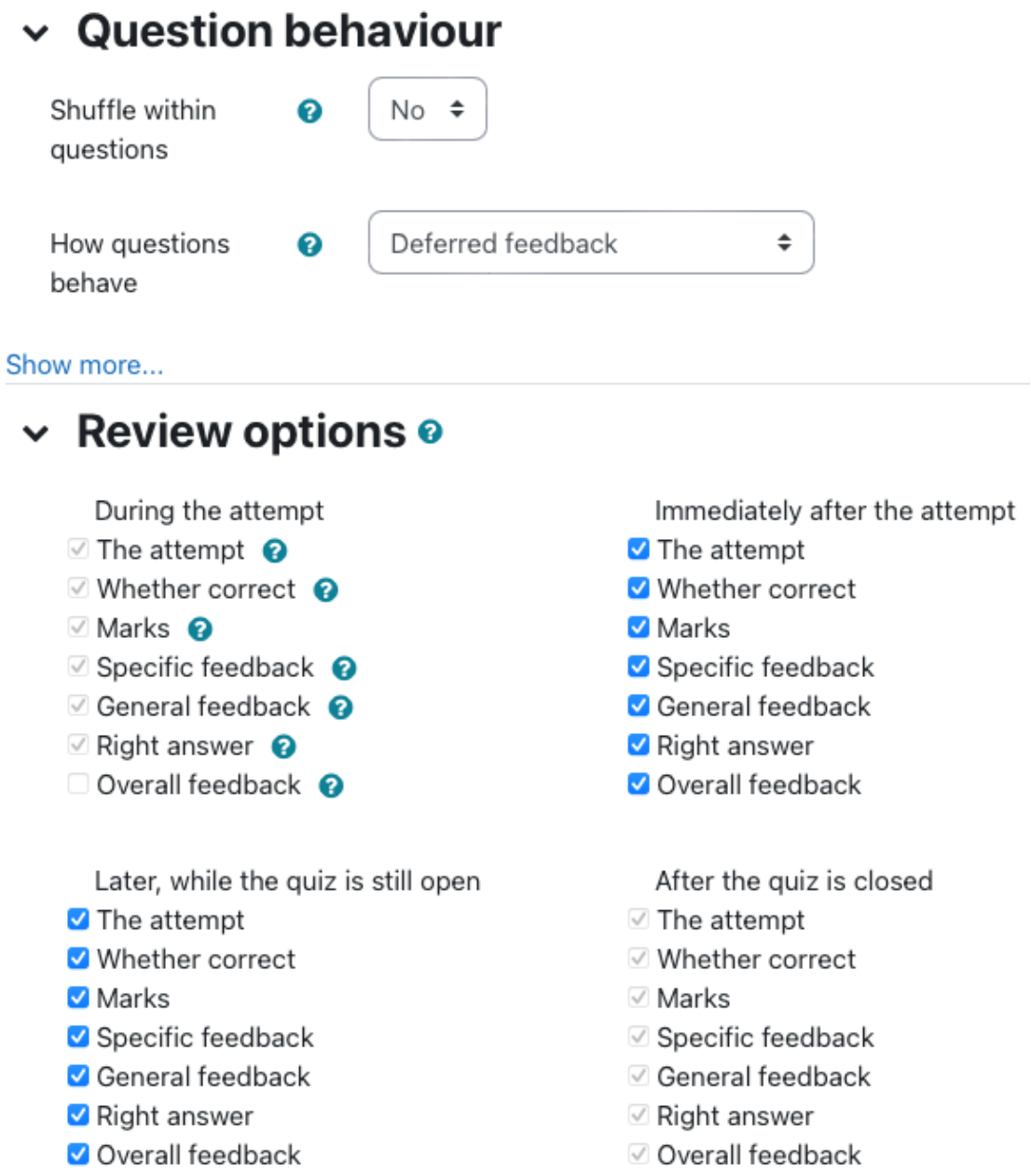
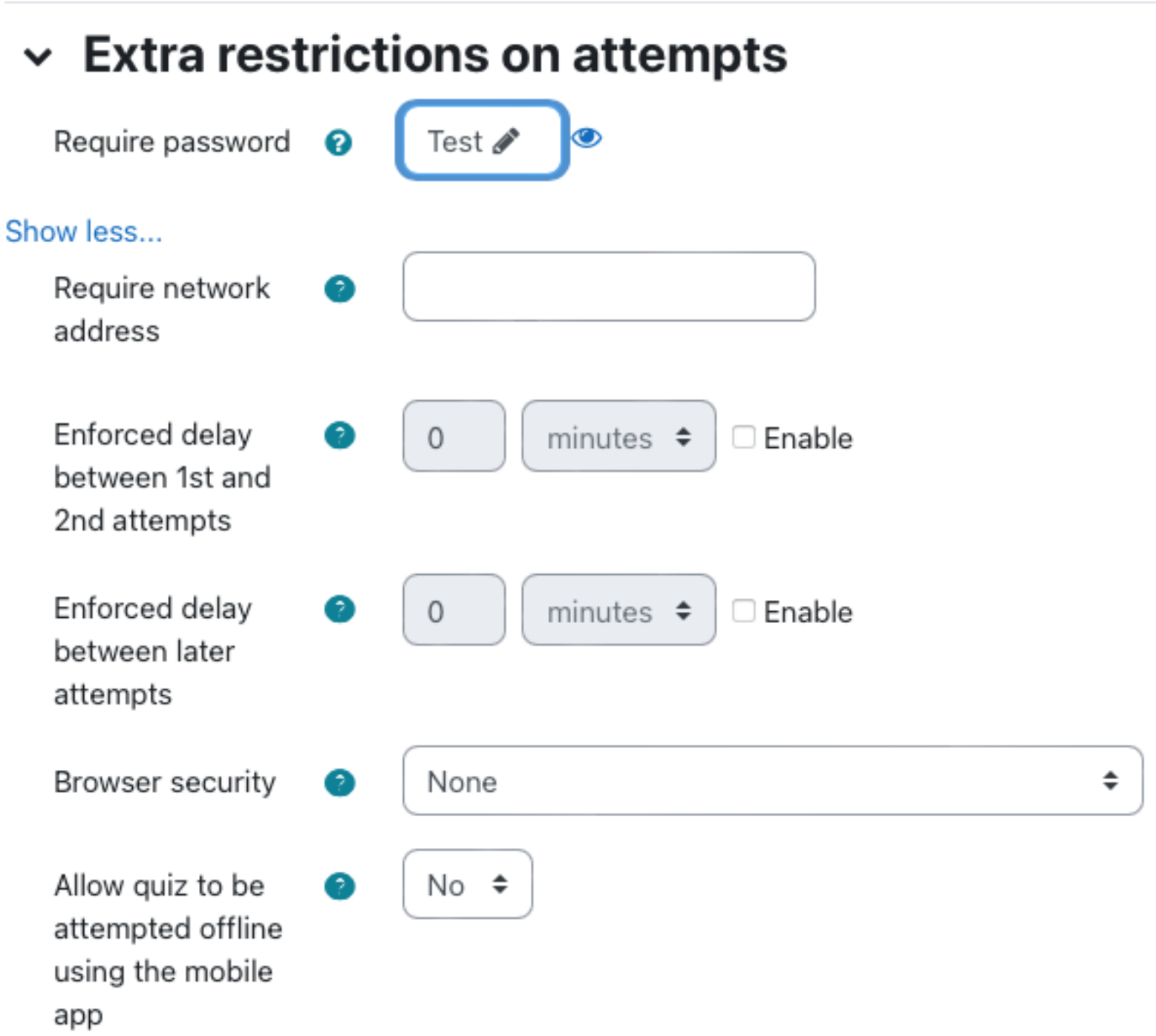
Create, configure the test and assigning it to students

- In the Question behavior section:
  - Opt for mixing answers within a question (Yes) or for keeping the order of the answers within the questions (No). We recommend mixing the answers like this make it harder for students to communicate their answers to questions;
- In the Review Options section:
  - We recommend unchecking The attempt box in the last two columns (Later, while the quiz is still open, respectively After the quiz is closed) to prevent students from viewing the grid quiz (the questions and correct answers) after they have completed the assessment. Students will only be able to view this information immediately after the assessment is complete (approximately 5 minutes). Students will also be able to view their grade at any point in time after completing the assessment.



Create, configure the test and assigning it to students

- In the Extra restrictions on attemps section:
  - From this section you can configure a password for accessing the test (to set it, press the Pencil icon);



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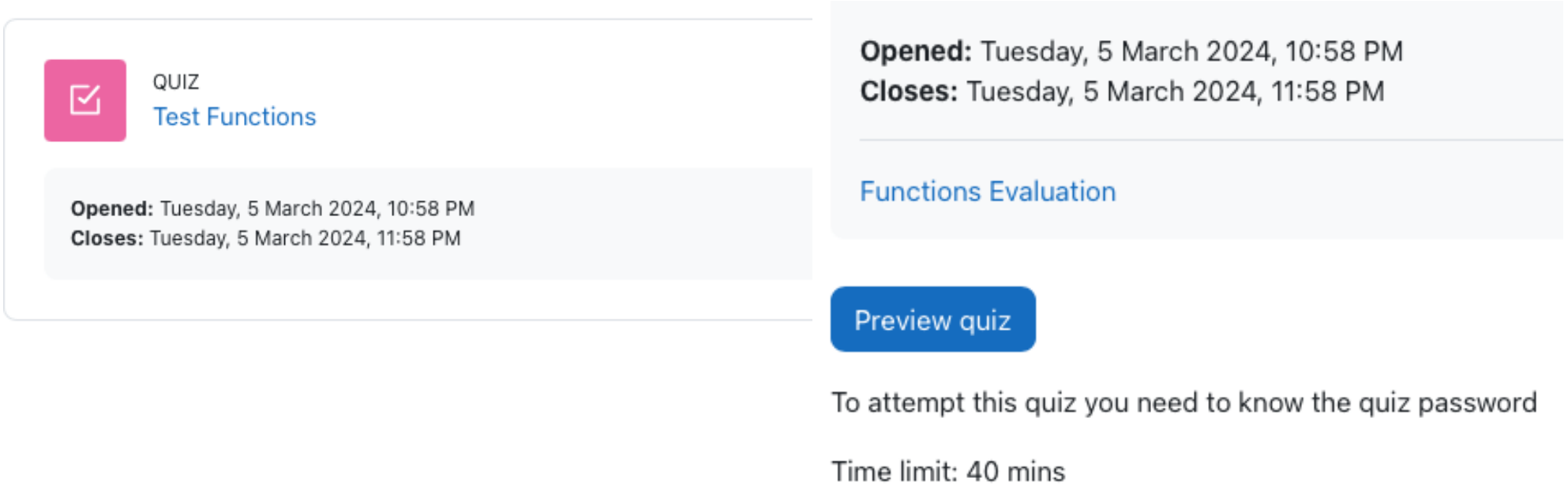
Create, configure the test and assigning it to students

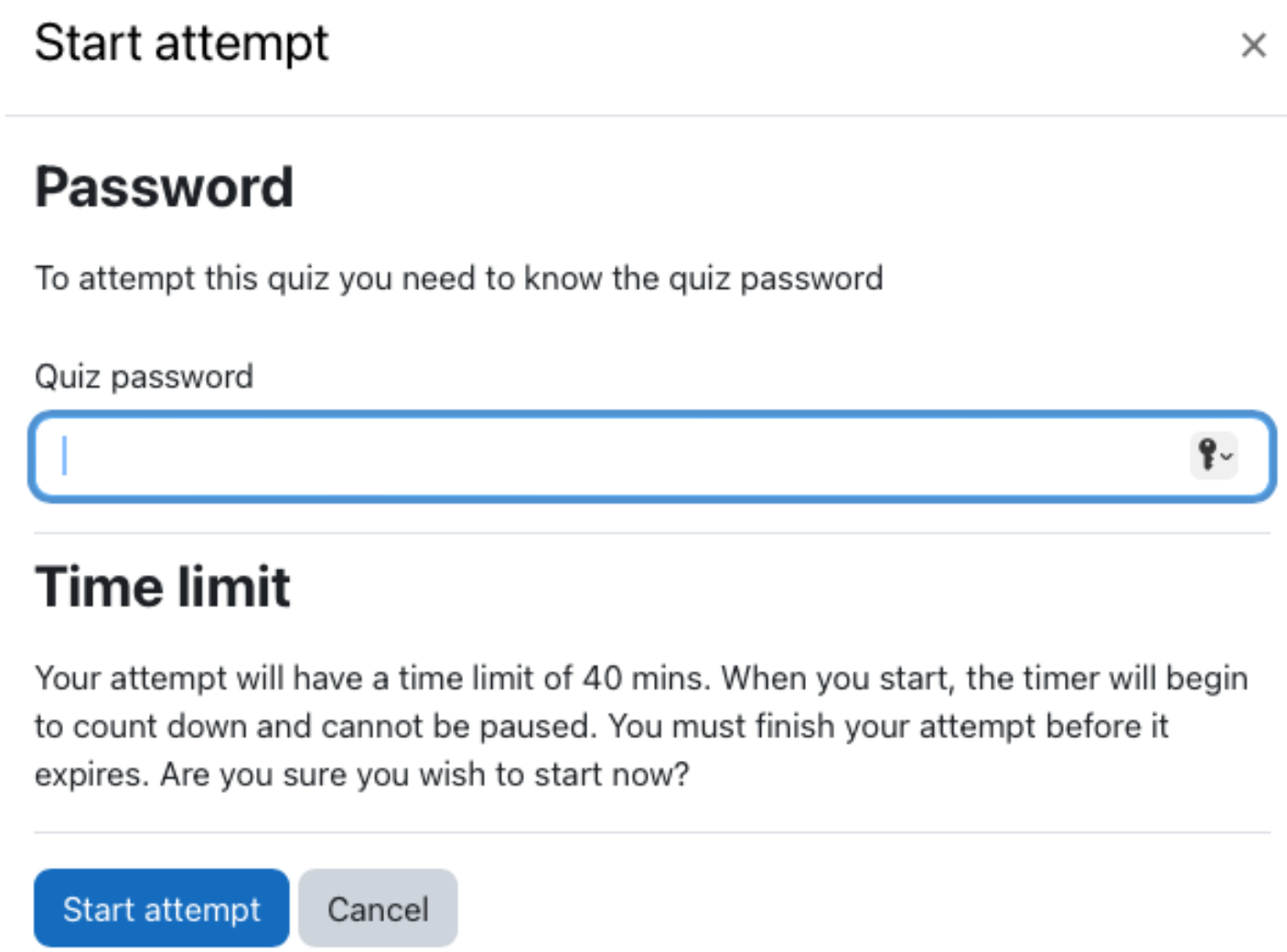
Save the changes and enter the stage of adding questions to the test. After you have created the test, it is time to assign it to the students enrolled in the course.

The student will know from the beginning between which dates and times he will be able to solve the test and how much time he has available from the moment the test starts until its completion.



Create, configure the test and assigning it to students

Students will access the test using the password provided, and begin the test.



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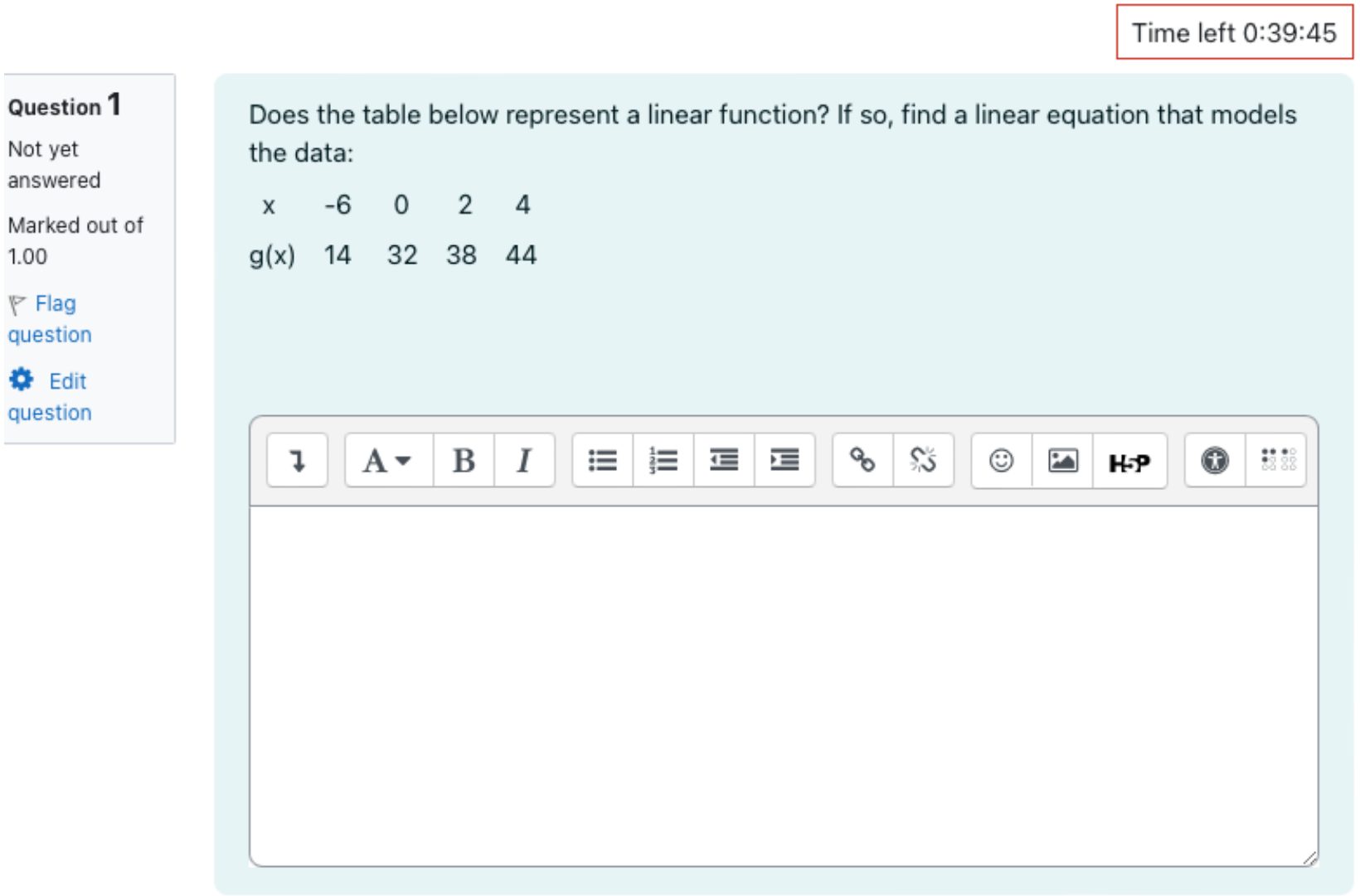
Create, configure the test and assigning it to students

The remaining time for solving the problems will be displayed on the page of each question, so that at any moment the student will be able to see how much time he has until the end of the test.



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Create, configure the test and assigning it to students

Time left 0:39:16

Question 2

Not yet answered

Marked out of 1.00

Flag question

Edit question

Graph the linear function $f(x) = -x + 6$.

geogebra.org

Maximum file size: 40 MB, maximum number of files: 1

Files

You can drag and drop files here to add them.

Accepted file types

Image files to be optimised, such as badges .gif .jpe .jpeg .jpg .png

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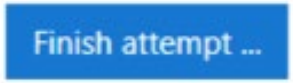
Previous page

Next page

Create, configure the test and assigning it to students

To complete the test, the student will press Finish attempt..., placed in the lower right area of the page.



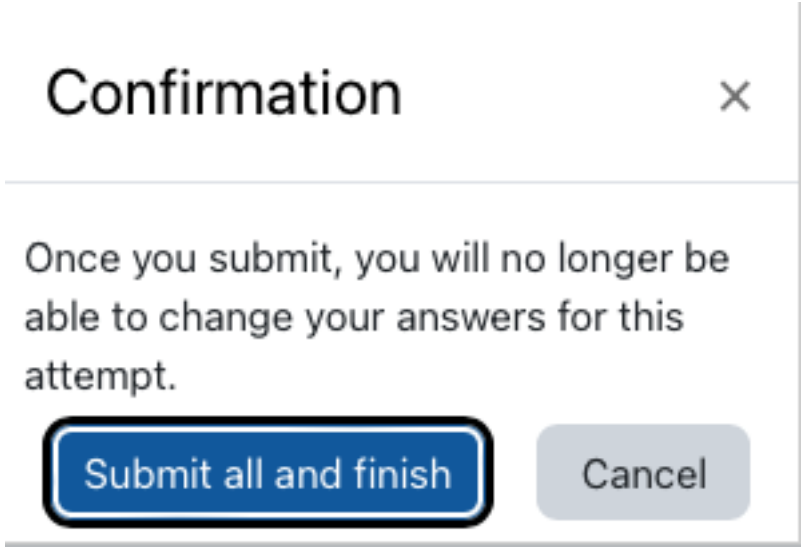
A summary of the answers is presented before the test is completed; At this point, the student can see the questions he has not answered yet, as well as how much time he has left.

Test Functions

Summary of attempt

Question	Status
1	Not yet answered
2	Answer saved
3	Not complete
4	Not complete
5	Not complete
6	Not complete
7	Not complete
8	Not complete

If he wants to complete, he will press Submit all and finish, followed by a confirmation from him.



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Platform Services benefits

- answers contain mathematical content;
- generates random structured questions;
- establish the mathematical properties of those answers;
- formative, summative and evaluative outcomes (i.e. feedback);
- stores all data for later analysis.
- Adaptability of online learning structures

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Let us now work in teams and get creative☺

Please answer the following questionnaire

THANK YOU FOR YOUR ATTENTION!

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