THE ROLE OF PROPER MOTIVATION IN MATHEMATICS EDUCATION

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Abstract

The role of proper motivation in mathematics education cannot be overstated as it serves as a catalyst for academic achievement and lifelong learning in mathematics. Educators must recognize the importance of motivation and strive to create an environment that inspires and empowers students to excel in mathematics. At technical universities, mathematics often emerges as a major obstacle to the successful completion of bachelor's degree programs, though mathematics stands as a cornerstone subject in all technical disciplines. The concept of Problem-Based Learning (PBL) has been adapted as an activating method to enhance student engagement in solving larger, real-world problems. The newly developed "MiniPBL" approach caters specifically to first-year bachelor's students, presenting them with small-scale problems that can be tackled using basic mathematical concepts and techniques taught in their curriculum, while these problems often revolve around critical environmental issues such as waste accumulation, water pollution, and deforestation.

Keywords: motivation, mathematics, active learning, miniPBL.

1 INTRODUCTION

The role of proper motivation in mathematics education is irreplaceable. By fostering a growth mindset, enhancing problem-solving skills, boosting confidence, and promoting deeper understanding, motivation serves as a catalyst for academic achievement and lifelong learning in mathematics. Educators must recognize the importance of motivation and strive to create an environment that inspires and empowers students to excel in mathematics. Recently, there has been an alarming decline in the mathematics performance of pupils and students across all levels of the educational system. This trend presents a significant challenge, particularly in technically oriented fields of study. At technical universities, mathematics often emerges as a major obstacle to the successful completion of bachelor's degree programs. Conversely, mathematics stands as a cornerstone subject in all technical disciplines, and without a solid foundation in mathematics, one's journey to becoming an expert in any technical domain becomes arduous.

Given this context, it becomes imperative to explore effective motivation strategies that can inspire students to recognize the importance of mathematics for their future careers, especially in the digital age. Among the traditional methods of motivation is the introduction of applied problems related to specific fields of study. This approach aims to demonstrate to students how mathematical methods are instrumental in solving practical problems. However, beyond this, fostering motivation to learn mathematics can serve as a potent tool in addressing broader challenges facing humanity, particularly those related to environmental sustainability. In this regard, the concept of Problem-Based Learning has been adapted as an activating method to enhance student engagement in solving larger, real-world problems, either individually or in teams. The newly developed "MiniPBL" approach caters specifically to first-year bachelor's students, presenting them with small-scale problems that can be tackled using basic mathematical concepts and techniques taught in their curriculum. These problems often revolve around critical environmental issues such as waste accumulation, water pollution, and deforestation of planet.

2 METHODOLOGY

The MiniPBL method involves distributing prepared sheets to students, comprising a brief contextual introduction to the posed problem, a detailed problem description with several tasks to solve, learning and solving guide, information about expected outcomes, and description of assessment forms. Teachers receive a corresponding sheet outlining the detailed solution, estimated time required for solving, competencies to be developed, and potential use of ICT tools, expected outcomes, and teaching guidance. An example of a miniPBL sheet for teachers dealing with environmental problem of deforestation and explaining the growth of trees that is related to concept of differential equations and methods for their solutions is presented in Fig. 1.

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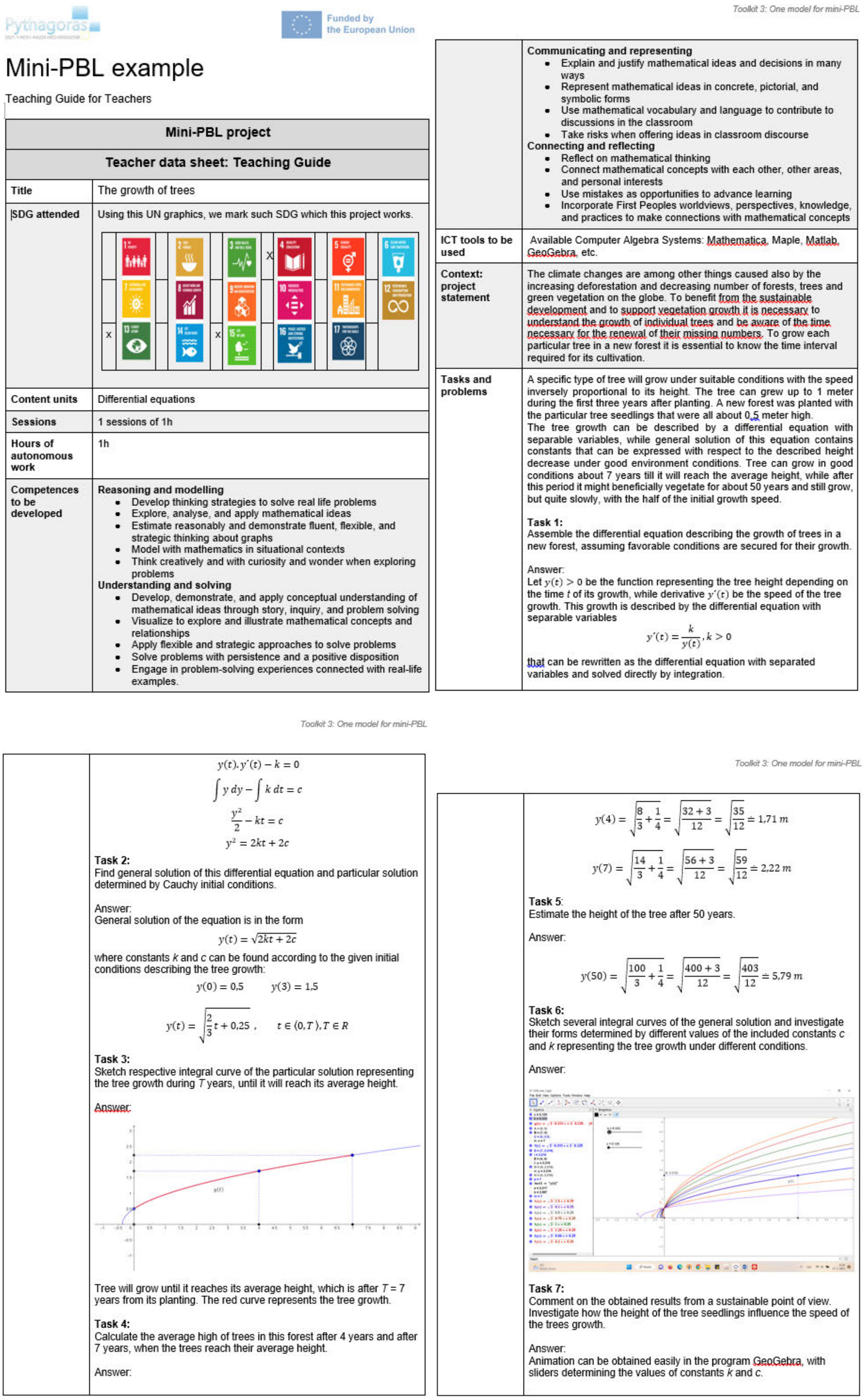


Pythagoras — Funded by the European Union

Mini-PBL example

Teaching Guide for Teachers

Mini-PBL project	
Teacher data sheet: Teaching Guide	
Title	The growth of trees
SDG attended	Using this UN graphics, we mark such SDG which this project works.
Content units	Differential equations
Sessions	1 sessions of 1h
Hours of autonomous work	1h
Competences to be developed	**Reasoning and modelling** • Develop thinking strategies to solve real life problems • Explore, analyse, and apply mathematical ideas • Estimate reasonably and demonstrate fluent, flexible, and strategic thinking about graphs • Model with mathematics in situational contexts • Think creatively and with curiosity and wonder when exploring problems **Understanding and solving** • Develop, demonstrate, and apply conceptual understanding of mathematical ideas through story, inquiry, and problem solving • Visualize to explore and illustrate mathematical concepts and relationships • Apply flexible and strategic approaches to solve problems • Solve problems with persistence and a positive disposition • Engage in problem-solving experiences connected with real-life examples. **Communicating and representing** • Explain and justify mathematical ideas and decisions in many ways • Represent mathematical ideas in concrete, pictorial, and symbolic forms • Use mathematical vocabulary and language to contribute to discussions in the classroom • Take risks when offering ideas in classroom discourse **Connecting and reflecting** • Reflect on mathematical thinking • Connect mathematical concepts with each other, other areas, and personal interests • Use mistakes as opportunities to advance learning • Incorporate First Peoples worldviews, perspectives, knowledge, and practices to make connections with mathematical concepts
ICT tools to be used	Available Computer Algebra Systems: Mathematica, Maple, Matlab, GeoGebra, etc.
Context: project statement	The climate changes are among other things caused also by the increasing deforestation and decreasing number of forests, trees and green vegetation on the globe. To benefit from the sustainable development and to support vegetation growth it is necessary to understand the growth of individual trees and be aware of the time necessary for the renewal of their missing numbers. To grow each particular tree in a new forest it is essential to know the time interval required for its cultivation.
Tasks and problems	A specific type of tree will grow under suitable conditions with the speed inversely proportional to its height. The tree can grew up to 1 meter during the first three years after planting. A new forest was planted with the particular tree seedlings that were all about 0,5 meter high. The tree growth can be described by a differential equation with separable variables, while general solution of this equation contains constants that can be expressed with respect to the described height decrease under good environment conditions. Tree can grow in good conditions about 7 years till it will reach the average height, while after this period it might beneficially vegetate for about 50 years and still grow, but quite slowly, with the half of the initial growth speed.

Task 1:
Assemble the differential equation describing the growth of trees in a new forest, assuming favorable conditions are secured for their growth.

Answer:
Let $y(t) > 0$ be the function representing the tree height depending on the time t of its growth, while derivative $y'(t)$ be the speed of the tree growth. This growth is described by the differential equation with separable variables

$$y'(t) = \frac{k}{y(t)}, k > 0$$

that can be rewritten as the differential equation with separated variables and solved directly by integration.

$$y(t).y'(t) - k = 0$$
$$\int y\,dy - \int k\,dt = c$$
$$\frac{y^2}{2} - kt = c$$
$$y^2 = 2kt + 2c$$

Task 2:
Find general solution of this differential equation and particular solution determined by Cauchy initial conditions.

Answer:
General solution of the equation is in the form

$$y(t) = \sqrt{2kt + 2c}$$

where constants k and c can be found according to the given initial conditions describing the tree growth:

$$y(0) = 0{,}5 \qquad y(3) = 1{,}5$$

$$y(t) = \sqrt{\frac{2}{3}t + 0{,}25}\,, \qquad t \in \langle 0, T \rangle, T \in R$$

Task 3:
Sketch respective integral curve of the particular solution representing the tree growth during T years, until it will reach its average height.

Answer:

Tree will grow until it reaches its average height, which is after $T = 7$ years from its planting. The red curve represents the tree growth.

Task 4:
Calculate the average high of trees in this forest after 4 years and after 7 years, when the trees reach their average height.

Answer:

$$y(4) = \sqrt{\frac{8}{3} + \frac{1}{4}} = \sqrt{\frac{32+3}{12}} = \sqrt{\frac{35}{12}} \doteq 1{,}71\,m$$

$$y(7) = \sqrt{\frac{14}{3} + \frac{1}{4}} = \sqrt{\frac{56+3}{12}} = \sqrt{\frac{59}{12}} \doteq 2{,}22\,m$$

Task 5:
Estimate the height of the tree after 50 years.

Answer:

$$y(50) = \sqrt{\frac{100}{3} + \frac{1}{4}} = \sqrt{\frac{400+3}{12}} = \sqrt{\frac{403}{12}} \doteq 5{,}79\,m$$

Task 6:
Sketch several integral curves of the general solution and investigate their forms determined by different values of the included constants c and k representing the tree growth under different conditions.

Answer:

Task 7:
Comment on the obtained results from a sustainable point of view. Investigate how the height of the tree seedlings influence the speed of the trees growth.

Answer:
Animation can be obtained easily in the program GeoGebra, with sliders determining the values of constants k and c.

Figure 1. MiniPBL learning guide for teachers.

The title declares the main intention of the project and it is the first spark to wake the interest of students. Therefore, it should be direct, clear, motivating and descriptive of the real-life issue which it addresses.

The project may cover more units, but preferably it should be just one or two not to design a too long activity. The key concept is to focus on the review processes to recover students at risk so that they do not fail, and remark the connection between the content units of the course. An appropriate number of classroom sessions should be estimated, needed to explain the problem and sketch the solution strategy. Students must know in advance that, in general, all the projects will require autonomous work.

It is important not to generate an overly demanding project. Too much work out of classroom affects students´ motivation, impacts negatively on the next mini-PBL proposed, and causes undue stress to students who are therefore solving their miniPBL in rush, without deep reflections, careful writing, checking results by different ways, discussing topic with classmates, etc. The worst consequence of an overtaking proposal is the cheating between students. You can introduce ways to avoid or reduce these bad practices, but this is not an easy task for teachers to manage additional control of the students' ethics performance.

Choose carefully which from the mathematical competencies described and elaborated by SEFI Mathematics Special Interest Group [1] might be developed solving the applied problem using mathematical methods.

1 Thinking mathematically:

This competency comprises knowledge of the kind of questions that are dealt with in mathematics and the types of answers mathematics can and cannot provide, and the ability to pose such questions. It includes the recognition of mathematical concepts and an understanding of their scope and limitations as well as extending the scope by abstraction and generalisation of results. This also includes an understanding of the certainty mathematical considerations can provide.

2 Reasoning mathematically:

This competency includes on the one hand the ability to understand and assess an already existing mathematical argumentation (chain of logical arguments), in particular to understand the notion of proof and to recognise the central ideas in proofs. It also includes the knowledge and ability to distinguish between different kinds of mathematical statements: definition, if-then-statement, iff-statement, etc. On the other hand it includes the construction of chains of logical arguments and hence of transforming heuristic reasoning into own proofs, which supports logical reasoning.

3 Posing and solving mathematical problems:

This competency comprises the ability to identify and specify mathematical problems, be they pure or applied, open-ended or closed, and also the ability to solve mathematical problems including knowledge of the adequate algorithms. What really constitutes a problem is not well defined and it depends on personal capabilities whether or not a question is considered as a problem. This has to be borne in mind, for example when identifying problems for a certain group of students.

4 Modelling mathematically:

This competency also has essentially two components: the ability to analyse and work with known models, find properties, investigate range and validity, relate to modeled reality and ability “to perform active modelling“, which means to structure the part of reality that is of interest, set up a correct mathematical model and transform the questions of interest into mathematical questions. Then to answer the questions mathematically, interpret the results in reality and investigate the validity of the model, monitor and control the whole modeling process.

5 Representing mathematical entities:

This competency includes the ability to understand and use mathematical representations, be they symbolic, numeric, graphical and visual, verbal, material objects, dynamic interpretations, various models, etc. and to know their relations, advantages and limitations. It also includes the ability to choose and switch between representations based on this knowledge.

6 Handling mathematical symbols and formalism:

This competency includes the ability to understand symbolic and formal mathematical language and its relation to natural language as well as the translation between both. It also includes the rules of formal mathematical systems and the ability to use and manipulate symbolic statements and expressions according to these rules.

7 Communicating in, with, and about mathematics:

This competency includes on the one hand the ability to understand mathematical statements (oral, written or other) made by others and on the other hand the ability to express oneself mathematically in different ways and using different means as ICT, paper and pencil, oral explanation, creation of models, etc.

8 Making use of aids and tools:

This competency includes knowledge about the aids and tools that are available as well as their potential and limitations. Additionally, it includes the ability to use them thoughtfully and efficiently. ICT – CAS, dynamic maths softwares (Maple, Matlab, Mathematica, GeoGebra) or any other packages, including AI.

In a broader context, we might expect students to develop various general competencies necessary for their future work as engineers, since these kinds of activities are one of the most favorable to unfold competencies by the combination of tasks and problems promoting thus more the “know how to do" than the “know by repetition”.

Reasoning and modeling

Develop thinking strategies to solve puzzles and play games

Explore, analyze, and apply mathematical ideas using reason, technology, and other tools

Estimate reasonably and demonstrate fluent, flexible, and strategic thinking about number

Model with mathematics in situational contexts

Think creatively and with curiosity and wonder when exploring problems

Understanding and solving

Develop, demonstrate, and apply conceptual understanding of mathematical ideas through play, story, inquiry, and problem solving

Visualize, explore and illustrate mathematical concepts and relationships

Apply flexible and strategic approaches to solve problems

Solve problems with persistence and a positive disposition

Engage in problem-solving experiences connected with place, story, cultural practices, and perspectives relevant to local communities and other cultures

Communicating and representing

Explain and justify mathematical ideas and decisions in many ways

Represent mathematical ideas in concrete, pictorial, and symbolic forms

Use mathematical vocabulary and language to contribute to discussions in the classroom

Take risks when offering ideas in classroom discourse

Connecting and reflecting

Reflect on mathematical thinking

Connect mathematical concepts with each other, other areas, and personal interests

Use mistakes as opportunities to advance learning

Incorporate First Peoples worldviews, perspectives, knowledge, and practices to make connections with mathematical concepts

In the section ICT tools to be used it is recommended to list the features which can be used in the miniPBL resolution. It is not necessary to list software applications that could be used, because this search is part of the project tasks that must be solved by students. A list like this might be provided:

- Graphics: explicit, implicit, 2D, 3D,...
- Solving equations and/or systems: graphically, numerically, algebraically,…
- Calculus calculator: derivatives, integrals,…
- Vector and matrix calculator: graphically, numerically.

Context project statement is section where the project is presented to the student. The introduction, the core topic and all the information will help students to allocate the tasks and problems posted later. Here one may include the references, graphics, news from media, official reports (UN, EU, OCDE, UNICEF,...), Artificial Intelligence and any source which helps to acquire a wider idea about the SDG - Sustainable Development Goals problem addressed. This effort to “put in context” the problem will connect mathematics with real-life, with Earth challenges and humanity needs. By teaching mathematics using SDG we support the reflection of our students, future professionals, on the major challenges of Mankind and Earth, thus raising responsible citizens.

Section Tasks and problems will present all the activities and tasks to be done. This can be done in any format: classical problems, inquiries, modeling, generalization, etc. Here we need to be stimulating, focus more on the competences to develop than the difficulty or the range of contents to cover. Problem tasks should be listed with increasing complexity, urging students to apply previous results to the next ones, and should cover all selected content units (spiral curriculum) while drawing students’ attention to the connection between these units.

Section expected outcomes must be as clear as possible. Students should know perfectly what we expect from them. Solving miniPBL they are training abilities that will be useful in their professional life: the deep analysis of a problem, the design of the solution and the presentation of the results which must be a well-structured process. Provide a guide of minimum output expected:

- Graphics fitting the solution
- Tables of data used/obtained in solutions
- Numerical results explained and put in context
- Capture of ICT tools solutions used
- Sequence of steps followed
- Remark computations done by hand and done by ICT tools
- Provide complete answer to questions
- All the results must be presented in the context of the problems.

In the guide for teaching, we should develop all the key facts and procedures which will guide the teachers to provide the advice, help and hints needed to present and launch the mini-PBL to students. Here we can inform teachers about the mathematical insights we desire to be achieved by the students along the construction and solution of the mini-PBL project.

One of the main items, which used to worry students and always affects their perception of the activity, is the form of assessment. We recommend the alternative assessment that starts with outcomes, proceeds with projects, products, and performances that map to the outcomes, and completes the loop with assessment and feedback to students. Alternative assessment provides tools to assess projects effectively. It assesses acquisition of knowledge and skills in ways other than the conventional methods such as traditional paper-and-pencil tests. It actively involves students in a process that combines what is taught, how it is taught, and how it is evaluated. Alternative assessment gives responsibility to learners for directing and managing their own learning and achievements. It is authentic, often in real-life environments, with real-world challenges and interdisciplinary in nature, emphasizing on specific knowledge as well as general skills such as transfer of information across settings. It involves negotiation and interpersonal skills as well as decision making skills, mastery of a task before progressing to the next task and assessment of the periodic performance mastery. It can be based on performance during presentation of the miniPBL solution, or based on a scoring scale incorporating a set of essential criteria for the task and appropriate levels of performance for each criterion used. Examined students are required to review and select items that best demonstrate their learning, presenting a portfolio that might be paper-based, computer-based or a combination of both.

Student sheet with the miniPBL problem illustrated in Fig. 2 and dealing with the same topic of the tree growth must contain a special section entitled Guide for learning. Some examples of comments instructing students are as follows:

- Read carefully the problem statement and the tasks posted. Always maintain a global view of all the projects.
- Identify, or try to do a first draft match, the content units of your lecture notes involved in every task.
- Take your lecture notes open and review before starting to solve the problems.
- Match output expected with the tasks posted, at least as first draft approach. This will give you ideas about which tools are needed (ICT tools, hand calculations, data collection,…).
- Follow the order of the tasks, try to increase the knowledge of the problem while you are solving the activities.
- Always think that might be different ways to solve the problem.
- Use ICT tools to avoid hard computations and check whether your solutions are correct in different ways if possible.
- The solutions are always part of a context, express a final solution totally integrated in the problem posted.
- Be sure you answer the complete questions. Partial solving could generate missing information for the following questions.
- Always try to solve the questions by yourself.
- If the project can be done in groups, discuss in groups to confirm and detect failures or weaknesses, confront strategies, discuss presentation format, etc. Working in groups doesn't mean to work less, but to work better.

Learning Guide for Students

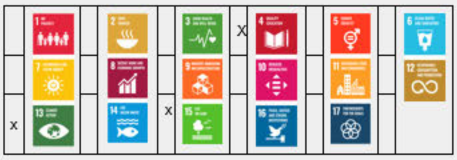
Mini-PBL project	
Student data sheet: Learning Guide	
Title	The growth of trees
SDG attended	Using this UN graphic, we mark such SDG which this project works. 
Content units	Ordinary differential equations of order I
Sessions	1 sessions of 1h
Hours of autonomous work	1h
ICT tools to be used	Available Computer Algebra Systems: Mathematica, Maple, Matlab, GeoGebra, etc.
Context: project statement	The climate changes are among other things caused also by the increasing deforestation and decreasing number of forests, trees and green vegetation on the globe. To benefit from the sustainable development and to support vegetation growth it is necessary to understand the growth of individual trees and be aware of the time necessary for the renewal of their missing numbers. To grow each particular tree in a new forest it is essential to know the time interval required for its cultivation.
Tasks and problems	A specific type of tree will grow under suitable conditions with the speed inversely proportional to its height. The tree can grew up to 1 meter during the first three years after planting. A new forest was planted with the particular tree seedlings that were all about 0,5 meter high. The tree growth can be described by a differential equation with separable variables, while general solution of this equation contains constants that can be expressed with respect to the described height decrease under good environment conditions. Tree can grow in good conditions about 7 years till it will reach the average height, while after this period it might beneficially vegetate for about 50 years and still grow, but much slowlier, with respect of the initial growth speed.
	Task 1: Assemble the differential equation describing the growth of trees in a new forest, assuming favorable conditions are secured for their growth. **Task 2:** Find general solution of this differential equation and particular solution determined by Cauchy initial conditions. **Task 3:** Sketch respective integral curve of the particular solution representing the tree growth during *T* years, until it will reach its average height. **Task 4:** Calculate the average high of trees in this forest after 4 years and after 7 years, when the trees reach their average height. **Task 5:** Estimate the height of the tree after 50 years. **Task 6**: Sketch several integral curves of the general solution and investigate their forms determined by different values of the included constants *c* and *k* representing the tree growth under different conditions. **Task 7:** Comment on the obtained results from a sustainable point of view. Investigate how the height of the tree seedlings influence the speed of the trees growth.
Outcomes expected	- Graphics fitting the solution; - Numerical results explained and put in context; - Capture of ICT tools solutions used; - Sequence of steps followed; - Remark computations done by hand and done by ICT tools; - Provide complete answer to questions; - All the results must be presented in the context of the problem;
Guide for learning	- Read carefully the problem statement and the tasks posted. Always maintain a global view of all the projects. - Identify, or try to do a first draft match, the content units of your lecture notes involved in every task. - Take your lecture notes open and review before starting to solve the problems. - Match output expected with the tasks posted, at least as first draft approach. - Follow the order of the tasks, try to increase the knowledge of the problem while you are solving the activities. - Always think that maybe there are different ways to solve a problem. - Use ICT tools to avoid hard computations and check your solutions are correct in different ways if possible. - The solutions are always part of a context, expressing such a final solution totally integrated in the problem posted. - Be sure you answer the complete questions. - Always try to solve the questions by yourself. - If the project can be done in groups, discuss with the groups the proposed problem, to confirm and detect fails or weaknesses, confront strategies, discuss presentation format, etc. Working in groups doesn't mean work less but work better.
Assessment	- Final report; - Oral presentation; - Peer-assessment: students will apply peer-assessment for their periodic performance using online peer assessment tools used and available at the respective institution.

Figure 2. MiniPBL learning guide for students.

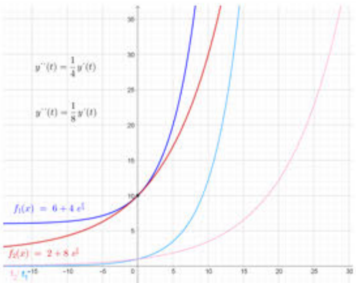
3 RESULTS

This method has been piloted in the Mathematics I course at the Faculty of Mechanical Engineering, Slovak University of Technology in Bratislava, Slovakia [2] and evaluated within the Strategic partnership program of the Erasmus+ project Pythagoras [3]. Project is focused on enhancing mathematics teaching and making it more inclusive, efficient, enjoyable, and relevant by linking mathematics with real-life cases pertinent to students' fields of study. This approach proved to be very effective. Students were engaged and showed interest to learn about methods for solving differential equations, in order to understand how the solve several highly current and acute problems of the sustainable development and life on our planet Earth. They worked in small teams to elaborate the solution and highly appreciated the direct application of mathematical methods to everyday problems.

There were prepared 3 different miniPBL tasks related to environmental problems. The one presented in Fig. 1 is an urgent task related to the uncontrolled deforestation of the planet caused by intolerable reduction of the planet´s forests caused by chaotic cutting down of the Amazon rainforests for profit. Ordinary differential equation with separable variables might be used as mathematical model representing the tree growth, while various constants are used to model the growth under different conditions and for different types of trees. Students were interested in modelling the newly planted forest growth related to properly chosen types of trees, the correct height of the seedlings used and suitable climate condition supporting their growth. They were quite surprised by the length of the time period necessary to cultivate a new forest. Devastating deforestation has dire consequences for drying out the soil, the lack of underground water supplies and the rapid reduction of the green areas on the planet. Mathematical models confirmed that only restrictive measures, a strict ban on cutting down living productive forests all over the planet and intensive reafforestation of uprooted dried soils and devastated lands can slow down the danger of drying up and loss of fertile soil on the planet and prevent the deterioration of the current situation.

The next miniPBL was about accumulation of the waste on the Earth and impact of possible actions taken to slow down this process, as e.g. recycling and other measures aimed to reduce the waste accumulation acceleration. Mathematical model of the waste accumulation might be represented by linear homogeneous differential equation of order I, while more detailed model might be found using homogeneous differential equation of order II modelling the acceleration of the waste growth. Both models were analysed with respect to special initial conditions and by modification of integration constants appearing in the general solutions of these equations and representing possible environmental measures. Both mathematical models clearly showed that any adopted environmental measures cannot lead to the complete diminishing of the accumulated waste on the planet, but they can considerably improve the planet pollution and have beneficial effect on the climate changes. The amount of waste could “stop growing” only in case that the industrial production will be reduced, while all accumulated waste will be rapidly and immediately recycled.

The third miniPLBL illustrated in Fig. 3 was related to the severe problem of water pollution. Ordinary differential equation of order II was assembled, which mathematically describes the process of increasing percentage of the planet polluted water resources before adoption of ecological measures and after their implementation. Particular solutions of this equation show the impact of implemented measures, which are mathematically determined by Cauchy initial conditions. The process of the water pollution increase in time is then obviously manifested by integral curves as graphs of these particular solutions. Modelling different situations choosing different constants in particular solutions which might represent adopted ecological measures (purification of polluted water resources, no pollution by toxic substances in wastewater, decrease in toxic industrial production, etc.) leads to the reasonable analysis of the current unfavourable situation. The increasing amount of polluted water sources could be stowed down only if no toxic materials and wastes were produced, all waste water would undergo the required treatment, dirty industrial production would be reduced, while all currently polluted water sources would be rapidly reduced by and immediate cleanup. The planet is in real danger of a lack of clean drinking water, while the situation in some areas is dangerously critical. Mankind is responsible for this undesirable situation and must take a decisive stand and adopt corrective actions. Mathematical models of the water pollution acceleration are one of the most convincing warning indicators.



Under both circumstancies the higher water pollution is inevoidable, as derivatives of both particular solutions are positive functions.

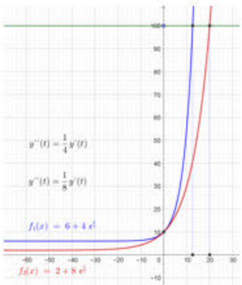
$$y_P{}'(t) = e^{\frac{1}{4}t}, \qquad t \in \langle 0, T\rangle, T \in R$$

$$y_P{}'(20) = e^{\frac{1}{8}t}, \ t \in \langle 0, T\rangle, T \in R$$

Task 4:
Calculate, how much time is left before 100% of planet water resources are completely polluted in both scenarios, without or with the adoption of strict functioning measures saving the planet clean drinking water.
Answer:

$$y_P{}'(12{,}5) = 97\%, \qquad t = 12{,}5$$

$$y_P{}'(20) = 99\%, \ t = 20$$



There is left less than 13 years before the total plane water resources are polluted in the first scenario, and about 20 years in the other one.

Task 5:
Find, what would be the necessary measures in decrease of pollution acceleration in order to stop the water pollution in 10 years at 20 %.
Answer:

$$y''(t) = \frac{1}{k}y'(t)$$

$$y''(t) - \frac{1}{k}y'(t) = 0$$

$$r^2 - \frac{1}{k}r = 0 \Rightarrow r_1 = 0, r_2 = \frac{1}{k}$$

$$y_P = c_1 + c_2 e^{\frac{1}{k}t}, \ t \in \langle 0, T\rangle, T \in R$$

$$y_P{}' = \frac{1}{k}c_2 e^{\frac{1}{k}t}, \ t \in \langle 0, T\rangle, T \in R$$

Initial conditions

$$y(0) = 10, \ y'(0) = 1$$

determine the values of constants c_1, c_2

$$c_1 + c_2 = 10, \frac{1}{k}c_2 = 1$$

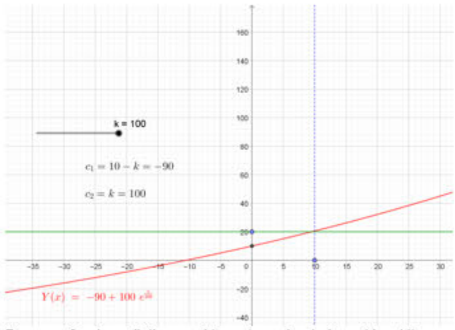
$$c_2 = k$$

$$c_1 = 10 - k$$

$$y_P = 10 - k + ke^{\frac{1}{k}t}, \ t \in \langle 0, T\rangle, T \in R$$

$$y(10) = 20 \Rightarrow 10 - k + ke^{\frac{10}{k}} = 20 \Rightarrow k \doteq 100$$

$$y_P = -90 + 100e^{\frac{1}{100}t}, \ t \in \langle 0, T\rangle, T \in R$$



Process of water pollution must be not accelerated, or at least its acceleration must be decreased to one hundred of the pollution speed!

Figure 3. MiniPBL – water pollution analysis of results.

4 CONCLUSIONS

Analysis of results and students' feedback from the implemented initiatives introducing described activities to university freshmen within the basic Mathematics I course at the bachelor study programs highlighted the efficacy of the MiniPBL approach in promoting mathematical understanding and engagement among students. Working on the 3 designed miniPBLs our students acquired skills in solving differential equations of various types and orders and learned to model the course of processes by choosing integral constants in the general solutions of these equations. This was achieved by solving real world problems that addressed students also as human beings living on the Earth which is in serious ecological danger. In addition to the achievements in mathematics understanding and the gain in calculation skills, our students also appreciated the acquired knowledge related to ecological behaviour towards the sustainability of life on the planet.

Teaching mathematics based on SDG - Sustainable Development Goals [4] principles results not only in the development of students´ critical thinking and development of their mathematical literacy and skills. It also brings educated and responsible inhabitants of our planet, who will take care of its further development, maintaining life on the planet while preserving the diversity of all living species. We believe that the main goal of education to raise responsible global citizens and cultivate humanity has been successfully launched.

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