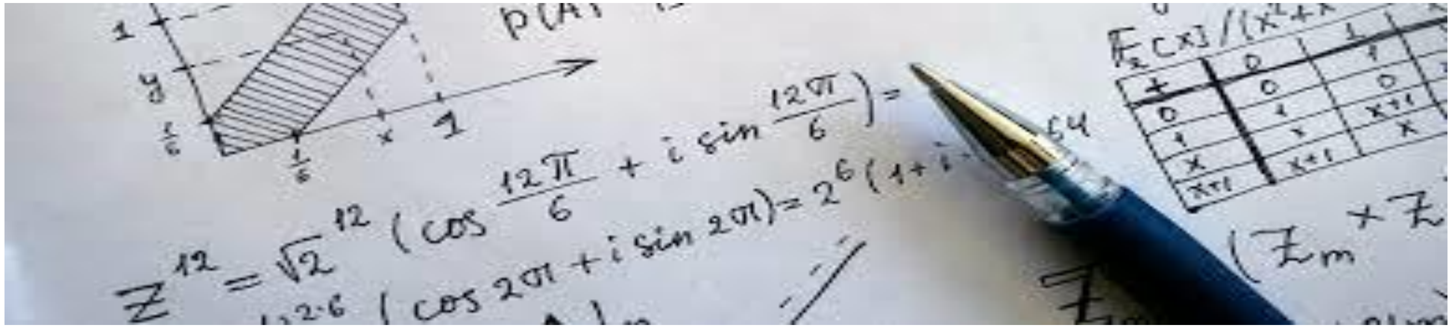

Introducing miniPBL in Mathematical Subjects

Daniela Velichová, Jana Gabková

SLOVAK UNIVERSITY OF TECHNOLOGY IN BRATISLAVA

Co-funded by the European Union

2nd Project Meeting
28 – 29 September, 2023
University of La Laguna

Pythagoras
2021-1-RO01-KA220-HED-000032258

„In situ“

SLOVAK UNIVERSITY OF TECHNOLOGY IN BRATISLAVA
FACULTY OF MECHANICAL ENGINEERING

Faculty of Mechanical Engineering STU

Department of Mathematics and Physics

- Yearly about 300 university freshmen
- Accepted without entry exams
- Constant decline in mathematical knowledge
- Weakly motivated students – reasons?
- High drop of students after the first semester – insufficient knowledge!

„In situ“

Basic courses at bachelor programes:

- **Mathematics I, Mathematics II**, Mathematics III
  Numerical Mathematics, Basics of Statistics
  Constructive Geometry, Linear Algebra, Differential Equations

Optional courses at master programes:

- Statistical Analyses, Applied Mathematics, Algebraic Structures
  Systems of Differential Equations

Selective courses at Phd programes:

- Optimisation Methods, Selected Parts for Mathematics
  Special Differential Equations

Applications as motivation?

SLOVAK UNIVERSITY OF TECHNOLOGY IN BRATISLAVA
FACULTY OF MECHANICAL ENGINEERING

Tested in **eduScrum** within the project **DrIVE MATH 2017-2020**

- **winter semester** of the academic year **2018/19**
- basic **Mathematics I** course for the first year engineering students at the bachelor study programmes
- students solved applications of mathematics in a team
- utilised applications of mathematics were selected from the lecture notes for specialised subjects taught at the faculty, while some of these were introduced in the lecture notes as examples of solved problems

Cohorts of students

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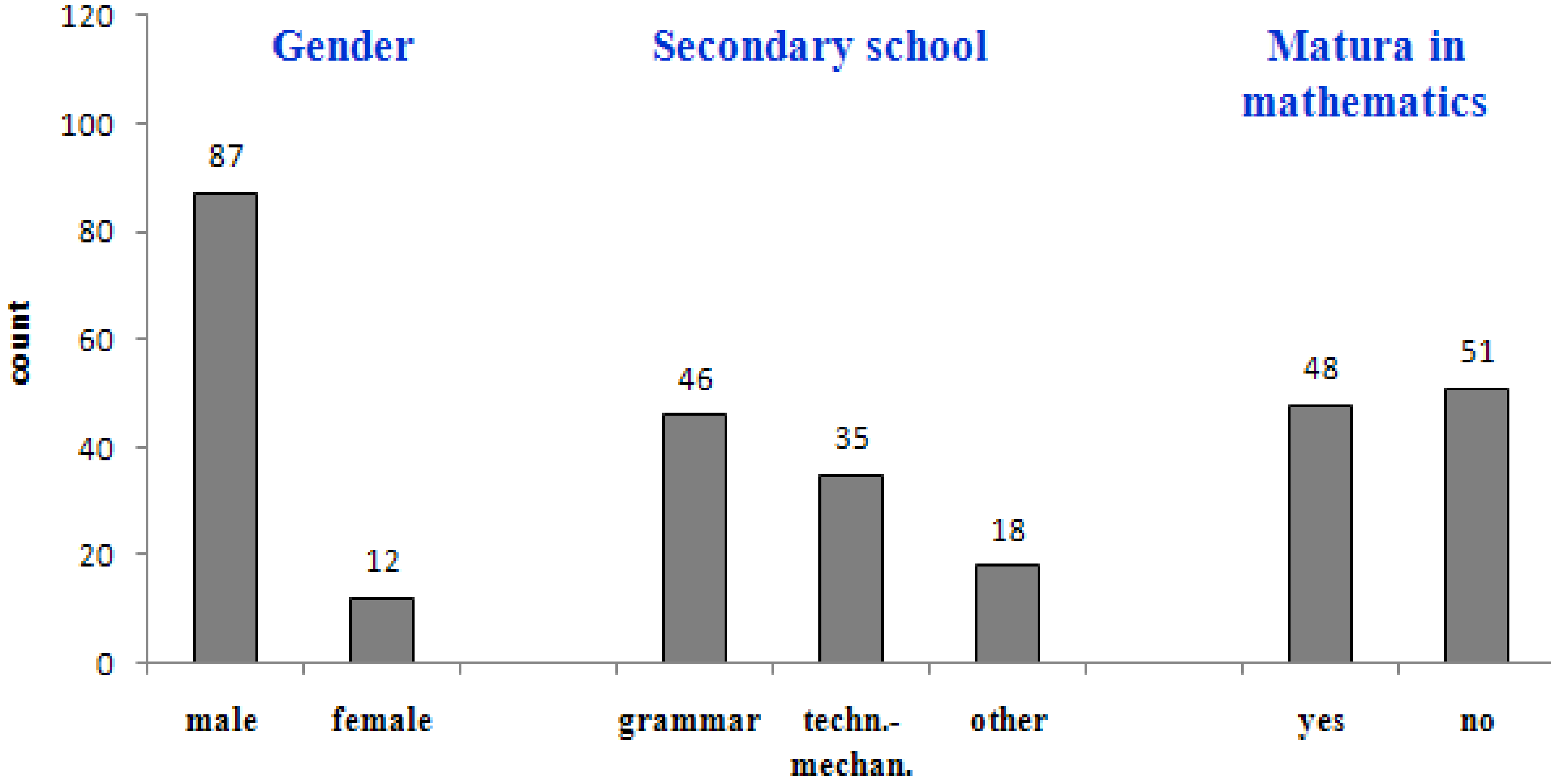
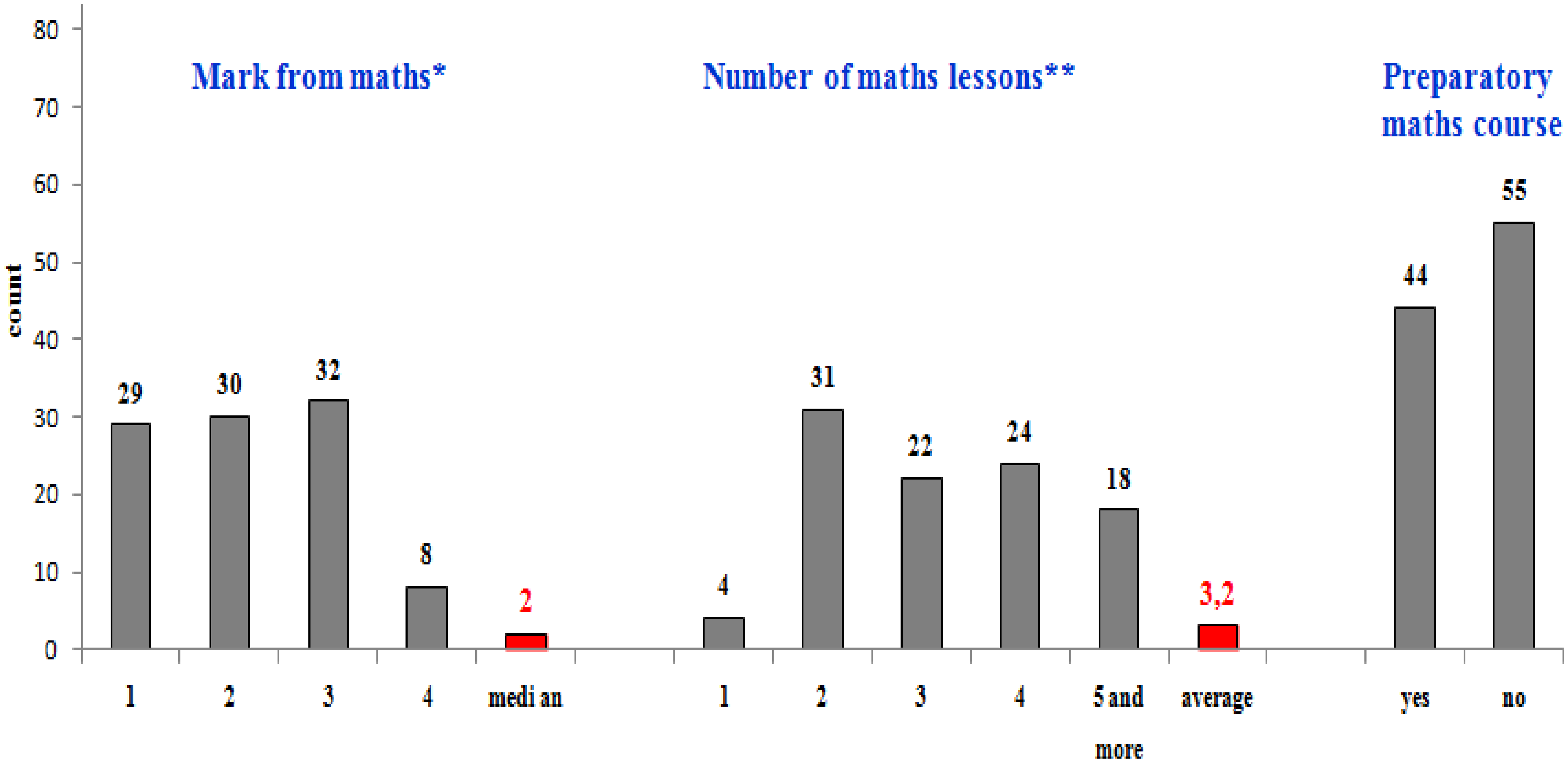


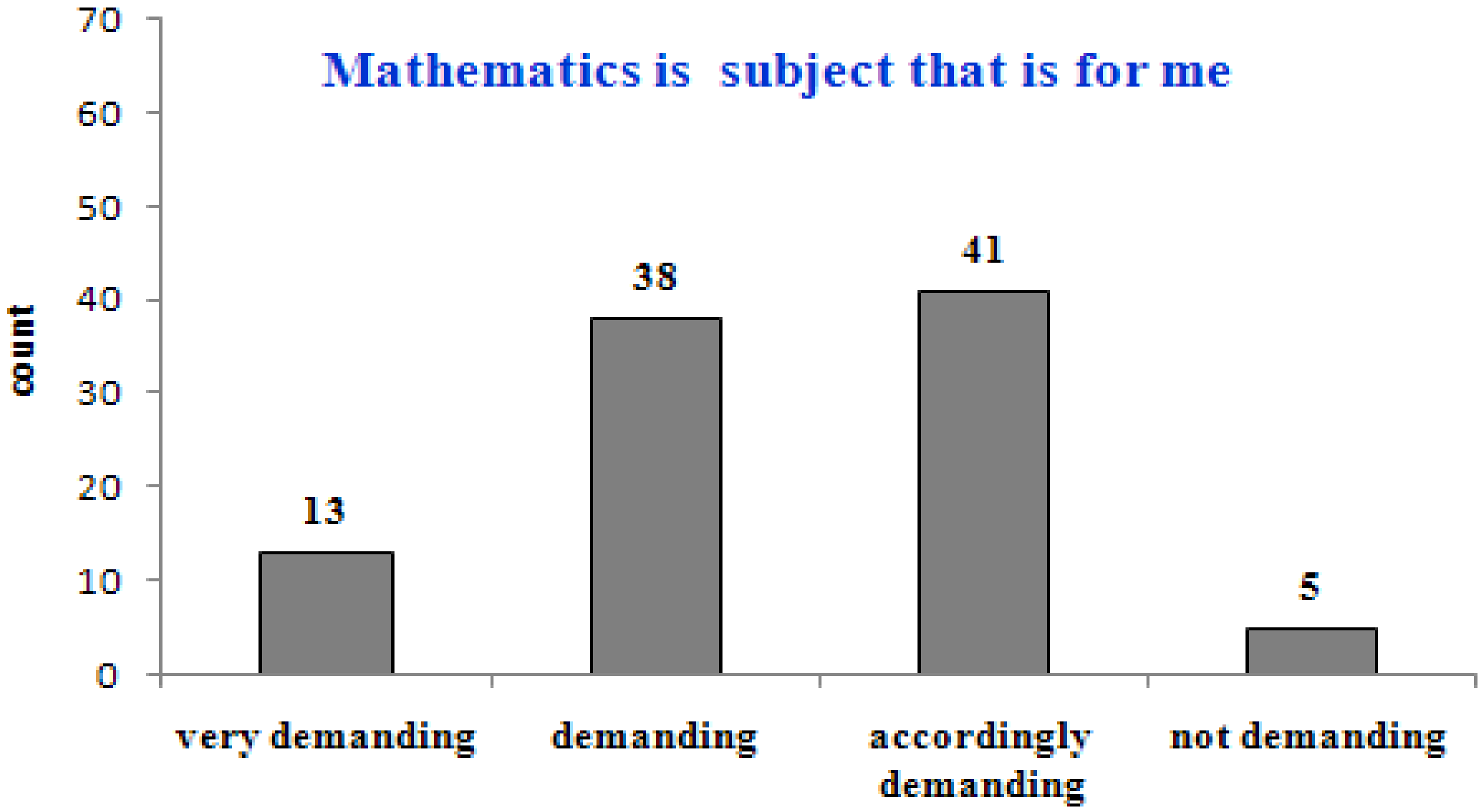
Cohorts of students

SLOVAK UNIVERSITY OF TECHNOLOGY IN BRATISLAVA
FACULTY OF MECHANICAL ENGINEERING



Cohorts of students

SLOVAK UNIVERSITY OF TECHNOLOGY IN BRATISLAVA
FACULTY OF MECHANICAL ENGINEERING



Cohorts of students

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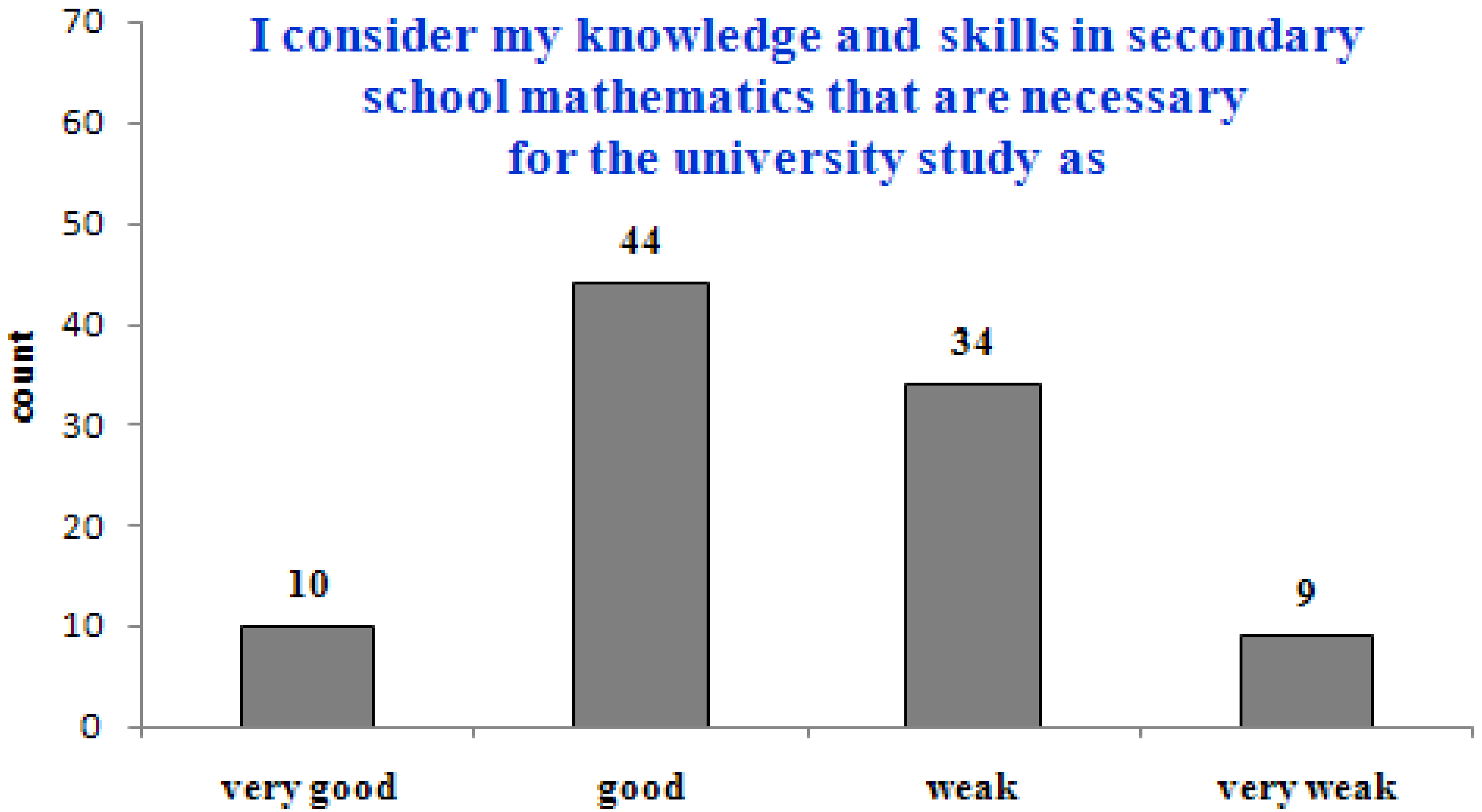

eduScrum results

STU SjF
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FACULTY OF MECHANICAL ENGINEERING

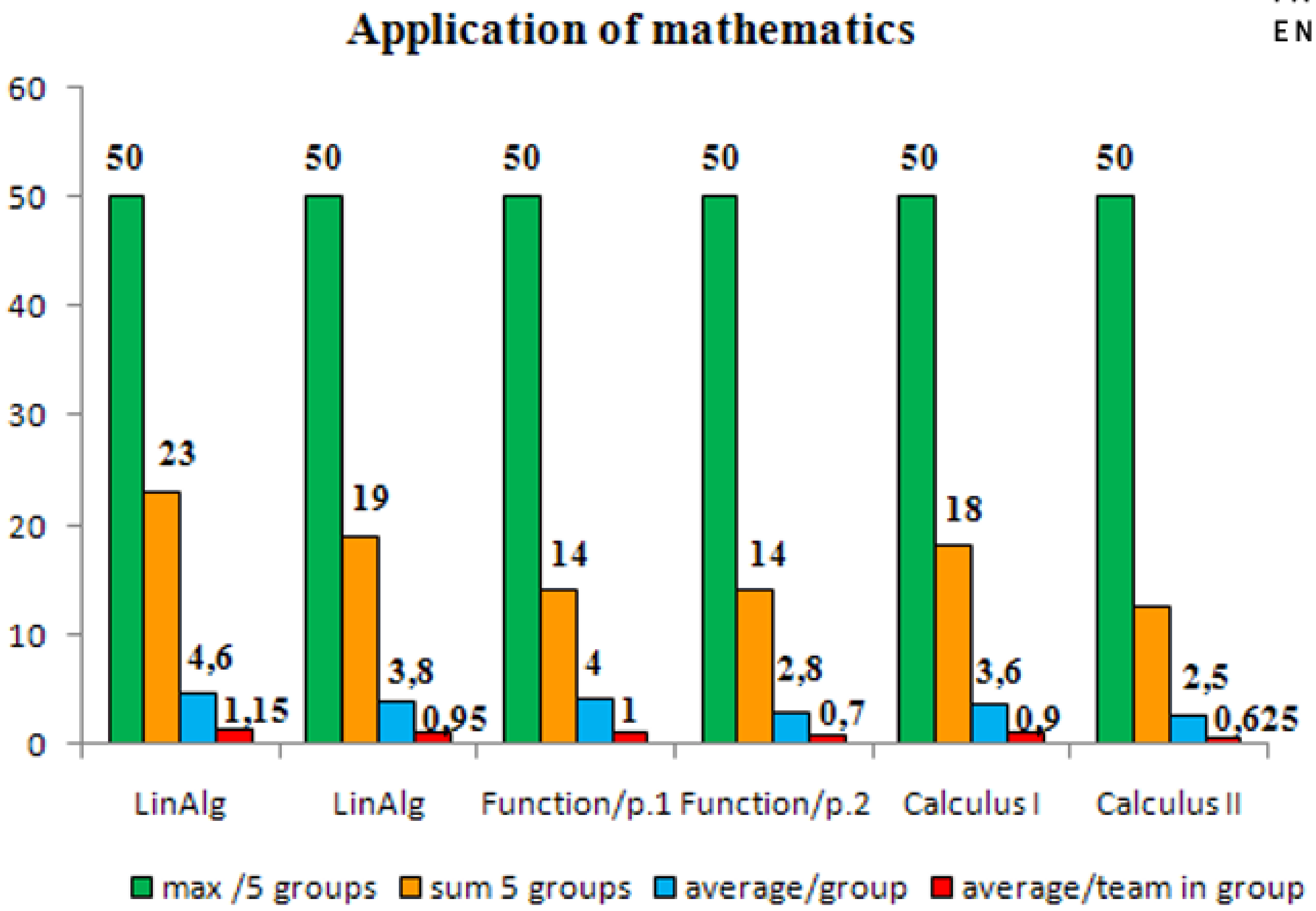

Fig. Complexity of application of mathematics – groups / teams

eduScrum results

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FACULTY OF MECHANICAL ENGINEERING

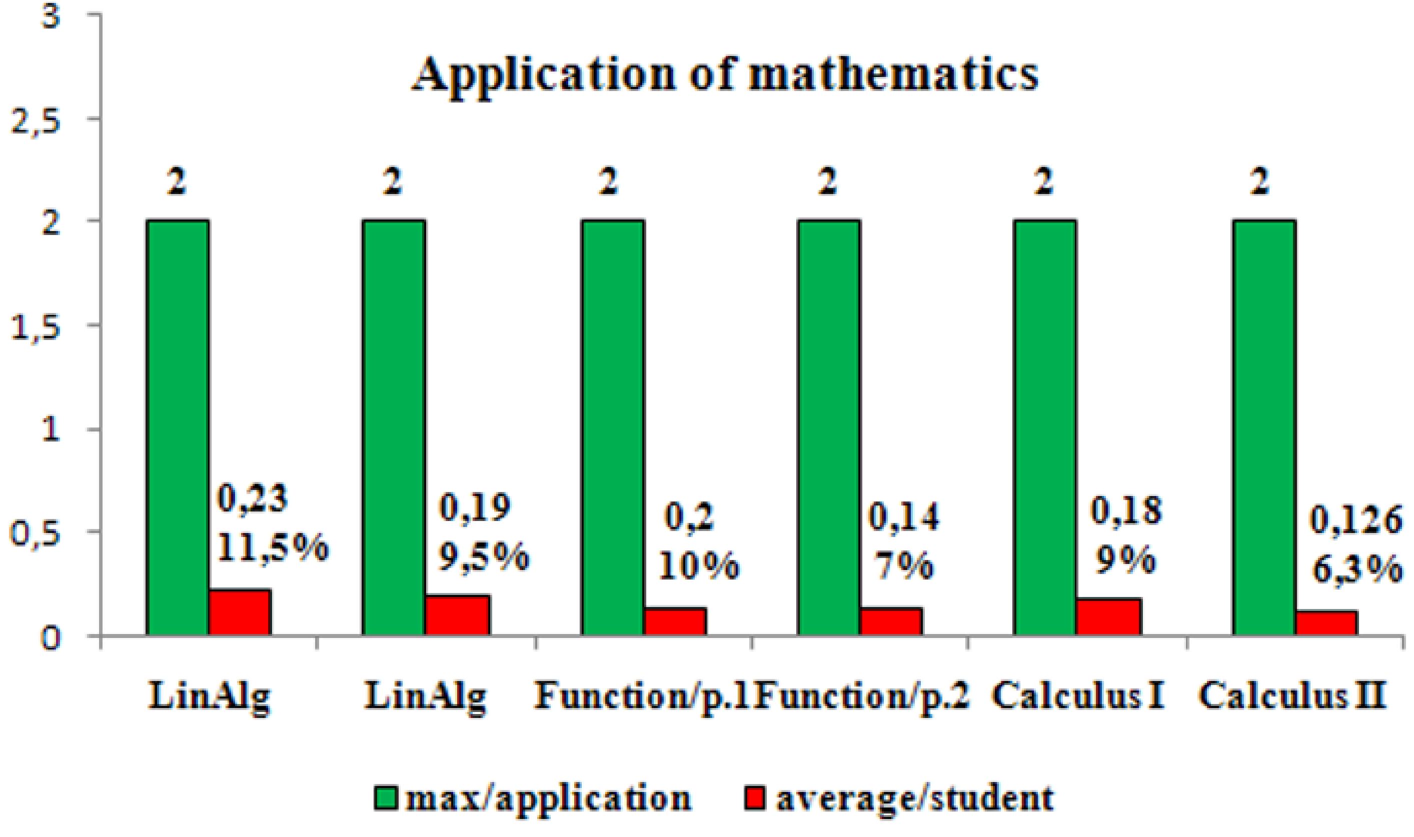

Fig. Complexity of application of mathematics – student

eduScrum results

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FACULTY OF MECHANICAL ENGINEERING

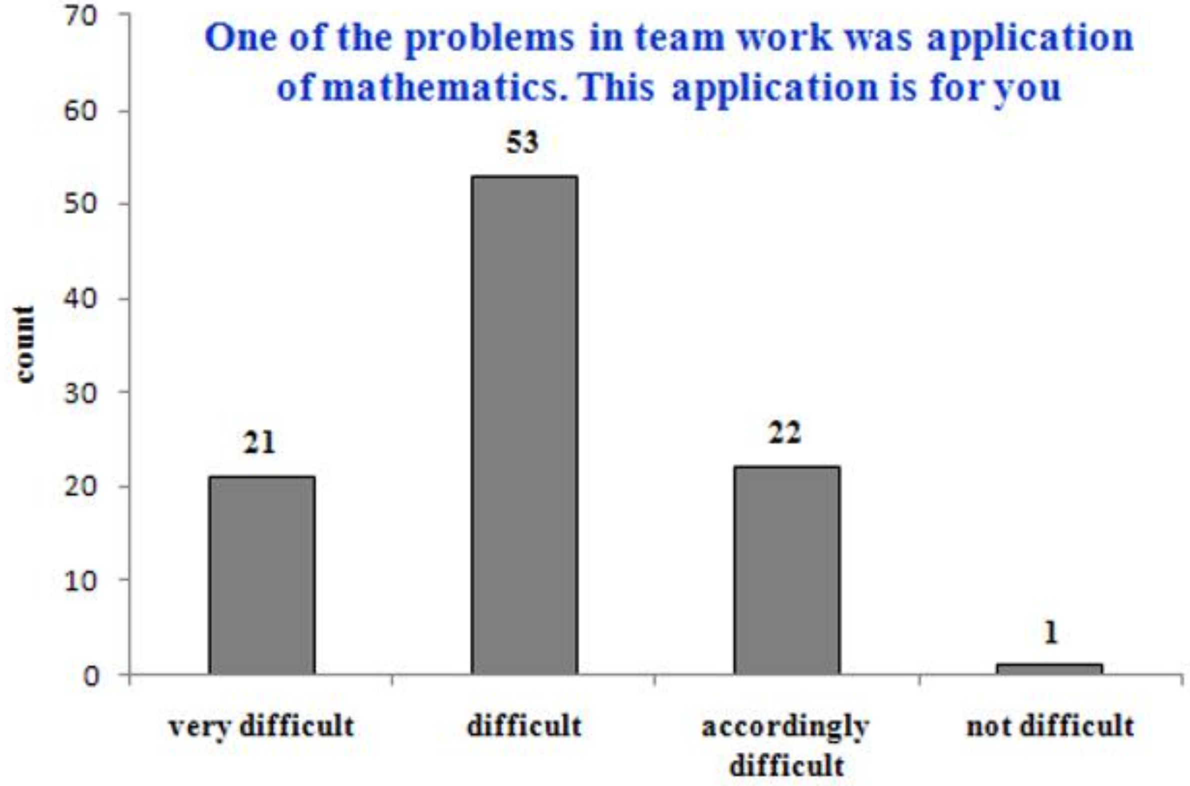

Fig. Complexity of application of mathematics

According to feedback received from questionnaire: **Applied tasks caused difficulties to 74% of students** ("very difficult" and "difficult").

eduScrum results

STU SjF
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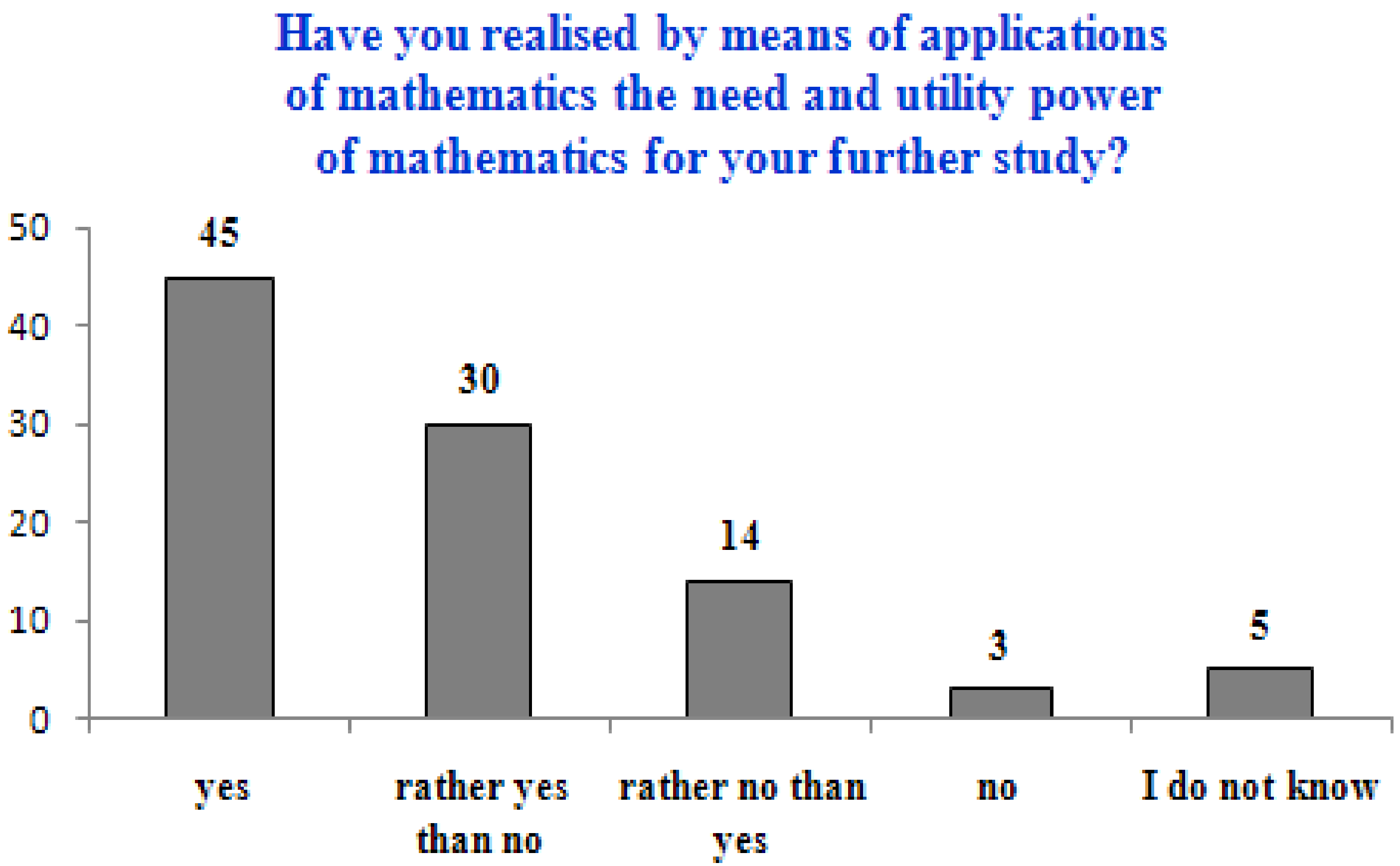
FACULTY OF MECHANICAL ENGINEERING



Motivation

SLOVAK UNIVERSITY OF TECHNOLOGY IN BRATISLAVA
FACULTY OF MECHANICAL ENGINEERING

- To develop miniPBL examples dealing with important environmental problems
- To use simplified mathematical models to show possible environmentally negative development with crucial impact on the Planet
- To introduce miniPBL in 3 different ways at the bachelor study groups
- To analyse experience with the introduction of this innovative teaching scenario

miniPBL1 - The growth of trees

The climate changes are among other things caused also by the increasing deforestation and decreasing number of forests, trees and green vegetation on the globe.

To benefit from the sustainable development and to support vegetation growth it is necessary to understand the growth of individual trees and be aware of the time necessary for the renewal of their missing numbers.

To grow each particular tree in a new forest it is essential to know the time interval required for its cultivation.

miniPBL1 - Introduction

A specific type of tree will grow under suitable conditions with the speed inversely proportional to its height.

The tree growth can be described by a differential equation with separable variables.

$$y'(t) = \frac{k}{y(t)}, k > 0, t \in \langle 0, T \rangle, T \in R$$

General solution of this equation contains constants that can be expressed with respect to the determined height decrease of the tree under good environmental conditions.

$$y(t) = \sqrt{2(k.t + c)}, k > 0, t \in \langle 0, T \rangle, T \in R$$

miniPBL1 - Introduction

Mathematics 2 – lecture on ODR I

- introductory information and presentation of problem
- explanation of the mathematical model and its solution
- discussion about possible meaning of constants present in the general solution
- illustrations (in GeoGebra) of the impact of constant variations on the form of integral curves
- students´ response on the usage of mathematics to solve currently crucial problems

Few students cooperated actively and overall reactions were positive!

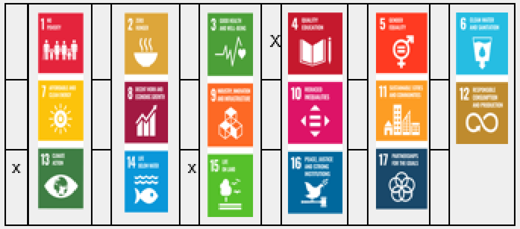
miniPBL1 - Introduction

Pythagoras
2021-1-RO01-KA220-HED-000032258

Funded by the European Union

Mini-PBL example

Teaching Guide for Teachers

Mini-PBL project	
Teacher data sheet: Teaching Guide	
Title	The growth of trees
SDG attended	Using this UN graphics, we mark such SDG which this project works. 
Content units	Differential equations
Sessions	1 sessions of 1h
Hours of autonomous work	1h
Competences to be developed	**Reasoning and modelling** • Develop thinking strategies to solve real life problems • Explore, analyse, and apply mathematical ideas • Estimate reasonably and demonstrate fluent, flexible, and strategic thinking about graphs • Model with mathematics in situational contexts • Think creatively and with curiosity and wonder when exploring problems **Understanding and solving** • Develop, demonstrate, and apply conceptual understanding of mathematical ideas through story, inquiry, and problem solving • Visualize to explore and illustrate mathematical concepts and relationships • Apply flexible and strategic approaches to solve problems • Solve problems with persistence and a positive disposition • Engage in problem-solving experiences connected with real-life examples.

	Communicating and representing • Explain and justify mathematical ideas and decisions in many ways • Represent mathematical ideas in concrete, pictorial, and symbolic forms • Use mathematical vocabulary and language to contribute to discussions in the classroom • Take risks when offering ideas in classroom discourse **Connecting and reflecting** • Reflect on mathematical thinking • Connect mathematical concepts with each other, other areas, and personal interests • Use mistakes as opportunities to advance learning • Incorporate First Peoples worldviews, perspectives, knowledge, and practices to make connections with mathematical concepts
ICT tools to be used	Available Computer Algebra Systems: Mathematica, Maple, Matlab, GeoGebra, etc.
Context: project statement	The climate changes are among other things caused also by the increasing deforestation and decreasing number of forests, trees and green vegetation on the globe. To benefit from the sustainable development and to support vegetation growth it is necessary to understand the growth of individual trees and be aware of the time necessary for the renewal of their missing numbers. To grow each particular tree in a new forest it is essential to know the time interval required for its cultivation.
Tasks and problems	A specific type of tree will grow under suitable conditions with the speed inversely proportional to its height. The tree can grew up to 1 meter during the first three years after planting. A new forest was planted with the particular tree seedlings that were all about 0,5 meter high. The tree growth can be described by a differential equation with separable variables, while general solution of this equation contains constants that can be expressed with respect to the described height decrease under good environment conditions. Tree can grow in good conditions about 7 years till it will reach the average height, while after this period it might beneficially vegetate for about 50 years and still grow, but quite slowly, with the half of the initial growth speed. **Task 1:** Assemble the differential equation describing the growth of trees in a new forest, assuming favorable conditions are secured for their growth. Answer: Let $y(t) > 0$ be the function representing the tree height depending on the time t of its growth, while derivative $y'(t)$ be the speed of the tree growth. This growth is described by the differential equation with separable variables $$y'(t) = \frac{k}{y(t)}, k > 0$$ that can be rewritten as the differential equation with separated variables and solved directly by integration.

miniPBL1 - Introduction

$$y(t).y'(t) - k = 0$$

$$\int y\,dy - \int k\,dt = c$$

$$\frac{y^2}{2} - kt = c$$

$$y^2 = 2kt + 2c$$

Task 2:
Find general solution of this differential equation and particular solution determined by Cauchy initial conditions.

Answer:
General solution of the equation is in the form

$$y(t) = \sqrt{2kt + 2c}$$

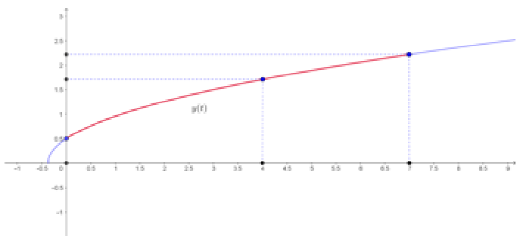
where constants k and c can be found according to the given initial conditions describing the tree growth:

$$y(0) = 0{,}5 \qquad y(3) = 1{,}5$$

$$y(t) = \sqrt{\frac{2}{3}t + 0{,}25}\ , \qquad t \in \langle 0, T\,\rangle, T \in R$$

Task 3:
Sketch respective integral curve of the particular solution representing the tree growth during T years, until it will reach its average height.

Answer:



Tree will grow until it reaches its average height, which is after $T = 7$ years from its planting. The red curve represents the tree growth.

Task 4:
Calculate the average high of trees in this forest after 4 years and after 7 years, when the trees reach their average height.

Answer:

Toolkit 3: One model for mini-PBL

$$y(4) = \sqrt{\frac{8}{3} + \frac{1}{4}} = \sqrt{\frac{32+3}{12}} = \sqrt{\frac{35}{12}} \doteq 1{,}71\ m$$

$$y(7) = \sqrt{\frac{14}{3} + \frac{1}{4}} = \sqrt{\frac{56+3}{12}} = \sqrt{\frac{59}{12}} \doteq 2{,}22\ m$$

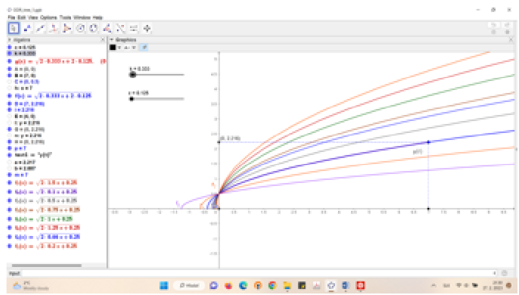
Task 5:
Estimate the height of the tree after 50 years.

Answer:

$$y(50) = \sqrt{\frac{100}{3} + \frac{1}{4}} = \sqrt{\frac{400+3}{12}} = \sqrt{\frac{403}{12}} \doteq 5{,}79\ m$$

Task 6:
Sketch several integral curves of the general solution and investigate their forms determined by different values of the included constants c and k representing the tree growth under different conditions.

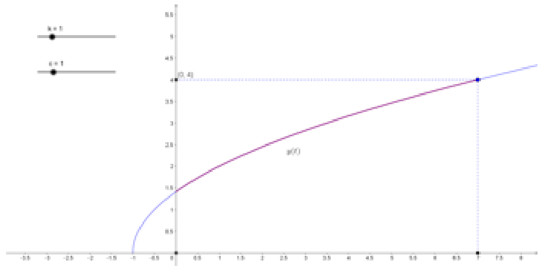
Answer:



Task 7:
Comment on the obtained results from a sustainable point of view. Investigate how the height of the tree seedlings influence the speed of the trees growth.

Answer:
Animation can be obtained easily in the program GeoGebra, with sliders determining the values of constants k and c.

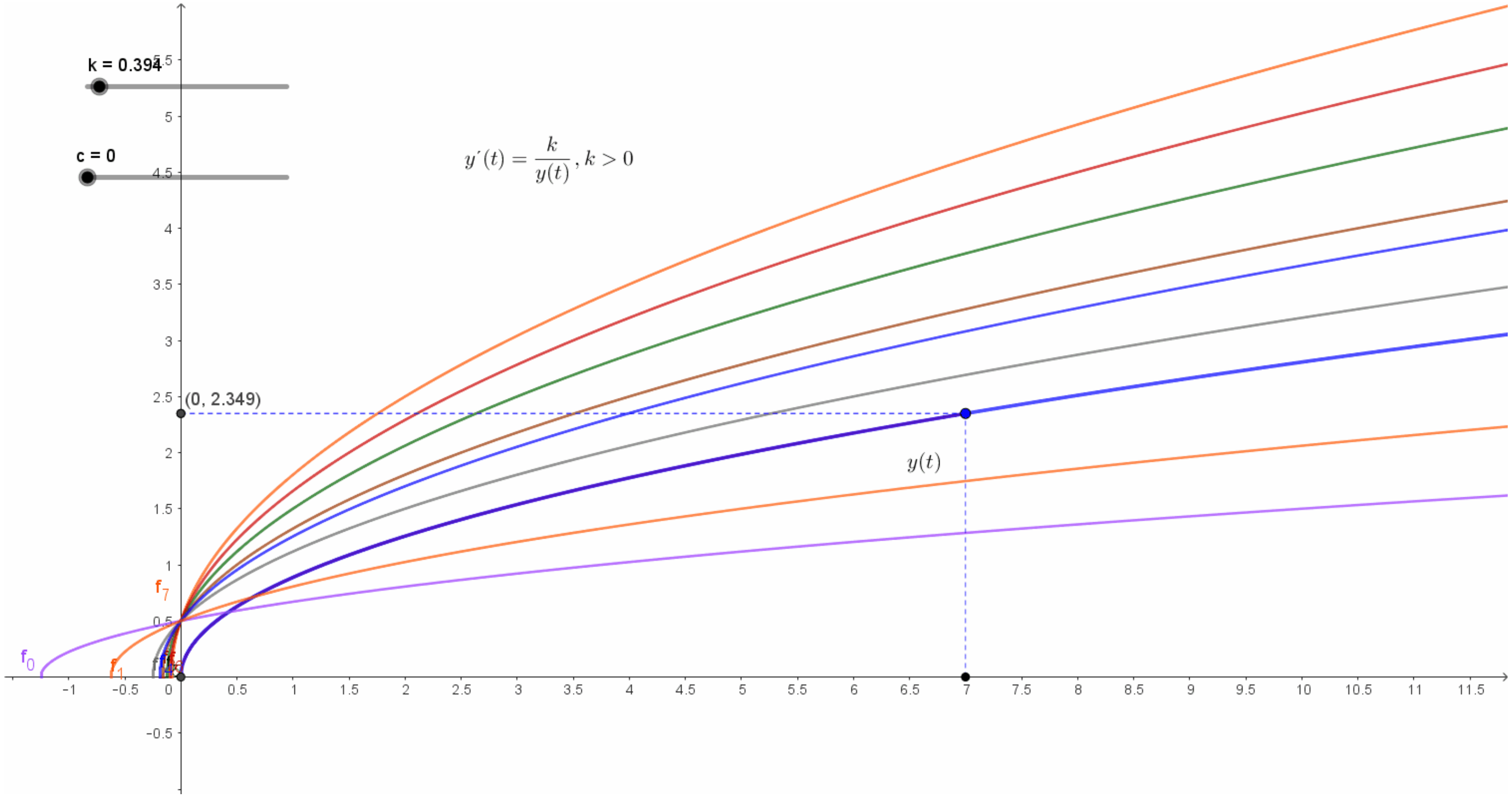
miniPBL1 - Introduction

	
Outcomes expected	- Graphics fitting the solution; - Numerical results explained and put in context; - Capture of ICT tools solutions used; - Sequence of steps followed; - Remark computations done by hand and done by ICT tools; - Provide complete answer to questions; - All the results must be presented in the context of the problem;
Guide for Learning	At the beginning of the course, the students need guides on new activities, and feel your support on a well-structured pack of suggestions on how to address the problems posted. Namely: - Read carefully the problem statement and the tasks posted. Always maintain a global view of all the projects. - Identify, or try to do a first draft match, the content units of your lecture notes involved in every task. - Take your lecture notes open and review before starting to solve the problems. - Match output expected with the tasks posted, at least as first draft approach. - Follow the order of the tasks, try to increase the knowledge of the problem while you are solving the activities. - Always think that maybe there are different ways to solve a problem. - Use ICT tools to avoid hard computations and check your solutions are correct in different ways if possible. - The solutions are always part of a context, expressing such a final solution totally integrated in the problem posted. - Be sure you answer the complete questions. - Always try to solve the questions by yourself. - If the project can be done in groups, discuss with the groups the proposed problem, to confirm and detect fails or weaknesses, confront strategies, discuss presentation format, etc. Working in groups doesn't mean work less but work better.
Guide for Teaching	Some hints needed to present and launch the mini-PBL to students - Do a small Introduction concerning Energy consumption, added to the Climate Change crisis we are currently living in. - Do a small introduction about the relations between power and energy, with the basic equations.

Toolkit 3: One model for mini-PBL

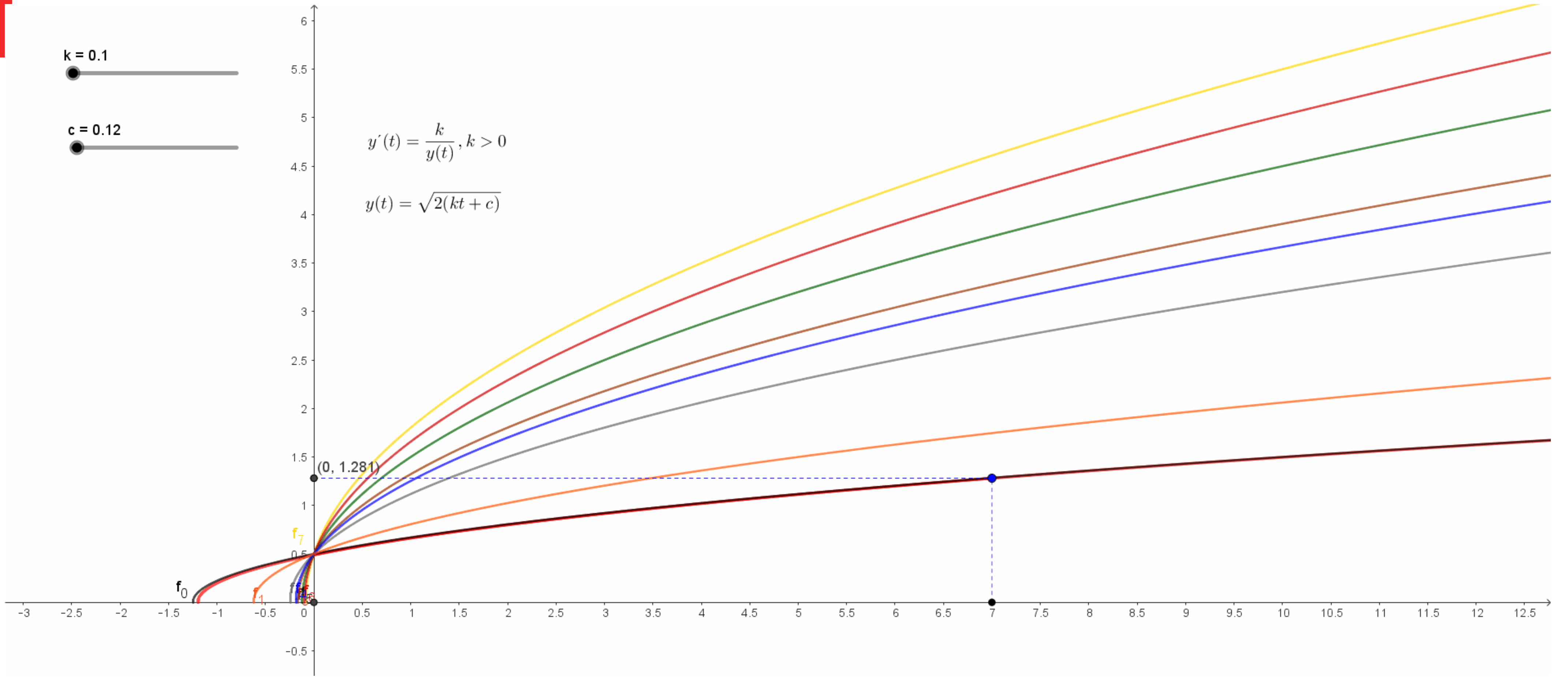
	- Students will form groups of 4 students and solve the mini-PBL using the eduScrum methodology. - The students should do each exercise in a sequential order, starting from Task 1. - The students should be able to thoroughly read and interpret the numerical results from a mathematical and the real-life example point of view. They should include also a discussion of the climate change crisis and enumerate some strategies they could apply at home or even at university to save resources, namely reduce energy consumption. They should also mention how this mini-PBL helps them identify the Sustainable Development Goals 4 And 7.
Assessment	- Final report; - Oral presentation; - Peer-assessment: students will apply peer-assessment for their periodic performance using online peer assessment tools used and available at the respective institution.
Others: References	Active Learning Calculus I (colorado.edu) https://eduscrum.org/about-us-and-how-we-try-to-make-it-happen/ More refs on active-learning tools: https://scholar.google.com/citations?hl=en&user=Aw39XwEAAAAJ&view_op=list_works&sortby=pubdate https://www.youtube.com/watch?v=mQ_mbDAB1us (there are more examples online)

miniPBL1 - Introduction



Illusrations of several integral curves as particular solutions of the differential equation for various values of constants *c* and *k*

miniPBL1 - Introduction



Different forms of integral curves as particular solutions of the differential equation for various values of constant *c* representing environmental conditions.

miniPBL2 - The waste reduction

The production of waste is increasing with the speed directly proportional to its quantity, due to increasing industrial production and poor environmental measures.

To decrease the waste accumulation it is necessary to adopt various ecological measures, as recycling of the produced waste, decrease in the production, etc. leading to the decreased speed of its accumulation and decrease of its growth acceleration.

These adopted environmental measures cannot lead to the complete diminishing of the accumulated waste on the planet, but they can considerably improve the planet pollution and have beneficial effect on the climate changes.

miniPBL2 - The waste reduction

The accumulation of waste is increasing with the speed equal to twice its actual quantity. $y'(t) = 2y(t)$

After adopting strong ecological measures, the speed of the accumulation was decreased to be equal to the actual waste quantity, while acceleration of this growth was decreased to one quarter of the former speed. $y''(t) = \frac{1}{4}y(t)$

The process of waste accumulation can be represented by ordinary differential equation, of order I before measures and order II after the regulations.

After 2 years there can be visible remarkable improvements and considerable decrease in the accumulated waste.

miniPBL2 - Introduction

Mathematics 2 – two lectures on ODR II

- L1 – introductory information and presentation of problem
- L1 – explanation of the mathematical model and its solution
- L1 – students asked to find answers to presented Tasks 2 - 6
- L2 – discussion about general solution and possible meaning of constants present in there
- L2 – illustrations in programme GeoGebra
- L2 – discussion about ecological restrictions, which could be mathematically described by constants in the general solution, and finding solution of Task 7 together

About 20% of students contributed to the discussions!

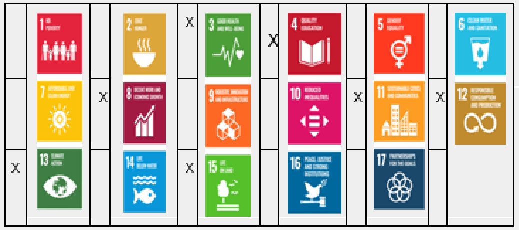
miniPBL2 - Introduction

Pythagoras 2021-1-RO01-KA220-HED-000032258

Funded by the European Union

Mini-PBL example

Teaching Guide for Teachers

Mini-PBL project	
Teacher data sheet: Teaching Guide	
Title	The waste reduction
SDG attended	Using this UN graphics, we mark such SDG which this project works. 
Content units	Ordinary differential equations of order II with constant coefficients
Sessions	1 sessions of 1h
Hours of autonomous work	1h
Competences to be developed	**Reasoning and modelling** • Develop thinking strategies to solve real life problems • Explore, analyse, and apply mathematical ideas • Estimate reasonably and demonstrate fluent, flexible, and strategic thinking about graphs • Model with mathematics in situational contexts • Think creatively and with curiosity and wonder when exploring problems **Understanding and solving** • Develop, demonstrate, and apply conceptual understanding of mathematical ideas through story, inquiry, and problem solving • Visualize to explore and illustrate mathematical concepts and relationships • Apply flexible and strategic approaches to solve problems • Solve problems with persistence and a positive disposition • Engage in problem-solving experiences connected with real-life

	examples. **Communicating and representing** • Explain and justify mathematical ideas and decisions in many ways • Represent mathematical ideas in concrete, pictorial, and symbolic forms • Use mathematical vocabulary and language to contribute to discussions in the classroom • Take risks when offering ideas in classroom discourse **Connecting and reflecting** • Reflect on mathematical thinking • Connect mathematical concepts with each other, other areas, and personal interests • Use mistakes as opportunities to advance learning • Incorporate First Peoples worldviews, perspectives, knowledge, and practices to make connections with mathematical concepts
ICT tools to be used	Available Computer Algebra Systems: Mathematica, Maple, Matlab, GeoGebra, etc.
Context: project statement	The production of waste is increasing with the speed directly proportional to its quantity, due to increasing industrial production and poor environmental measures. To decrease the waste accumulation it is necessary to adopt various ecological measures, as recycling of the produced waste, decrease in the production leading to the decreased speed of its accumulation and decrease of its growth acceleration. These adopted environmental measures cannot lead to the complete diminishing of the accumulated waste on the planet, but they can considerably improve the planet pollution and have beneficial effect on the climate changes.
Tasks and problems	The accumulation of waste is increasing with the speed equal to twice its actual quantity. After adopting strong ecological measures, the speed of the accumulation was decreased to be equal to the actual waste quantity, while acceleration of this growth was decreased to one quarter of the former speed. Consider the amount of the waste to be 1 unit at the time, when the measures were imposed, $t = 0$. The process of waste accumulation can be represented by ordinary differential equation, of order 1 before measures, and order II after the regulations. Already after 2 years there can be visible remarkable improvements and considerable decrease in the accumulated waste. **Task 1:** Assemble both differential equations describing the accumulation of waste before measures and after their imposition. Answer: Let $y(t) > 0$ be the amount of accumulated waste in the time t, while derivative $y'(t)$ be the speed of the waste growth before impact of the ecological measures. This growth is described by the differential equation od order I $$y'(t) = 2y(t)$$ that can be rewritten as the homogeneous differential equation of order I and solved directly.

miniPBL2 - Introduction

$$y'(t) - 2y(t) = 0$$

Acceleration of the waste accumulation is then expressed as

$$y''(t) = 2y'(t) = 4y(t)$$

therefore the solution can be obtained from the differential equation of order II with constant coefficients

$$y''(t) - 4y(t) = 0$$

leading to the same general solution as differential equation of order I.

After impact of the ecological measures, derivative $y'(t)$ would be represented as

$$y'(t) = y(t)$$

while the acceleration of this speed will be reduced to

$$y''(t) = \frac{1}{4}y'(t) = \frac{1}{4}y(t)$$

Such waste accumulation process can be described by the differential equation od order II with constant coefficients

$$y''(t) - \frac{1}{4}y(t) = 0$$

Task 2:
Find general solutions of both differential equations and their particular solutions determined by Cauchy initial conditions.

Answer:
General solution of the ODR I is in the form

$$Y(t) = ce^{2t}, c \in R, t \in \langle 0, T\rangle$$

where value of constant c in the particular solution can be determined from the given initial condition describing the waste accumulation as Cauchy initial problem:

$$Y(0) = 1 \Rightarrow 1 = ce^0 \Rightarrow c = 1$$

$$Y_P(t) = e^{2t}, \quad t \in \langle 0, T\rangle, T \in R$$

General solution of the respective ODR II can be determined from the characteristic equation

$$r^2 - 4 = 0 \Rightarrow r_1 = -2, r_2 = 2$$

in the form

$$y(t) = c_1e^{-2t} + c_2e^{+2t}, \quad c_1, c_2 \in R, t \in \langle 0, T\rangle$$

Values of constants c_1, c_2 for the particular solution can be calculated from the given initial conditions describing the waste accumulation as Cauchy initial problem:

$$y(0) = 1, \ y'(0) = 2$$

$$Y_P(t) = 0.e^{-2t} + 1.e^{+2t} = e^{2t}, \quad t \in \langle 0, T\rangle, T \in R$$

Toolkit 3: One model for mini-PBL

General solution of the ODR II describing the accumulation process after some ecological restrictions are implied can be determined form the characteristic equation

$$r^2 - \frac{1}{4} = 0 \Rightarrow r_1 = -\frac{1}{2}\ r_2 = \frac{1}{2}$$

in the form

$$y(t) = c_1e^{-\frac{1}{2}t} + c_2e^{+\frac{1}{2}t}, \quad c_1, c_2 \in R, t \in \langle 0, T\rangle, T \in R$$

Values of constants c_1, c_2 for the particular solution can be calculated from the given initial conditions describing the waste accumulation as Cauchy initial problem:

$$y(0) = 1, \ y'(0) = 1$$

$$y_P(t) = -\frac{1}{2}e^{-\frac{1}{2}t} + \frac{3}{2}e^{+\frac{1}{2}t}, \quad t \in \langle 0, T\rangle, T \in R$$

Task 3:
Calculate the amount of accumulated waste after 2 years under both circumstances, compare these values, and estimate the impact of the ecological measures.
Answer:

ODR I

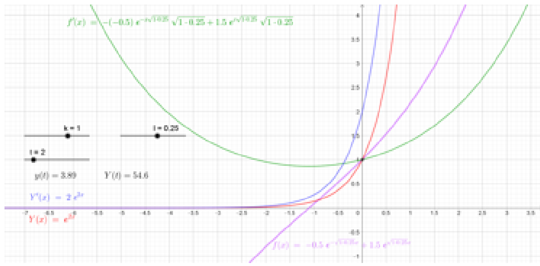
$$Y_p(2) = e^4 \doteq 54{,}6\ units$$

ODR II

$$y_P(2) = -\frac{1}{2}e^{-1} + \frac{3}{2}e^{1} \doteq -\frac{1}{6} + \frac{9}{2} \doteq \frac{-1+27}{6} \doteq \frac{13}{3} \doteq 3{,}9\ units$$

Achieved reduction of accumulated waste can be up to $50{,}7\ units$.

Task 4:
Sketch respective integral curves that are particular solutions of ODRs representing the waste accumulation under both circumstances and compare the waste growth during *T* years.
Answer:



miniPBL2 - Introduction

Both functions are increasing, as their derivatives are positive functions for $t \geq 0$.

$$Y_P{}' = 2e^{2t}, \qquad t \in \langle 0, T \rangle, T \in R$$

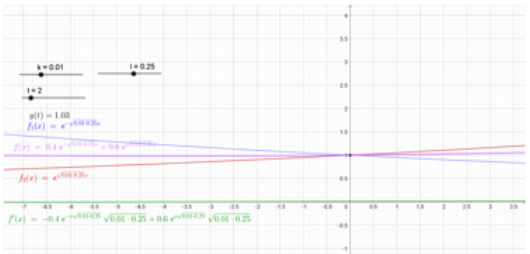
$$y_P{}' = \frac{1}{4}e^{-\frac{1}{2}t} + \frac{3}{4}e^{+\frac{1}{2}t}, \; t \in \langle 0, T \rangle, T \in R$$

Task 4:
Calculate the amount of waste after 10 years of functioning measures saving the planet ecology.
Answer:

$$y_P(10) = -\frac{1}{2}e^{-5} + \frac{3}{2}e^{5} \doteq 222{,}62 \; units$$

Task 5:
Estimate, what would be the necessary measures in order not to increase the amount of accumulated waste on the planet.
Answer:
Function $y_P(t)$ is increasing for $t \in \langle 0, T \rangle, T \in R$, as its derivative is positive function for $t \geq 0$.

$$y_P{}' = \frac{1}{4}e^{-\frac{1}{2}t} + \frac{3}{4}e^{+\frac{1}{2}t}, \; t \in \langle 0, T \rangle, T \in R$$



None of the chosen values of constants $k, l \in R$ representing adopted

Toolkit 3: One model for mini-PBL

measures that determine the growth speed and acceleration

$$y'(t) = k.y(t), y''(t) = l.y(t)$$

$$y''(t) = k.y'(t) = k.l.y(t)$$

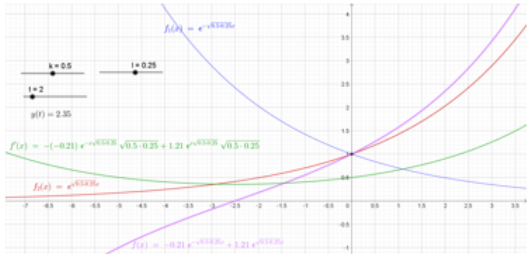
leading to the differential equation

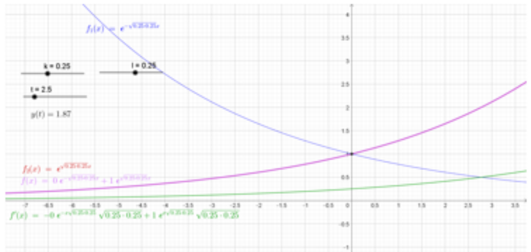
$$y'' - k - l - y = 0$$

could result in the waste accumulation decrease.

The amount of waste could "stop to be increasing" only in case that the industrial production will be decreased, while all accumulated waste will be rapidly and immediately recycled.

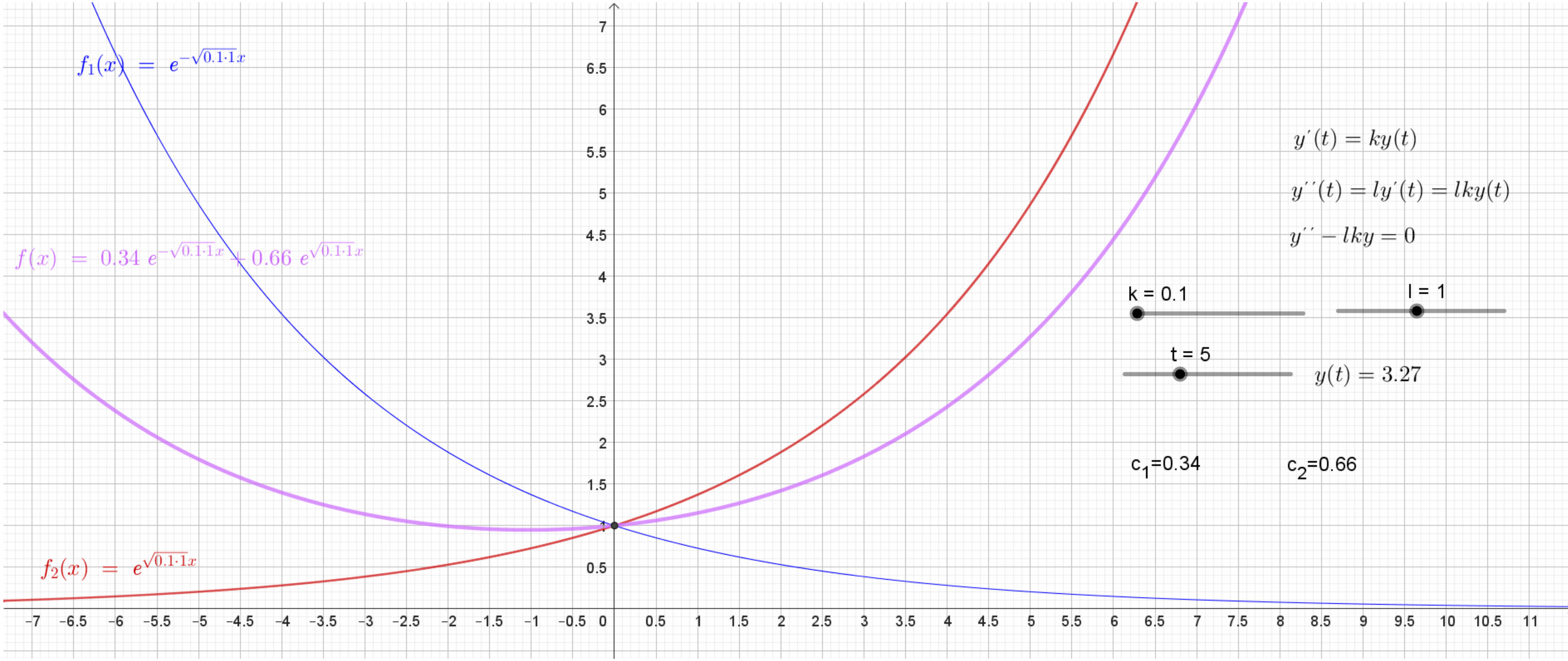
Task 6:
Sketch several integral curves of the general solution of ODR II and investigate their forms determined by different values of the included constants c_1, c_2 representing the ecological restrictions.
Answer:





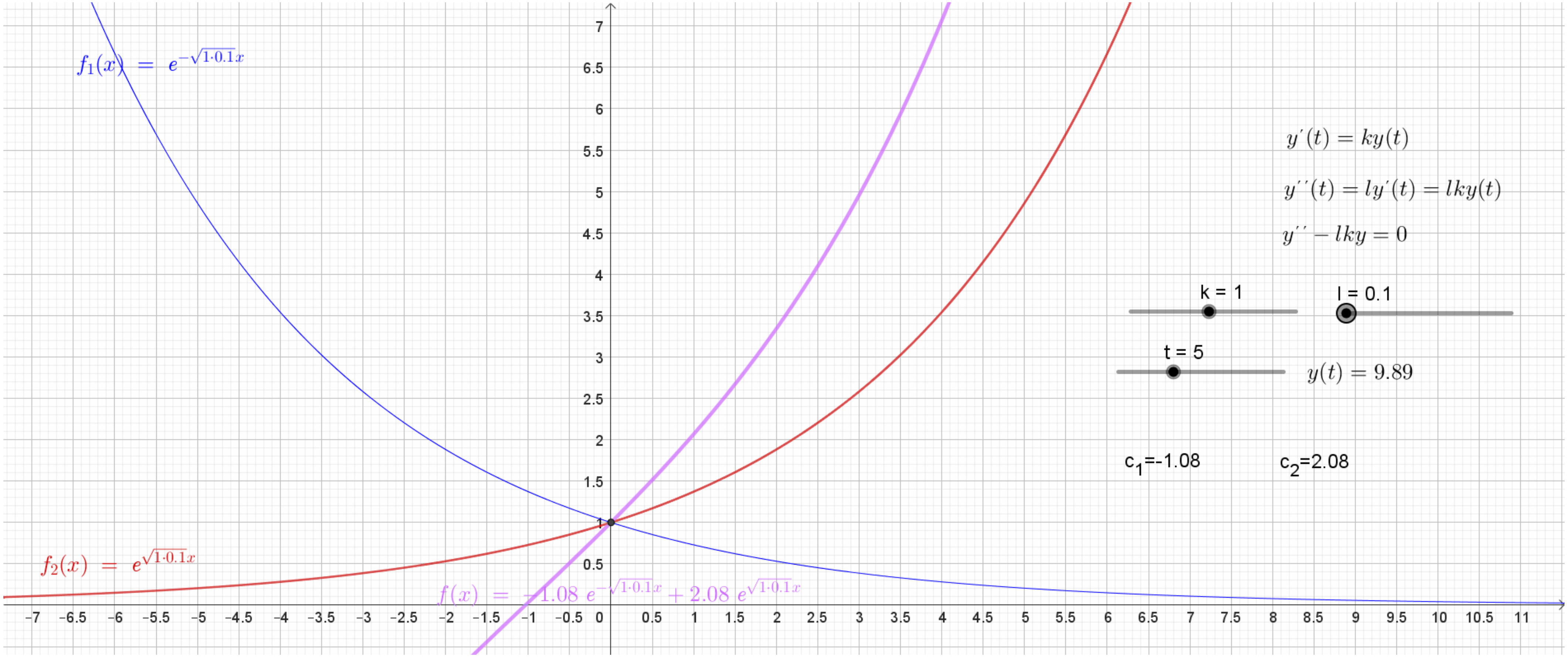
Task 7:
Comment on the obtained results from a sustainable point of view. Investigate how the values of the constants c_1, c_2 influence the speed of the waste decrease.

miniPBL2 - Introduction



Animated illustration in GeoGebra

miniPBL2 - Introduction



Animated illustration in GeoGebra

miniPBL3 - Water pollution

Overall pollution of planet water resources is dangerously increasing every year with the increasing speed and acceleration of this process.

The percentage of the polluted world ocean area is rapidly increasing every year.

Scientists are urgently pointing to this problem, which can lead to the global lack of clean drinking water resources and endanger the life on the planet.

The Buriganga River in Bangladesh, which flows past the southwest outskirts of the capital city Dhaka is one among the most warning examples.

miniPBL3 - Water pollution

Water pollution rate can be mathematically modelled as differential equation of order II with constant coefficients.

Striking evidence of the rapidly worsening situation can be presented by this mathematical model predicting the unavoidable consequences provided no actions will be taken to completely stop or at least to slow down this undesirable development.

Adopting strong ecological measures resulting in reduction of the water pollution process by half we could achieve quite visible and remarkable improvements just in few coming years.

$$y''(t) = \frac{1}{4} y'(t)$$

miniPBL3 - Introduction

Mathematics 2 – final lecture on ODR II

- introductory information and presentation of problem
- motivation to solve problem as individual project for extra BONUS POINTS (included into the final results)

Problem was then presented at the Blackboard in front of department, where we usually place some extra work or interesting problems to solve for more engaged students.

The student miniPBL working sheet with all instructions was uploaded to AIS document server, where all study materials for students are available.

About 5% of all students submitted their solutions, half of which were correct and complete!

miniPBL3 - Introduction

Pythagoras
2021-1-RO01-KA220-HED-000032258

Funded by
the European Union

Mini-PBL example

Teaching Guide for Teachers

Mini-PBL project	
Teacher data sheet: Teaching Guide	
Title	Water pollution
SDG attended	Using this UN graphics, we mark such SDG which this project works. 
Content units	Ordinary differential equations of order II with constant coefficients
Sessions	1 sessions of 1h
Hours of autonomous work	1h
Competences to be developed	**Reasoning and modelling** • Develop thinking strategies to solve real life problems • Explore, analyse, and apply mathematical ideas • Estimate reasonably and demonstrate fluent, flexible, and strategic thinking about graphs • Model with mathematics in situational contexts • Think creatively and with curiosity and wonder when exploring problems **Understanding and solving** • Develop, demonstrate, and apply conceptual understanding of mathematical ideas through story, inquiry, and problem solving • Visualize to explore and illustrate mathematical concepts and relationships • Apply flexible and strategic approaches to solve problems • Solve problems with persistence and a positive disposition • Engage in problem-solving experiences connected with real-life examples. **Communicating and representing** • Explain and justify mathematical ideas and decisions in many ways • Represent mathematical ideas in concrete, pictorial, and symbolic forms • Use mathematical vocabulary and language to contribute to discussions in the classroom • Take risks when offering ideas in classroom discourse **Connecting and reflecting** • Reflect on mathematical thinking • Connect mathematical concepts with each other, other areas, and personal interests • Use mistakes as opportunities to advance learning • Incorporate First Peoples worldviews, perspectives, knowledge, and practices to make connections with mathematical concepts
ICT tools to be used	Available Computer Algebra Systems: Mathematica, Maple, Matlab, GeoGebra, etc.
Context: project statement	The Buriganga River is a river in Bangladesh, which flows past the southwest outskirts of the capital city Dhaka. Its average depth is 7.6 metres and its length is only 18 kilometres, but it is economically very important to Dhaka. Launches and country boats provide connection to other parts of Bangladesh, a largely riverine country. The river is also the source of drinking water. Buriganga is afflicted by the noisome problem of pollution, and it ranks among the most polluted rivers in the world. More than 60,000 cubic metres of toxic chemical waste from mills and textile factories, household waste, medical waste, sewage, dead animals, plastics, and oil are some of the river most dangerous pollutants.  Source: SITA/AP, [Buriganga River - Wikipedia](https://en.wikipedia.org/wiki/Buriganga_River) https://en.wikipedia.org › wiki › Buriganga_River

miniPBL3 - Introduction

Overall pollution of planet water resources is dangerously increasing every year with the increasing speed and acceleration of this process, which can be mathematically modelled as differential equation of order II with constant coefficients. Scientists are urgently pointing to this problem, which can lead to the global lack of clean drinking water resources and endanger the life on the planet. Striking evidence of the rapidly worsening situation can be provided by the mathematical models predicting the unavoidable consequences provided no actions will be taken to completely stop or at least to slow this development.

Tasks and problems

The percentage of the polluted world ocean area is rapidly increasing every year. The latest findings of Canadian oceanographic scientists proved that in the ocean water one can find a higher portion of micro-plastics (six times more!) than is the amount of live plankton. Consider the percentage of the planet polluted water resources to be 10% at the time $t = 0$. What would be the situation in the following years, under the current circumstances, when the process of water pollution is accelerated by one fourth of the pollution speed? Adopting strong ecological measures resulting in reduction of the water pollution process by half, quite visible and remarkable improvements can be achieved in few coming years.

Task 1:
Assemble differential equation describing the process of increasing percentage of the planet polluted water resources before adaptation of ecological measures and after their imposition.

Answer:
Let $y(t) > 0$ be the percentage of polluted planet water resources at the time t, while acceleration of the pollution speed be the second derivative $y''(t)$ of this percentage (before impact of the ecological measures) and this equals to one fourth of the pollution speed. Such pollution spreading process can be well represented mathematically by the differential equation od order II

$$y''(t) = \frac{1}{4} y'(t)$$

that can be rewritten as the homogeneous differential equation of order II with constant coefficients

$$y''(t) - \frac{1}{4} y'(t) = 0$$

After impact of the ecological measures, the acceleration of the pollution process will be reduced to one half, $\frac{y''(t)}{2}$, which would be represented by differential equation

$$y''(t) - \frac{1}{8} y'(t) = 0$$

Task 2:
Find general solutions of both differential equations and their particular solutions determined by Cauchy initial conditions.

Answer:

Toolkit 3: One model for mini-PBL

General solution of the first respective ODR II can be determined from the characteristic equation

$$r^2 - \frac{1}{4} r = 0 \Rightarrow r_1 = 0, r_2 = \frac{1}{4},$$

$$y(t) = c_1 + c_2 e^{\frac{1}{4}t}, \quad c_1, c_2 \in R, t \in \langle 0, T \rangle, T \in R$$

General solution of the ODR II describing the pollution process after some ecological restrictions were implied can be determined from the characteristic equation

$$r^2 - \frac{1}{8} r = 0 \Rightarrow r_1 = 0, r_2 = \frac{1}{8}$$

in the form

$$y(t) = c_1 + c_2 e^{\frac{1}{8}t}, \quad c_1, c_2 \in R, t \in \langle 0, T \rangle, T \in R$$

Values of constants c_1, c_2 for the particular solutions can be calculated from the given initial conditions describing the water pollution as Cauchy initial problem:

$$y(0) = 10, \; y'(0) = 1$$

$$c_1 = 6, c_2 = 4$$

$$y_P(t) = 6 + 4e^{\frac{1}{4}t}, \quad t \in \langle 0, T \rangle, T \in R$$

or in the other circumstances

$$y(0) = 10, \; y'(0) = 1$$

$$c_1 = 2, c_2 = 8$$

$$y_P(t) = 2 + 8e^{\frac{1}{8}t}, \quad t \in \langle 0, T \rangle, T \in R$$

Task 3:
Calculate the amount of polluted planet waters percentage in several next years under both circumstances, compare these values in a table, and estimate the impact of the ecological measures.

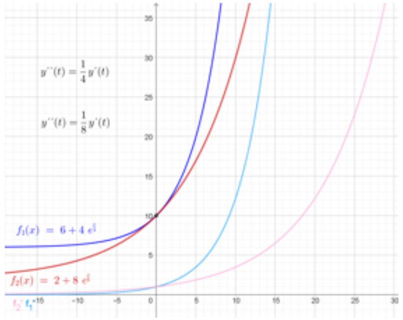
Answer:

t	0	2	4	6	8	10	12	**12,5**	14	16	18	**20**
1.	10	12,6	16,9	24	35,5	54,8	88,3	**97**				
2.	10	12,3	15,2	19	23,7	30	37,9	42,6	48	61	78	**99,5**

Achieved impact of the imposed ecological measures is only the reduction of water pollution accumulation in time, while all water resources on the planet will be polluted about 8 years later than in case that no measures will be implied.

Task 4:
Sketch respective integral curves that are particular solutions of ODRs representing the water pollution under both circumstances and compare the pollution growth acceleration during *T* years.
Answer:

miniPBL3 - Introduction



Under both circumstancies the higher water pollution is inevoidable, as derivatives of both particular solutions are positive functions.

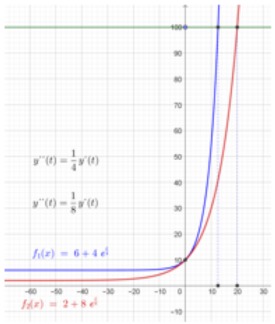
$$y_P'(t) = e^{\frac{1}{4}t}, \quad t \in \langle 0, T \rangle, T \in R$$

$$y_P'(20) = e^{\frac{1}{8}t}, \quad t \in \langle 0, T \rangle, T \in R$$

Task 4:
Calculate, how much time is left before 100% of planet water resources are completely polluted in both scenarios, without or with the adoption of strict functioning measures saving the planet clean drinking water.
Answer:

$$y_P'(12{,}5) = 97\%, \quad t = 12{,}5$$

$$y_P'(20) = 99\%, \quad t = 20$$



There is left less than 13 years before the total plane water resources are polluted in the first scenario, and about 20 years in the other one.

Toolkit 3: One model for mini-PBL

Task 5:
Find, what would be the necessary measures in decrease of pollution acceleration in order to stop the water pollution in 10 years at 20 %.
Answer:

$$y''(t) = \frac{1}{k} y'(t)$$

$$y''(t) - \frac{1}{k} y'(t) = 0$$

$$r^2 - \frac{1}{k} r = 0 \Rightarrow r_1 = 0, r_2 = \frac{1}{k}$$

$$y_P = c_1 + c_2 e^{\frac{1}{k}t}, \quad t \in \langle 0, T \rangle, T \in R$$

$$y_P' = \frac{1}{k} c_2 e^{\frac{1}{k}t}, \quad t \in \langle 0, T \rangle, T \in R$$

Initial conditions

$$y(0) = 10, \quad y'(0) = 1$$

determine the values of constants c_1, c_2

$$c_1 + c_2 = 10, \frac{1}{k} c_2 = 1$$

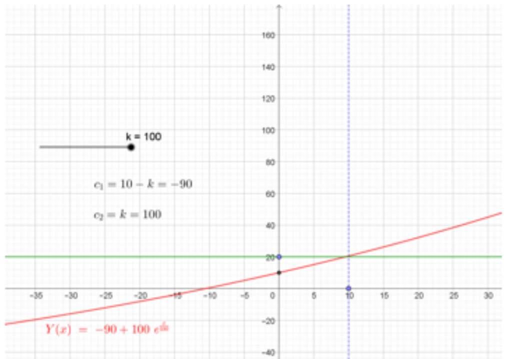
$$c_2 = k$$

$$c_1 = 10 - k$$

$$y_P = 10 - k + k e^{\frac{1}{k}t}, \quad t \in \langle 0, T \rangle, T \in R$$

$$y(10) = 20 \Rightarrow 10 - k + k e^{\frac{10}{k}} = 20 \Rightarrow k \doteq 100$$

$$y_P = -90 + 100 e^{\frac{1}{100}t}, \quad t \in \langle 0, T \rangle, T \in R$$



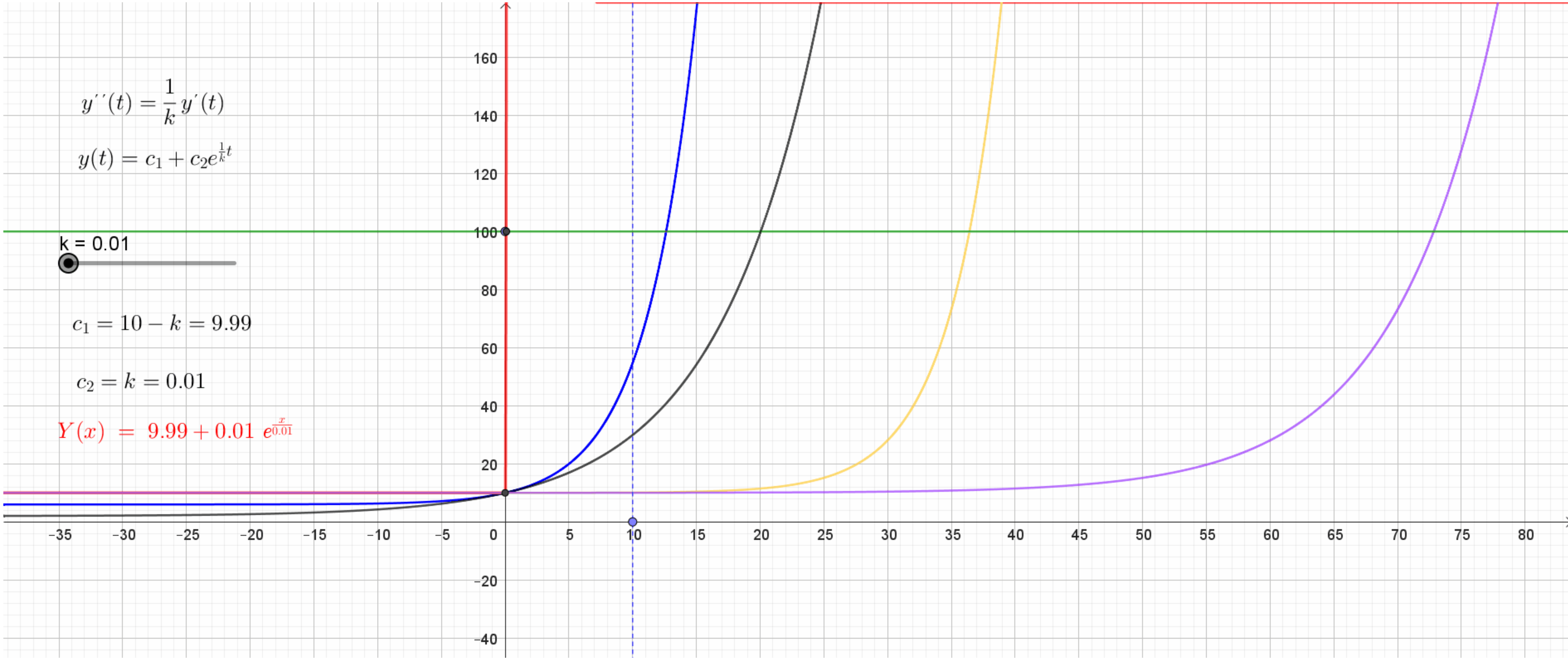
Process of water pollution must be not accelerated, or at least its acceleration must be decreased to one hundred of the pollution speed!

miniPBL3 - Introduction

	Task 6: Sketch several integral curves of the general solution of ODR II and investigate their forms determined by different values of the included constants c_1, c_2 representing the ecological restrictions. Answer: Animation can be obtained easily in the program GeoGebra, with sliders determining the values of constants k and corresponding c_1, c_2. **Task 7:** Comment on the obtained results from a sustainable point of view. Investigate how the values of the constants c_1, c_2 influence the speed and acceleration of the water pollution. Answer: The amount of polluted water resources could "stop to be increasing" only in case that no toxic materials and waste will be produced, all wastewater would undergo required treatment, the dirty industrial production would be decreased, while all currently polluted water resources would be rapidly and immediately cleaned.
Outcomes expected	- Graphics fitting the solution; - Numerical results explained and put in context; - Capture of ICT tools solutions used; - Sequence of steps followed; - Remark computations done by hand and done by ICT tools; - Provide complete answer to questions; - All the results must be presented in the context of the problem;
Guide for Learning	At the beginning of the course, the students need guides on new activities, and feel your support on a well-structured pack of suggestions on how to address the problems posted. Namely: - Read carefully the problem statement and the tasks posted. Always maintain a global view of all the projects. - Identify, or try to do a first draft match, the content units of your lecture notes involved in every task. - Take your lecture notes open and review before starting to solve the problems. - Match output expected with the tasks posted, at least as first draft approach. - Follow the order of the tasks, try to increase the knowledge of the problem while you are solving the activities. - Always think that maybe there are different ways to solve a problem. - Use ICT tools to avoid hard computations and check your solutions are correct in different ways if possible. - The solutions are always part of a context, expressing such a final solution totally integrated in the problem posted. - Be sure you answer the complete questions. - Always try to solve the questions by yourself. - If the project can be done in groups, discuss with the groups the proposed problem, to confirm and detect fails or weaknesses, confront strategies, discuss presentation format, etc. Working in groups doesn't mean work less but work better.

Guide for Teaching	Some hints needed to present and launch the mini-PBL to students - Do a small Introduction concerning Energy consumption, added to the Climate Change crisis we are currently living in. - Do a small introduction about the relations between power and energy, with the basic equations. - Students will form groups of 4 students and solve the mini-PBL using the eduScrum methodology. - The students should do each exercise in a sequential order, starting from Task 1. - The students should be able to thoroughly read and interpret the numerical results from a mathematical and the real-life example point of view. They should include also a discussion of the climate change crisis and enumerate some strategies they could apply at home or even at university to save resources, namely reduce energy consumption. They should also mention how this mini-PBL helps them identify the Sustainable Development Goals 4 And 7.
Assessment	- Final report; - Oral presentation; - Peer-assessment: students will apply peer-assessment for their periodic performance using online peer assessment tools used and available at the respective institution.
Others: References	Active Learning Calculus I (colorado.edu) https://eduscrum.org/about-us-and-how-we-try-to-make-it-happen/ More refs on active-learning tools: https://scholar.google.com/citations?hl=en&user=Aw39XwEAAAAJ&view_op=list_works&sortby=pubdate https://www.youtube.com/watch?v=mQ_mbDAB1us (there are more examples online)

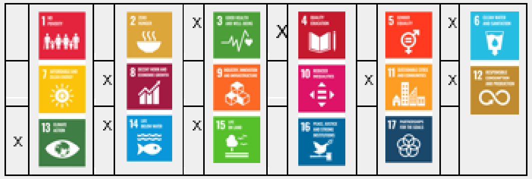
miniPBL3 - Introduction



Animated illustration of the environmental measures´ impact on the reduction of water pollution.

miniPBL3 - Introduction

Learning Guide for Students

Mini-PBL project	
Student data sheet: Learning Guide	
Title	The water pollution
SDG attended	Using this UN graphic, we mark such SDG which this project works. 
Content units	Ordinary differential equations of order II with constant coefficients
Sessions	1 sessions of 1h
Hours of autonomous work	2 hrs
ICT tools to be used	Available Computer Algebra Systems: Mathematica, Maple, Matlab, GeoGebra, etc.
Context: project statement	The Buriganga River is a river in Bangladesh, which flows past the southwest outskirts of the capital city Dhaka. Its average depth is 7.6 metres and its length is only 18 kilometres, but it is economically very important to Dhaka. Launches and country boats provide connection to other parts of Bangladesh, a largely riverine country. The river is also the source of drinking water. Buriganga is afflicted by the noisome problem of pollution, and it ranks among the most polluted rivers in the world. More than 60,000 cubic metres of toxic chemical waste from mills and textile factories, household waste, medical waste, sewage, dead animals, plastics, and oil are some of the river most dangerous pollutants.  Source: SITA/AP, Buriganga River - Wikipedia https://en.wikipedia.org › wiki › Buriganga_River

Tasks and problems	The percentage of the polluted world ocean area is rapidly increasing every year. The latest findings of Canadian oceanographic scientists proved that in the ocean water one can find a higher portion of micro-plastics (six times more!) than is the amount of live plankton. Consider the percentage of the planet polluted water resources to be 10% at the time t = 0. What would be the situation in the following years, under the current circumstances, when the process of water pollution is accelerated by one fourth of the pollution speed? Adopting strong ecological measures resulting in reduction of the water pollution process by half, quite visible and remarkable improvements can be achieved in few coming years. **Task 1:** Assemble differential equation describing the process of increasing percentage of the planet polluted water resources before adaptation of ecological measures and after their imposition. Answer: **Task 2:** Find general solutions of both differential equations and their particular solutions determined by Cauchy initial conditions. Answer: **Task 3:** Calculate the amount of polluted planet waters percentage in several next years under both circumstances, compare these values in a table, and estimate the impact of the ecological measures. Answer: **Task 4:** Sketch respective integral curves that are particular solutions of ODRs representing the water pollution under both circumstances and compare the pollution growth acceleration during T years. Answer: **Task 5**: Find, what would be the necessary measures in decrease of pollution acceleration in order to stop the water pollution in 10 years at 20 %. Answer: **Task 6:** Sketch several integral curves of the general solution of ODR II and investigate their forms determined by different values of the included constants c_1, c_2 representing the ecological restrictions. Answer: **Task 7:** Comment on the obtained results from a sustainable point of view. Investigate how the values of the constants c_1, c_2 influence the speed and acceleration of the water pollution. Answer:

miniPBL - Introduction

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FACULTY OF MECHANICAL ENGINEERING

In all 3 scenarios, these tasks were quite difficult to our students, as it was

- something completely different
- triple burden for students:

  1. to construct the proper mathematical model
  2. to solve the differential equation
  3. to interpret the results back to the practical problem solution.

miniPBL - Introduction

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Alternative way how to introduce miniPBL scenario into the continuous study at the faculty within mathematical subjects:

1. To modify the same problem used in various subjects
2. To adapt the applied problem with respect to actual knowledge and available mathematical/professional skills of students
3. To increase the tasks difficulty accordingly and continuously
4. To arrive gradually to complex solutions using mathematical models and suitable methods
5. **To awaken students´ awareness of the power of mathematics!**