Workshop 1

mini-PBL, ICT & SDG

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Pythagoras
2021-1-RO01-KA220-HED-000032258

Universidad
de La Laguna

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(Very short) Introduction

Project-based learning (PBL) is a teaching approach that **engages students in sustained, collaborative real-world investigations**. Projects are organized around a **driving question**, and students participate in a **variety of tasks** that seek to meaningfully address this question.

Projects can be complex tasks, based on **challenging questions, problems, events and or activities, that involve students in the design, implementation, reflection, problem–solving and decision making** that give students the opportunity to work collaboratively and independently over a period of time, that concludes with a realistic product, presentation, activity or event.

Characteristics:

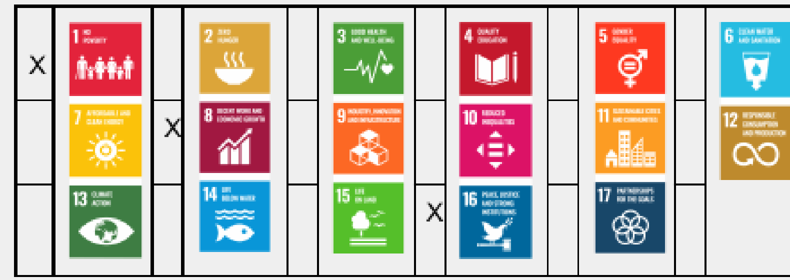
- Projects are **central** and not peripheral to the curriculum;
- PBL are focussed on questions or problems or activities that **"drive" students to encounter (and struggle with) the central concepts and principles of a discipline**;
- Projects involve students in a **constructive investigation**;
- Projects are **largely student driven**;
- Projects add value and are **realistic and authentic** (not school like).

Requiriments to students:

- **Think critically, creatively, and collaboratively**,
- **Access the knowledge** in the disciplines,
- Develop effective oral and written **communication skills**,
- **Apply their learning** by designing products and performances,
- Assess their **own learning**,
- Develop as a **self-directed, independent and interdependent learner**,
- Integrate **technology meaningfully**.

Mini-PBL project	
Teacher data sheet: Teaching Guide	
Title	The Title declares most of the project and is probably the first spark to wake the interest of students. It must be direct, clear, motivating and descriptive of the real-life issue which it addresses.
SDG attended	Using this UN graphics, we mark such SDG which this project works. 
Content units	The project may cover **2 or 3 content units**, as minimum. As the course advances, more units can be considered, but we may take care not to design a too long activity. The key concept to attend is the spiral curriculum strategy, focusing the review processes to recover students on risk to fail, and remark the connection between the content units of the course.
Sessions	Here we advance the number of sessions in the classroom we dedicate to work on this project. However, the students may know in advance that, in general, all the projects will require autonomous work, following the ECTS metric.
Hours of autonomous work	Here we may pay special attention and be careful not to generate an overtasking project. This is relevant since if you don't measure this autonomous part, the students' attitude and performance will b... seriously affected. The exceed of work out of classro... to another subjects, impact negatively on the ... proposed and, mo... PBL will be displa... ul writing, check... etc). The w... ating betwe... e this bad practi... orkflow for both s... nal control of the...
Competences to be developed	Your s... ur studen... ject should be listed. Recall always the sense of competences, don't mix with contents to be

This is a workshop..
So, let´s start to learn by working!

Mini-PBL project	
Student data sheet: Learning Guide	
Title	It must be direct, clear, motivating and descriptive of the real-life issue which it addresses.
SDG attended	Using this UN graphic, we mark such SDG which this project works. 
Content units	The project may cover **2 or 3 content units**, as minimum.
Sessions	Number of sessions in the classroom we dedicate to work on this project.
Hours of autonomous work	The students may know in advance that, in general, all the projects will require autonomous work, following the ECTS metric.
ICT tools to be used	As example, we should provide a list like this: - Graphics: explicit, implicit, 2D, 3D,... - Solving equations and/or systems: graphically, numerically, algebraically, ... - Calculus calculator: derivatives, integrals,.. - Vector and matrix calculator: graphically, numerically,..
Context: project estatement	This section is where the project is presented to the student. The introduction, the core topic and all the information will help students to allocate the tasks and problems posted late.
Tasks and problems	List the problems/tasks in difficulty crescent order. Activity 1: - *Problem 1.1* - *Problem 1.2* - ... Activity 2: ......

Exercise 1: Yeast (logistic) growth

In this exercise, you are student: To be solved in this workshop

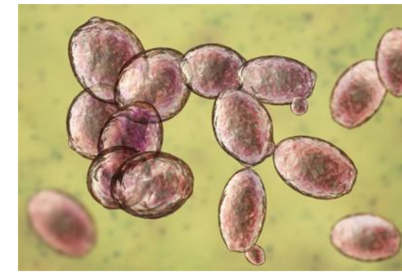
- **Level:** 1^{st}-2^{nd} year undergraduate
- **Studies:** STEM
- **Key math contents:** Data analysis: linear regression; ODE: Logistic equation; Fitting data.
- **SDG related:** 9 Industry, Innovation and Infrastructure; 3 Good Health and Well-Being; 7 Affordable and Clean Energy; 12 Responsible Consumption and Production
- **Competences:**

  Mathematics
  - *Thinking mathematically*
  - *Modeling mathematically*
  - *Posing and solving mathematical problems*
  - *Making use of aids and tools*

  Sustainability
  - *Systems thinking competency*
  - *Critical thinking competency*
  - *Integrated problem-solving competency*
- **ICT tools:** DESMOS, Google Calc, Generative AI
- **Activities:**
  - *On SDG dimension: 3*
  - *On mathematical dimension: 5*

Exercise 1: Yeast (logistic) growth

Yeasts are tiny but mighty organisms that **play a crucial role in many aspects of our lives**, especially in food production. Here's why they are important:



- **Food and Beverage Production:** Yeasts are essential for the fermentation process, which is used to produce a variety of alcoholic beverages like beer, wine, and cider. In these processes, yeast consumes sugars and converts them into alcohol and carbon dioxide. They are also key to baking, where yeast fermentation causes dough to rise, resulting in fluffy breads and pastries.

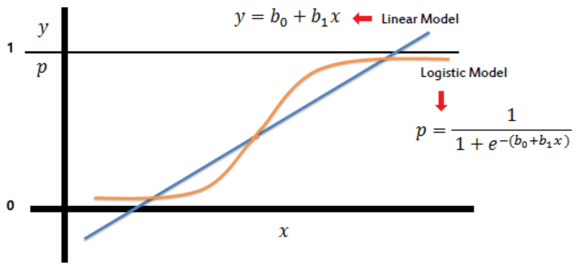
- **Nutritional Value:** Some types of yeast, like brewer's yeast, are a good source of vitamins, particularly B vitamins, and protein.
- **Medicine:** Yeasts are used in the production of some medications, such as insulin.

- **Biofuels:** Certain yeast strains can be used to produce biofuels, which are renewable energy sources derived from organic matter.
- **Scientific Research:** Because of their relatively simple cell structure and rapid reproduction rate, yeasts are widely used as model organisms in scientific research.

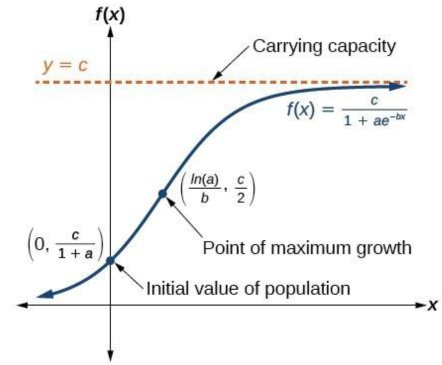
With the help of: Gemini

Yeasts, due to their well-defined growth patterns and ease of study, **have become a favorite subject for mathematical modeling of microbial growth**. Here are some common models used to describe yeast growth:

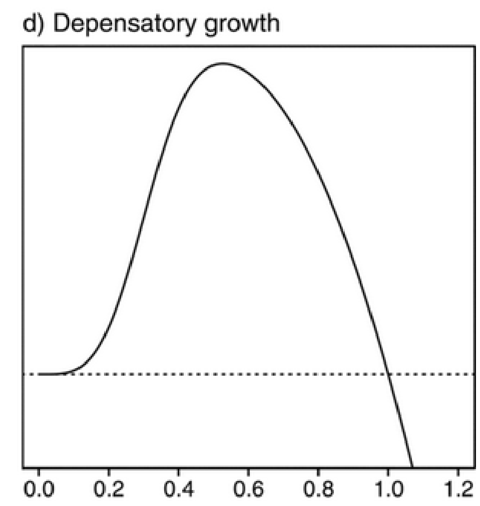
- **Logistic Model:** This classic model, developed by Pierre-François Verhulst, represents a population's growth initially at an exponential rate, but eventually slowing down and reaching a carrying capacity limited by available resources (https://pubmed.ncbi.nlm.nih.gov/35033338/).



- **Gompertz Model:** This model is useful for describing yeast colony growth on solid surfaces like agar plates. It captures the initial slow growth phase (lag phase), followed by an exponential growth phase, and finally a stationary phase where growth plateaus due to limitations (https://pubmed.ncbi.nlm.nih.gov/25099389/).



- **Mechanistic Models:** These more complex models delve deeper into the biological processes underlying yeast growth. They consider factors like nutrient uptake, cell division rates, and even toxin production by some yeast strains (https://www.sciencedirect.com/science/article/pii/S0166445X20304872).



The choice of model depends on the specific aspect of yeast growth being studied. **Simpler models like the Logistic model provide a good starting point**, while complex mechanistic models offer more detailed insights but require more data and computational power.

Consider the growth of a population of yeast. Researchers collected the data in Table 1 from a yeast population grown in liquid culture, measuring the population size (in number of individuals per mL of culture) at different points in time (in hours). Figure 1 is a scatter plot of these data.

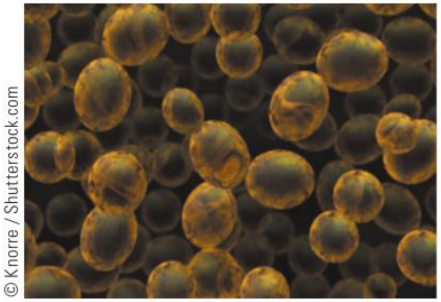


Table 1

Time (h)	Pop. size ($\times 10^6$/mL)	Time (h)	Pop. size ($\times 10^6$/mL)
0	0.200	19	209
1	0.330	20	190
2	0.500	21	210
3	1.10	22	200
4	1.40	23	215
5	3.10	24	220
6	3.50	25	200
7	9.00	26	180
8	10.0	27	213
9	25.4	28	210
10	27.0	29	210
11	55.0	30	220
12	76.0	31	213
13	115	32	200
14	160	33	211
15	162	34	200
16	190	35	208
17	193	36	230
18	190		

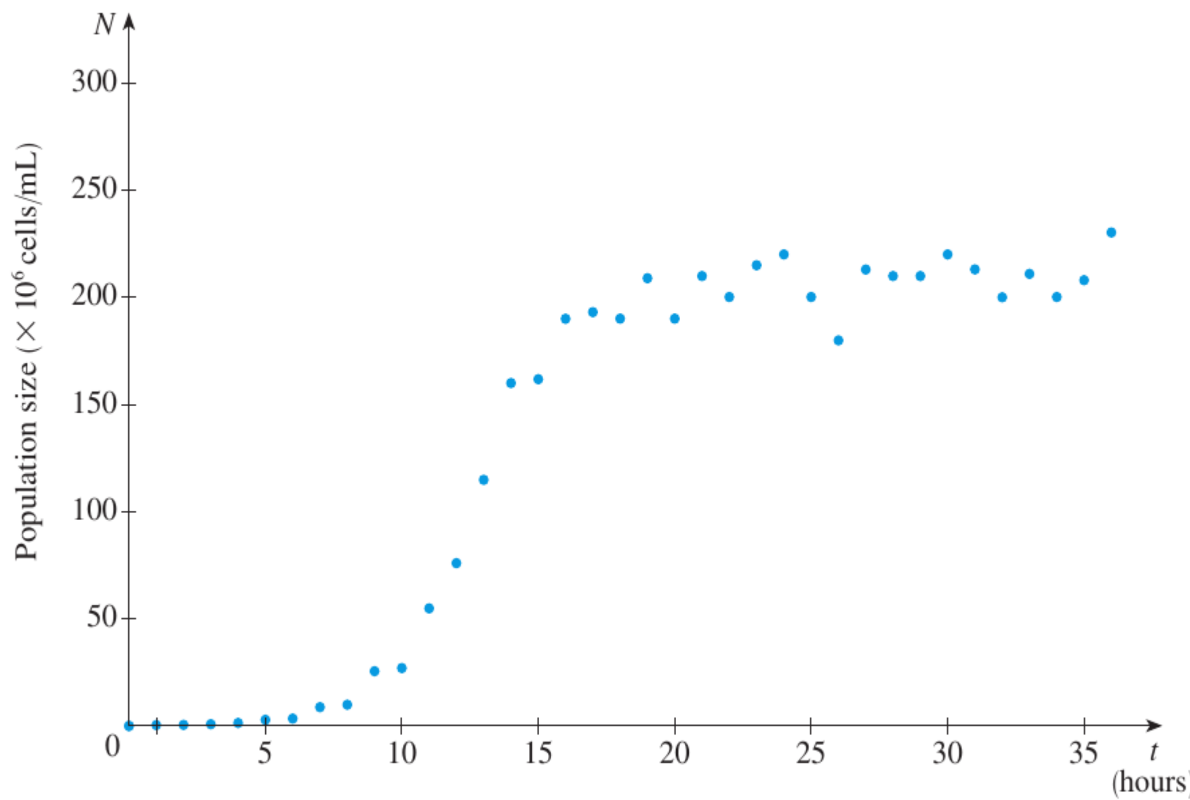


FIGURE 1 A scatter plot of the data in Table 1

1. B. K. Mable et al., "Masking and Purging Mutations following EMS Treatment in Haploid, Diploid, and Tetraploid Yeast (*Saccharomyces cerevisiae*)," *Genetical Research* 77 (2001): 9–26.

Logistic Differential Equation

The **Logistic Differential Equation** is a mathematical tool used to model population growth that takes into account resource limitations. Here are the key fundamentals:

$$\frac{dN}{dt} = r\left(1 - \frac{N}{K}\right)N \qquad N(0) = N_0$$

- **Population Change:** It describes the rate of change of a population (***dN/dt***) over time (dt). This rate of change is represented by a derivative.

- **Growth Proportionality:** The rate of change is proportional (***r***) to the current population size (***N***). This means that larger populations tend to grow faster (but not infinitely).

- **Resource Limitation:** The equation incorporates a factor that slows down growth as the population approaches a carrying capacity (***K***). This carrying capacity represents the maximum population sustainable with available resources (like food or space).

- **Logistic Term:** The term that considers resource limitation is typically **(*1 - N/K*)**. This term approaches zero as the population (***N***) gets closer to the carrying capacity (***K***), reducing the growth rate.

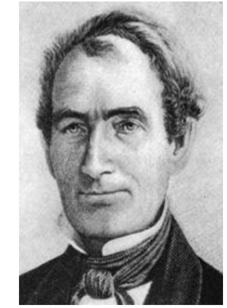
- **Exponential Growth:** When the population is small relative to the carrying capacity (***N << K***), the **(*1 - N/K*)** term is close to 1, and the growth rate is approximately proportional to the population size (***rN***), resembling exponential growth.

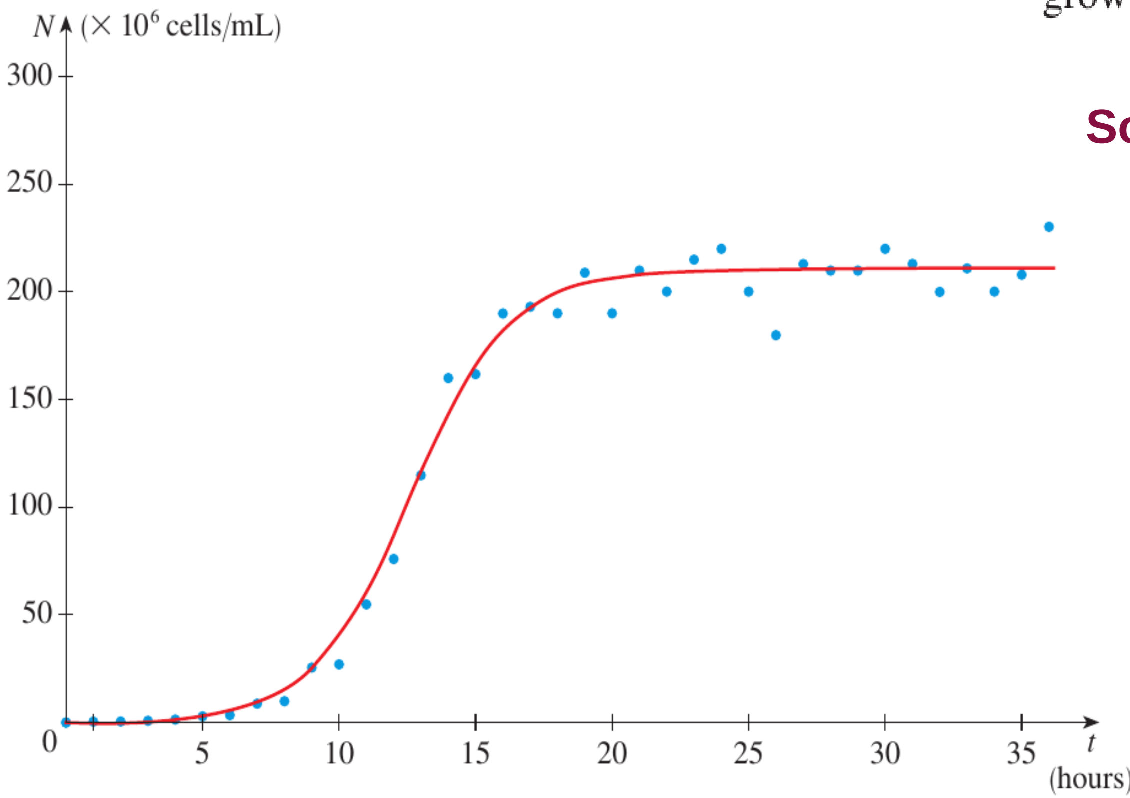
By solving this differential equation, you can obtain mathematical functions that describe how the population grows over time, reaching an equilibrium at the carrying capacity.

Logistic Equation

$$\frac{dN(t)}{dt}\frac{1}{N(t)} = r\left(1 - \frac{N}{K}\right)$$

The logistic growth equation was first proposed by Dutch mathematical biologist Pierre-François Verhulst in the 1840s as a model for world population growth.





Solution (without know the parameters):

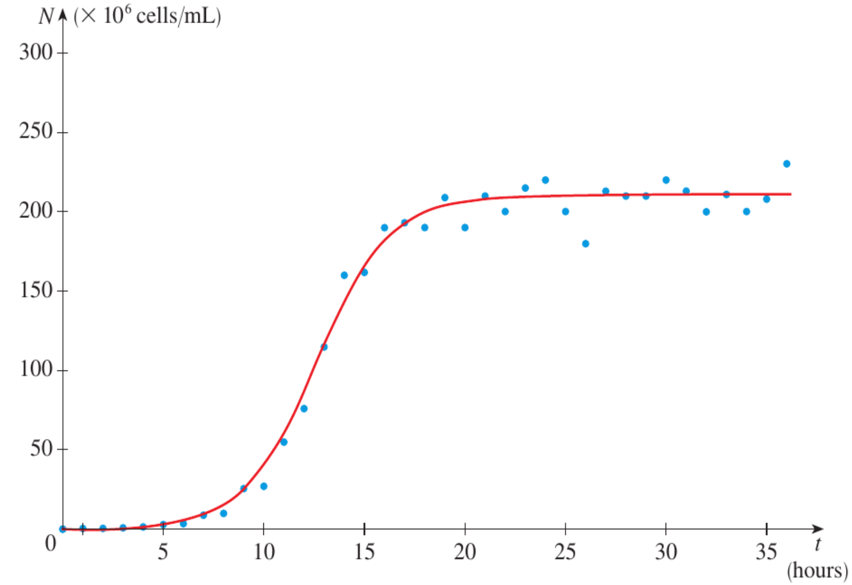
$$N(t) = \frac{KN_0}{N_0 + (K - N_0)e^{-rt}}$$

$$\lim_{t\to\infty} N(t) = K.$$

Resolution of the equation is the standard procedure.
Our aim is the construction of the equation (i.e., determinate ***r*** and ***K***) for this real case.

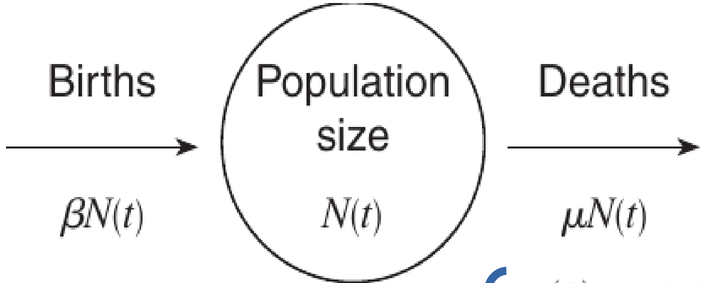
Goal of Excercise 1:

Design a mini-PBL based on the Logistic equation



- **What do we mathematically identify in these table?**
- **What math activities can we construct?**
- **What SDG can be attended by these mathematical activities?**
- **What activities on SDG dimension can we produce?**
- **Which competences can we address in our T&L process?**
- **…..*Your turn***

First analysis: Incomplete



$$\frac{dN(t)}{dt}\frac{1}{N(t)} = r$$

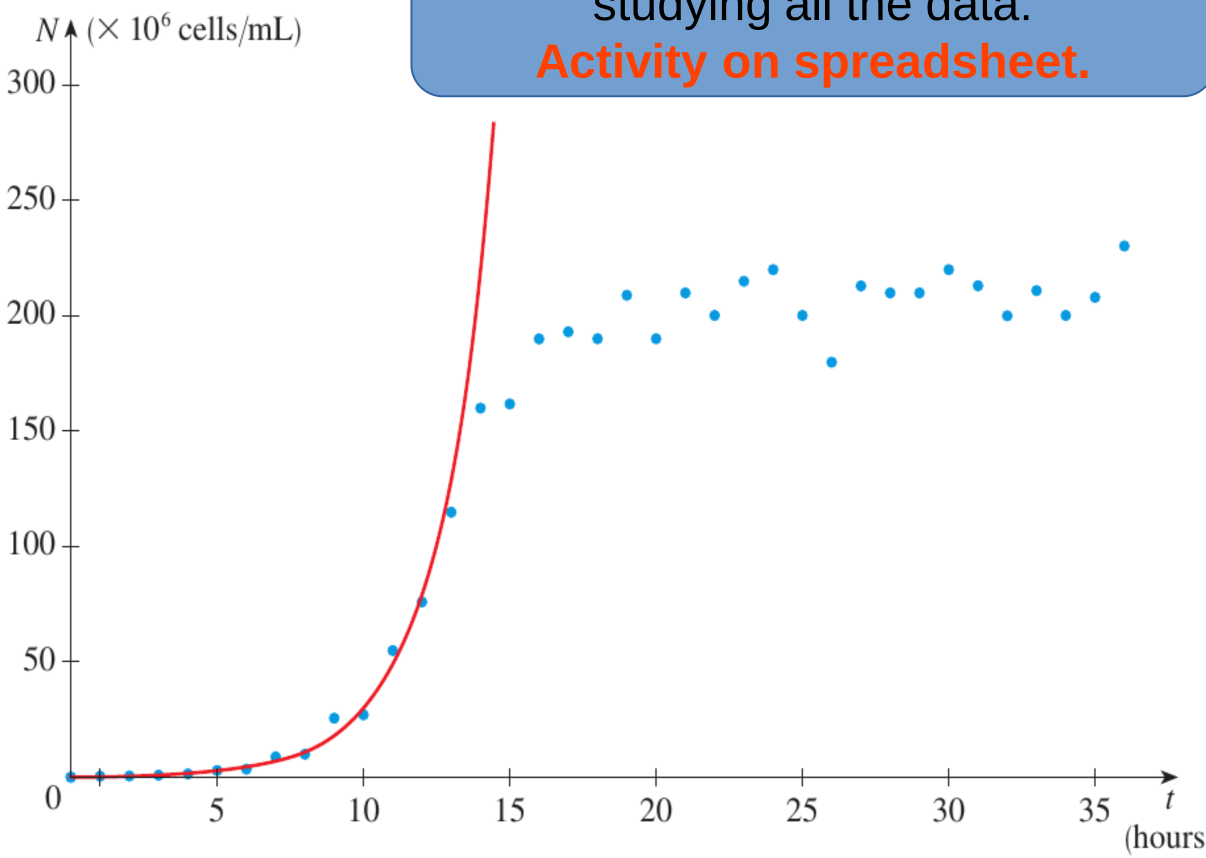
Growth rate per capita CONSTANT

Our view/solution:

$$\frac{dN(t)}{dt} = \beta N(t) - \mu N(t) = rN(t) \Rightarrow N(t) = Ce^{rt}$$

$$\left\{\begin{array}{l} N(0) = 0.200 \Rightarrow C = 0.200 \\ \dfrac{N(1)}{N(0)} = \dfrac{0.330}{0.200} = 1.65 = \dfrac{Ce^{r\cdot 1}}{Ce^{r\cdot 0}} = e^{r} \Rightarrow r = \ln 1.65 \approx 0.5. \end{array}\right\} \Rightarrow N(t) = 0.2e^{0.5t}$$

This value can also be obtained by studying all the data:
Activity on spreadsheet.



In perspective, an obvious biological reason for this discrepancy is that the model assumes that the per capita growth rate remains constant r, irrespective of population size.

In reality, **as population increases, overcrowding and resource depletion can be expected to cause the per capita growth rate to decrease as population N increases.**

Which model do we choose?

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

Logistic

To the exponential growth.... **...we apply a reduction over *r* when *N* increase**

Logistic Eq.: $\frac{dN}{dt} = r\left(1 - \frac{N}{K}\right)N \qquad N(0) = N_0$

What we have taught in previous courses

Solution: Separable Var. Equation

$$\int \frac{dN}{(1 - N/K)N} = \int r\,dt \quad \Rightarrow \quad \int \left(\frac{1}{N} + \frac{1}{K - N}\right) dN = \int r\,dt$$

$$\ln|N| - \ln|K - N| = rt + C$$

$$\ln\left|\frac{K - N}{N}\right| = -rt - C$$

$$\left|\frac{K - N}{N}\right| = e^{-rt-C} = e^{-C}e^{-rt}$$

$$\frac{K - N}{N} = Ae^{-rt}$$

$$N(t) = \frac{K}{1 + Ae^{-rt}}$$

$$\text{where } A = \frac{K - N_0}{N_0}$$

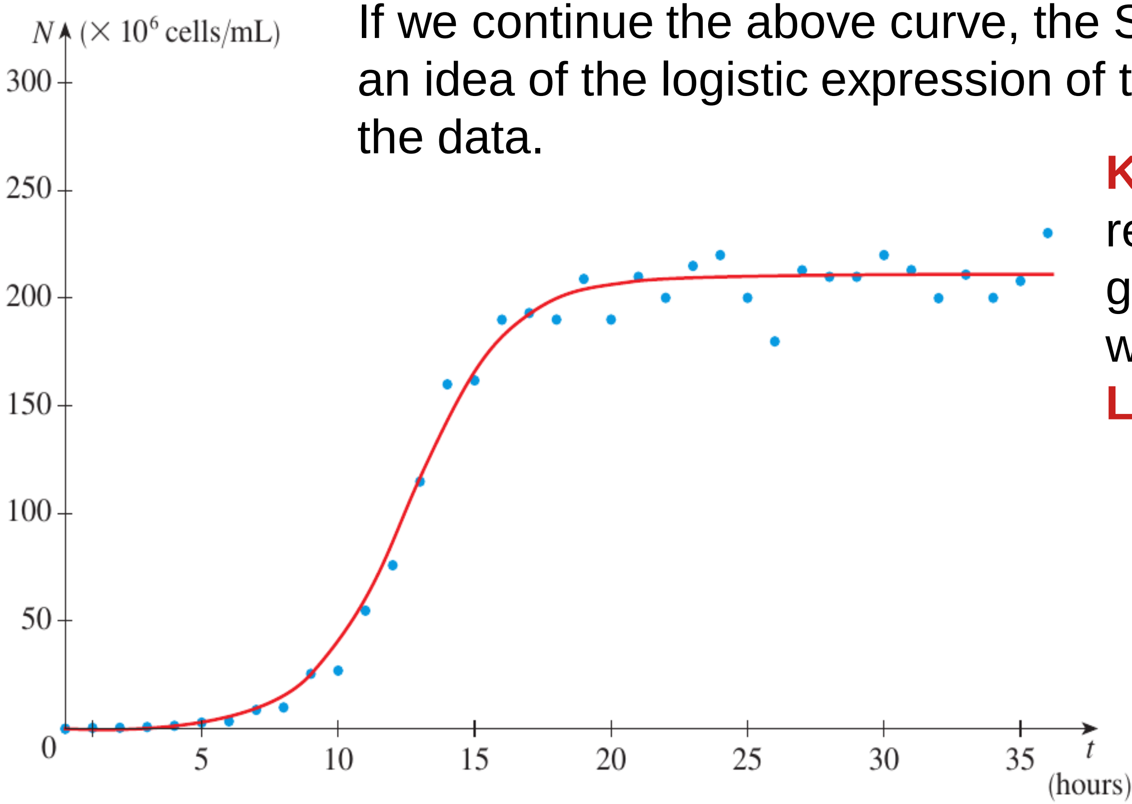
$$N(t) = \frac{KN_0}{N_0 + (K - N_0)e^{-rt}}$$

$$\lim_{t\to\infty} N(t) = K.$$

K is the limit capacity of the system

What parameters does give DESMOS?

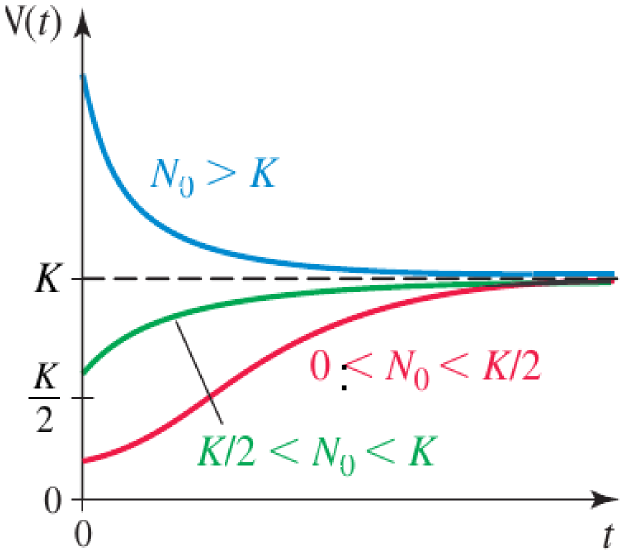
Second analysis: Logistic solution



If we continue the above curve, the S-shape that fits the data gives us an idea of the logistic expression of the best solution that approximates the data.

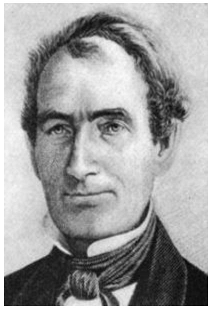
KEY IDEA 1: We look for a relationship where the per capita growth rate decreases as N increases, we choose linear decreasing model: **Logistic Eq.**

$$\frac{dN(t)}{dt}\frac{1}{N(t)} = r\left(1 - \frac{N}{K}\right)$$



Solution curves of the logistic equation $\frac{dN}{dt} = rN(1 - N/K)$ for different initial values N_0.

The logistic growth equation was first proposed by Dutch mathematical biologist Pierre-François Verhulst in the 1840s as a model for world population growth.



Aim: **How do we obtain (estimate) the fundamental parameters *r* and *K* of the equation from the data?**

KEY IDEA 2: We operate with the expression N/(K-N) without yet knowing the paramet

$$N(t) = \frac{KN_0}{N_0 + (K-N_0)e^{-rt}} \quad \Rightarrow \quad \frac{N(t)}{K-N(t)} = \frac{\frac{KN_0}{N_0+(K-N_0)e^{-rt}}}{K - \frac{KN_0}{N_0+(K-N_0)e^{-rt}}}$$

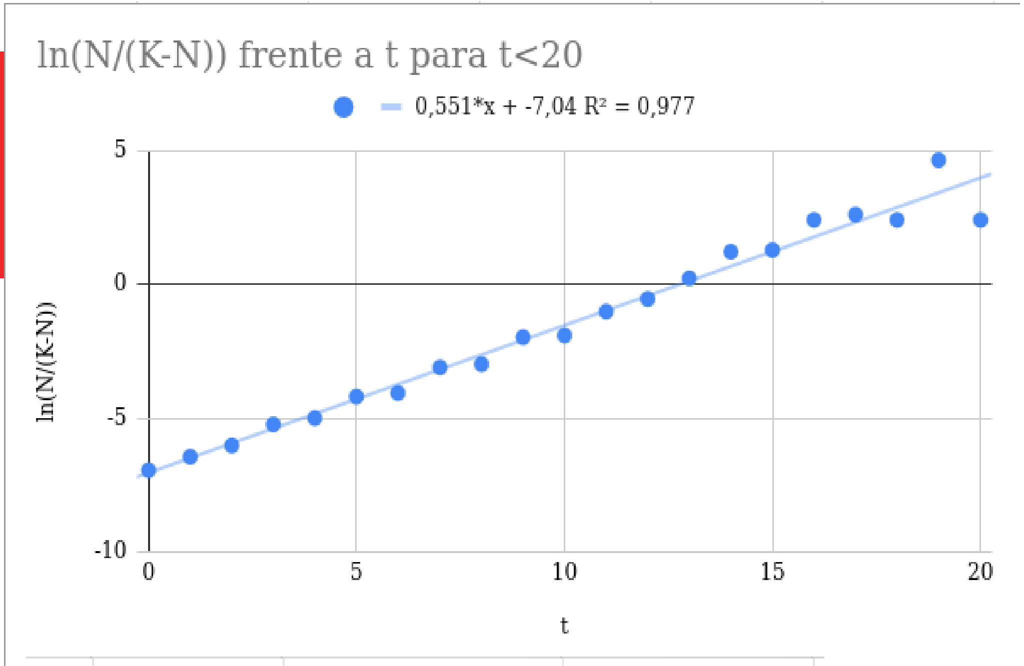
$$= \frac{N_0}{(K-N_0)e^{-rt}}$$

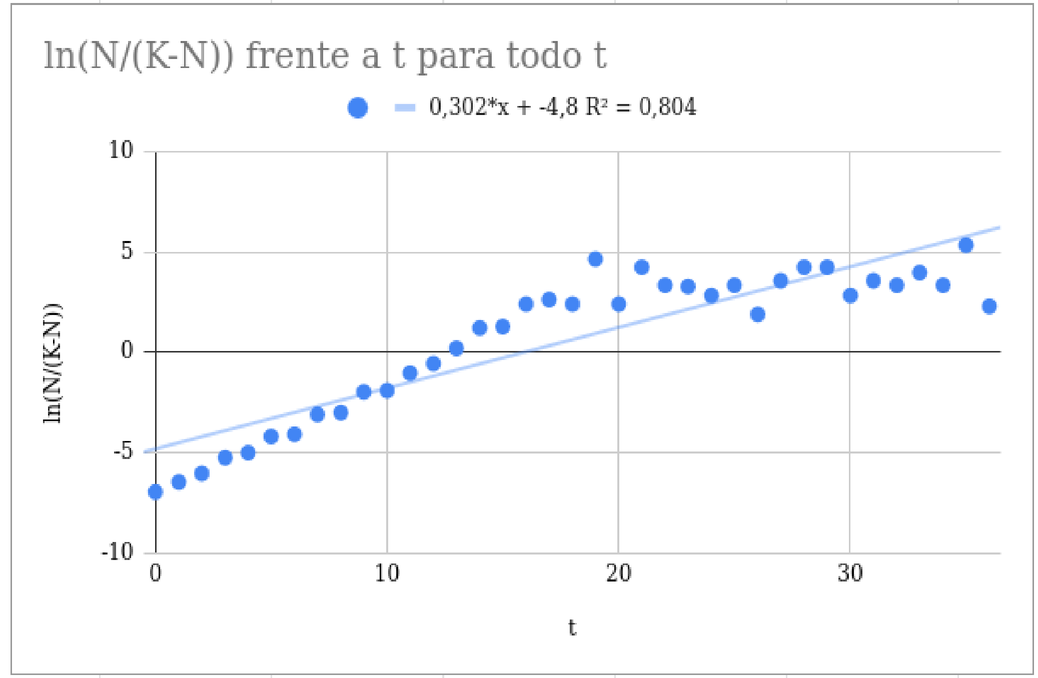
$$= \left(\frac{N_0}{K-N_0}\right)e^{rt}$$

$$\ln\left(\frac{N(t)}{K-N(t)}\right) = \ln\left(\frac{N_0}{K-N_0}\right) + rt = C + rt$$

We are now going to use the experimental data to look for these two parameters, *r* and *K*.
L

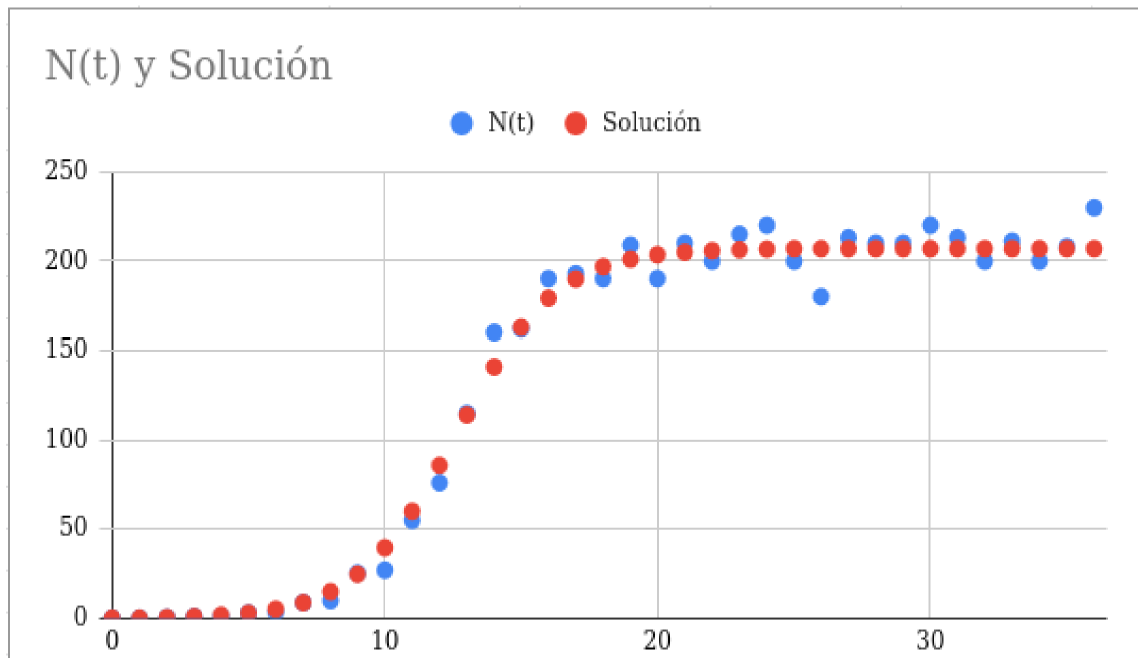
- **Step 1:** We calculate the table of ln(N/K-N) values for different values of K, which we get from the trend of the data for large t values.
- **Step 2:** We fit the data by regression and choose K so that we have the best estimate. This step is where the linear trend of the data materialises (what we were looking to confirm).
- **Step 3:** With K fixed, linear regression gives us the value of r that determines the logistic model that best fits the da





K=	**207**		
r=	**0,55**		
	N(t)	**ln(N/(K-N))**	**Solución**
0	0,2	-6,941190055	0,2
1	0,33	-6,439785943	0,3464051904
2	0,5	-6,023447593	0,5996720045
3	1,1	-5,232080434	1,037180124
4	1,4	-4,989460297	1,791115045
5	3,1	-4,186227566	3,084883085
6	3,5	-4,062903036	5,289086239
7	9	-3,091042453	8,998729427
8	10	-2,980618636	15,11525993
9	25,4	-1,967057292	24,86712058
10	27	-1,897119985	39,61175378
11	55	-1,016547336	60,2089107
12	76	-0,5444639829	86,01272848
13	115	0,2231435513	114,2668222
14	160	1,225026214	140,986728
15	162	1,280933845	162,9739873
16	190	2,413810728	179,0877427
17	193	2,623632859	189,9218052

$$N(t) = \frac{207 \cdot 0,2}{0,2 + (207 - 0,2)e^{-0,55t}}$$



Exercise 2: Air Quality Index

- **Level:** High School (2nd year) – 1st year undergraduate
- **Studies:** STEM
- **Key math contents:** ??
- **SDG related:** ??
- **Competences:** ??
- **ICT tools:** ??
- **Activities:** ??

In this exercise, you are professor:
To be completed in this workshop

Exercise 2: Air Quality Index

https://www.airnow.gov/aqi/aqi-basics/

Air Quality Index (AQI) Basics

Versión en Español

What is the U.S. Air Quality Index (AQI)?

The U.S. AQI is EPA's index for reporting air quality.

How does the AQI work?

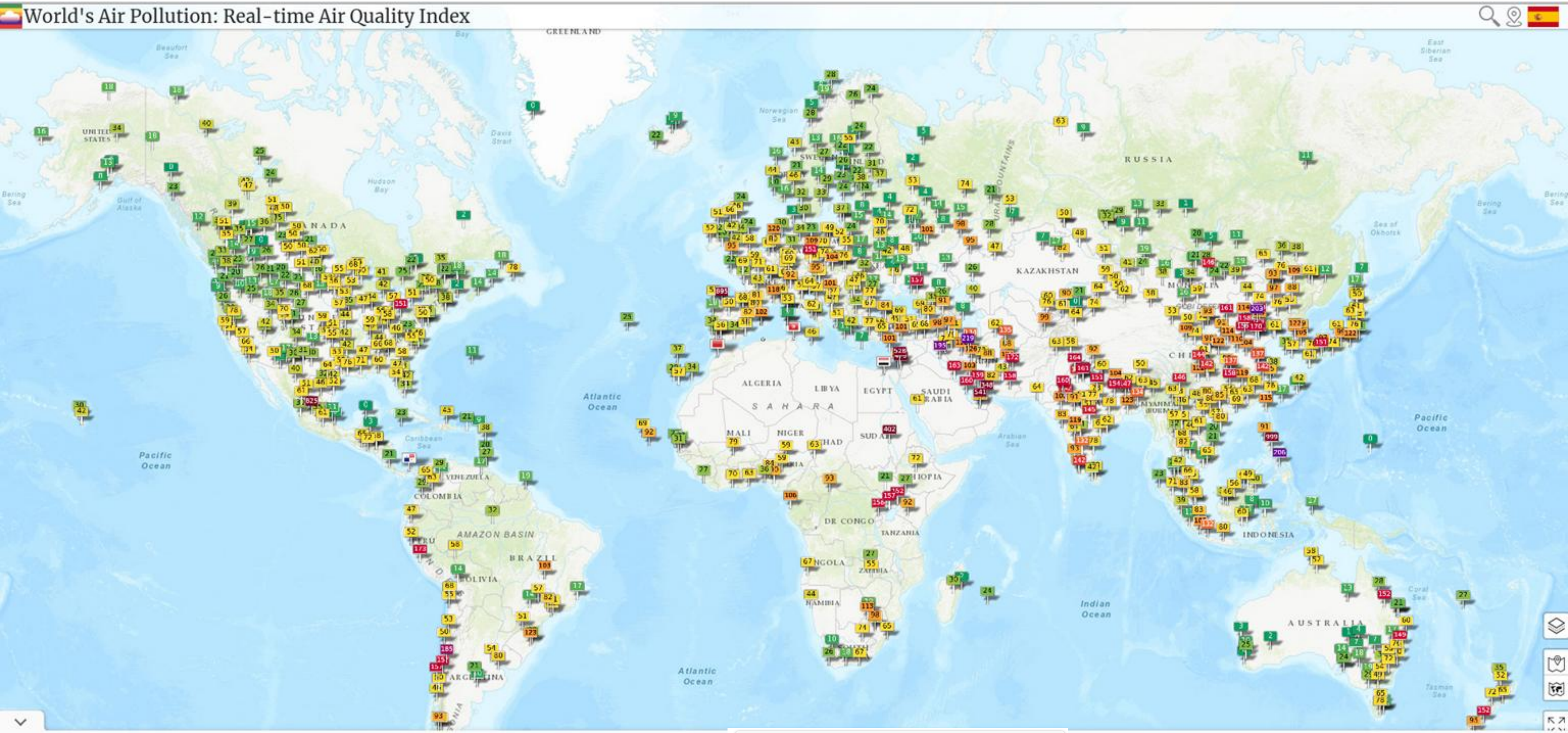
Think of the AQI as a yardstick that runs from 0 to 500. The higher the AQI value, the greater the level of air pollution and the greater the health concern. For example, an AQI value of 50 or below represents good air quality, while an AQI value over 300 represents hazardous air quality.

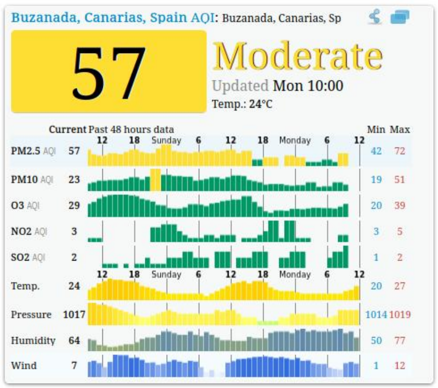
For each pollutant an AQI value of 100 generally corresponds to an ambient air concentration that equals the level of the short-term national ambient air quality standard for protection of public health. AQI values at or below 100 are generally thought of as satisfactory. When AQI values are above 100, air quality is unhealthy: at first for certain sensitive groups of people, then for everyone as AQI values get higher.

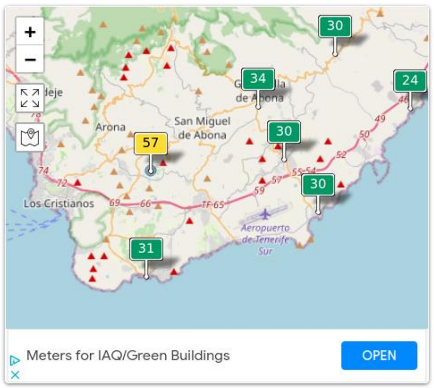
The AQI is divided into six categories. Each category corresponds to a different level of health concern. Each category also has a specific color. The color makes it easy for people to quickly determine whether air quality is reaching unhealthy levels in their communities.

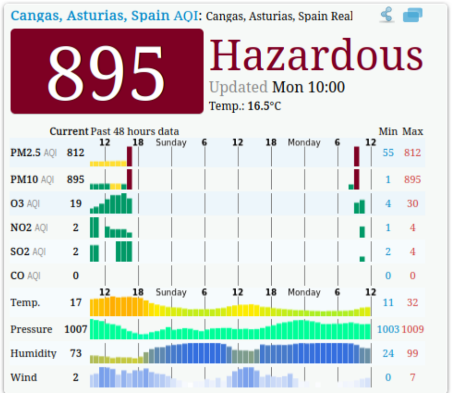
AQI Basics for Ozone and Particle Pollution

Daily AQI Color	Levels of Concern	Values of Index	Description of Air Quality
Green	Good	0 to 50	Air quality is satisfactory, and air pollution poses little or no risk.
Yellow	Moderate	51 to 100	Air quality is acceptable. However, there may be a risk for some people, particularly those who are unusually sensitive to air pollution.
Orange	Unhealthy for Sensitive Groups	101 to 150	Members of sensitive groups may experience health effects. The general public is less likely to be affected.
Red	Unhealthy	151 to 200	Some members of the general public may experience health effects; members of sensitive groups may experience more serious health effects.
Purple	Very Unhealthy	201 to 300	Health alert: The risk of health effects is increased for everyone.
Maroon	Hazardous	301 and higher	Health warning of emergency conditions: everyone is more likely to be affected.







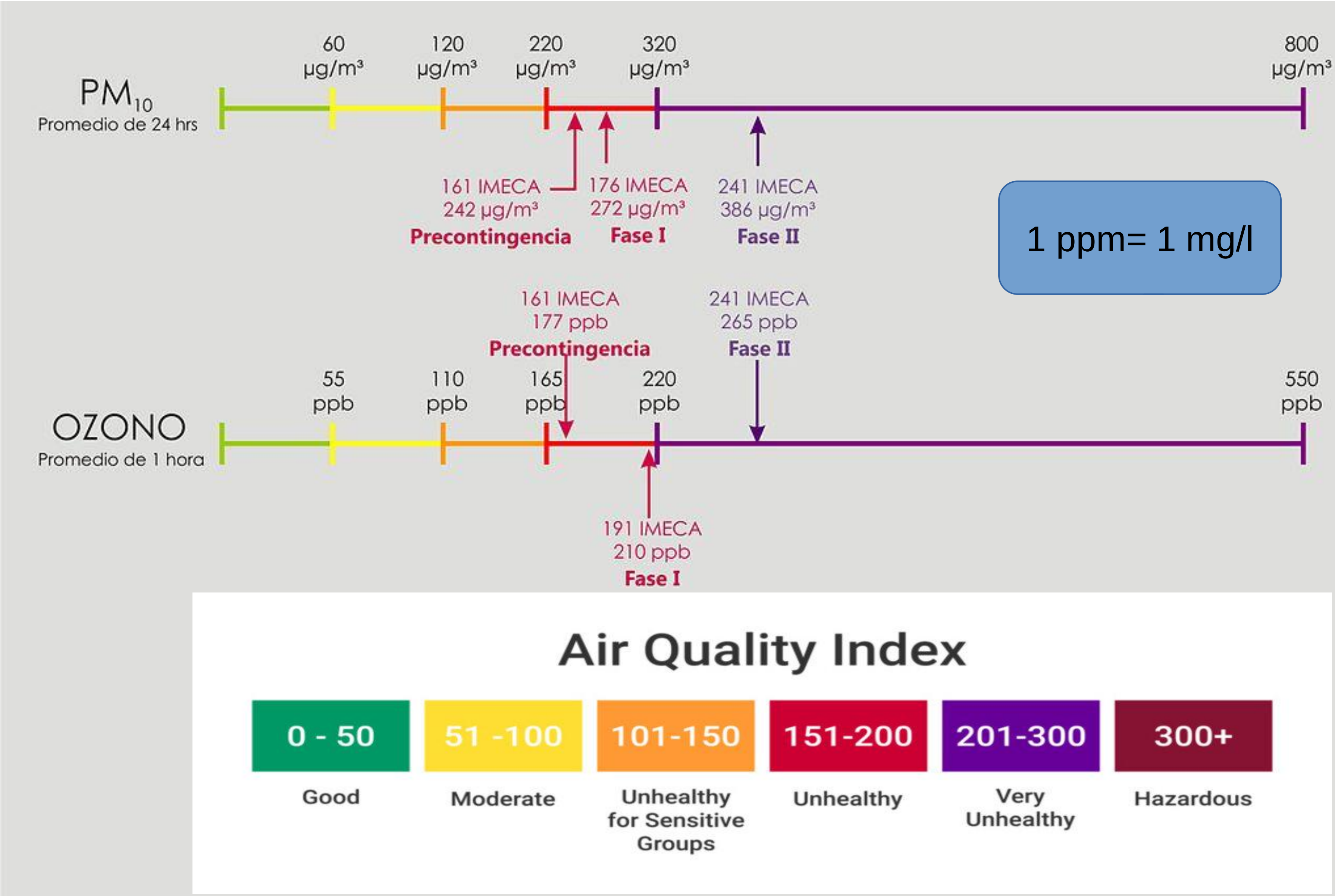


https://waqi.info/

https://aqicn.org/map/world/

https://www.iqair.com/world-air-quality-ranking

Air Quality Index (AQI)/IMECA Index (Mexico)



Parts per billion (ppb) is a unit of measure by which concentration is measured. It refers to the number of units of a given substance (agent, etc.) per billion (10^9) units of the whole.

Goal of Excercise 2:

Design a mini-PBL on the AQI scores

Tips:

- **What do we mathematically identify in these table?**
- **What math activities can we construct?**
- **What SDG can be attended by these mathematical activities?**
- **What activities on SDG dimension can we produce?**
- **Which competences can we address in our T&L process?**
- **…..*Your turn***

AQI intervals	Concentrations
Partículas menores a 10 micrómetros (PM_{10})	
PM10	
IMECA	µg/m^3
0-50	0-60
51-100	61-120
101-150	121-220
151-200	221-320
>200	>320
Partículas menores a 2.5 micrómetros ($PM_{2.5}$)	
$PM_{2.5}$	
IMECA	µg/m^3
0-50	0-15.4
51-100	15.5-40.4
101-150	40.5-65.4
151-200	65.5-150.4
>200	>150.4
Ozono (O_3)	
O_3	
IMECA	ppm
0-50	0-0.055
51-100	0.056-0.110
101-150	0.111-0.165
151-200	0.166-0.220
>200	>0.220
Dióxido de Nitrógeno (NO_2)	
NO_2	
IMECA	ppm
0-50	0-0.105
51-100	0.106-0.210
101-150	0.211-0.315
151-200	0.316-0.420
>200	>0.420
Dióxido de Azufre (SO_2)	
SO_2	
IMECA	ppm
0-50	0-0.065
51-100	0.066-0.130
101-150	0.131-0.195
151-200	0.196-0.260
>200	>0.260
Monóxido de Carbono (CO)	
CO	
IMECA	ppm
0-50	0-5.50
51-100	5.51-11.00
101-150	11.01-16.50
151-200	16.51-22.00
>200	>22.00

Our view/solution:

1. On SDG dimension: Let's take a look the situation all around the World on this AQI.

Activity 1.1: *What are the most harzadous zones in the Planet right now? And which one is closer to your location?*

Activity 1.2: *What are the most significant impact on health of the differents levels of the AQI?*

2. On mathematical dimension: To publish all the real-time indicators of AQI all around the World, it is necesary to establish the rules of translation from each pollutant measure into the AQI level. Our goal is construct such rules by simple linear equations.

Activity 2.1: *Let's take a look on the measures of the pollutants.*

a) *What does pollutant run in the biggest range?*

b) *Which one does in the smaller?*

c) *Have all the pollutant intervals with the same size?*

Activity 2.2: *Let's start studying two particular cases.*

a) ***O_2*** *and* ***SO_2*** *have a common singularity on the range of intervals tha correspond to each AQI.*

b) *Study and compare these two cases and describe this similarity.*

c) *What is the situation with respect the others pollutants?*

Activity 2.3:

a) *Construct the rule of transformation of the ranges of pollutant* ***O_2*** *anc* ***SO_2*** *to the corresponding AQI/IMECA scales.*

b) *Explain the relation of* ***Activity 2.2*** *with the results obtained.*

c) *Draw both equations using a graphic tool.*

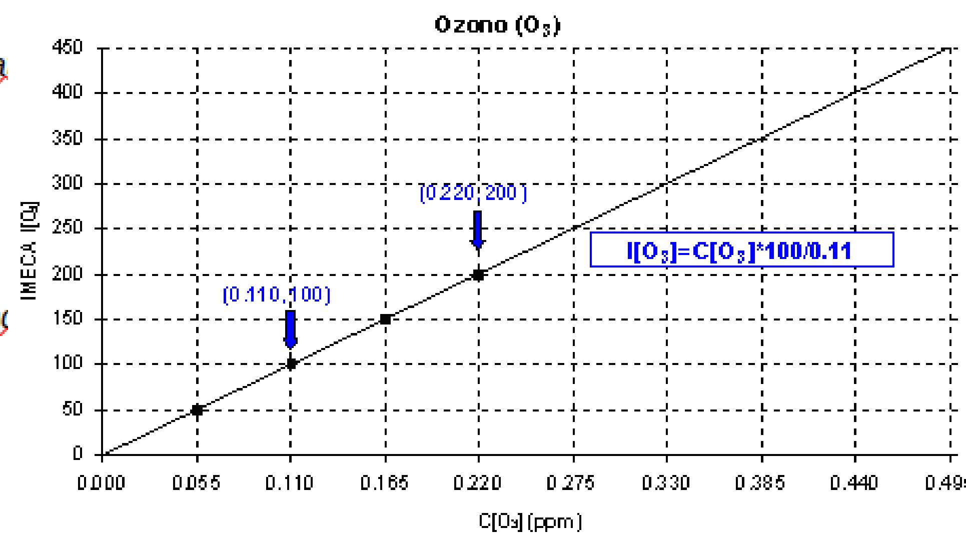
Activity 2.4:

a) *Construct the rule of transformation for* ***$PM_{2.5}$*** *(or* ***NO_2****)*

b) *What characteristics of the data make some equations so similar?*

c) *What are the equal/different parameters in each pollutant? What is the mathematical property of the intervals which justify these*

INTERVALOS (IMECA)		CONCENTRACIONES	ECUACIONES
Partículas menores a 10 micrómetros (PM10)			
PM10			
	IMECA	µg/m³	Ecuaciones
	0-50	0-60	$I[PM_{10}] = C[PM_{10}] * 5/6$
	51-100	61-120	
	101-150	121-220	I[PM10] = 40+C[PM10] * 0.5
	151-200	221-320	
	>200	>320	I[PM10] = C[PM10] * 5/8
Partículas menores a 2.5 micrómetros (PM2.5)			
$PM_{2.5}$			
	IMECA	µg/m³	Ecuaciones
	0-50	0-15.4	I[PM2.5] = C[PM2.5] * 50/15.4
	51-100	15.5-40.4	I[PM2.5] = 20.50 + C [PM2.5] * 49/24.9
	101-150	40.5-65.4	I[PM2.5] = 21.30 + C[PM2.5] * 49/24.9
	151-200	65.5-150.4	I[PM2.5] = 113.20 + C[PM2.5] * 49/84.9
	>200	>150.4	I[PM2.5] = C[PM2.5] * 201/150.5
Ozono (O3)			
O_3			
	IMECA	ppm	Ecuaciones
	0-50	0-0.055	
	51-100	0.056-0.110	
	101-150	0.111-0.165	I[O3] = C[O3] * 100/0.11
	151-200	0.166-0.220	
	>200	>0.220	



	51-100	5.51-11.00	I[CO] = 1.82 + C[CO] * 49/5.49
	101-150	11.01-16.50	I[CO] = 2.73 + C[CO] * 49/5.49
	151-200	16.51-22.00	I[CO] = 3.64 + C[CO] * 49/5.49
	>200	>22.00	I[CO] = C[CO] * 201/22.01