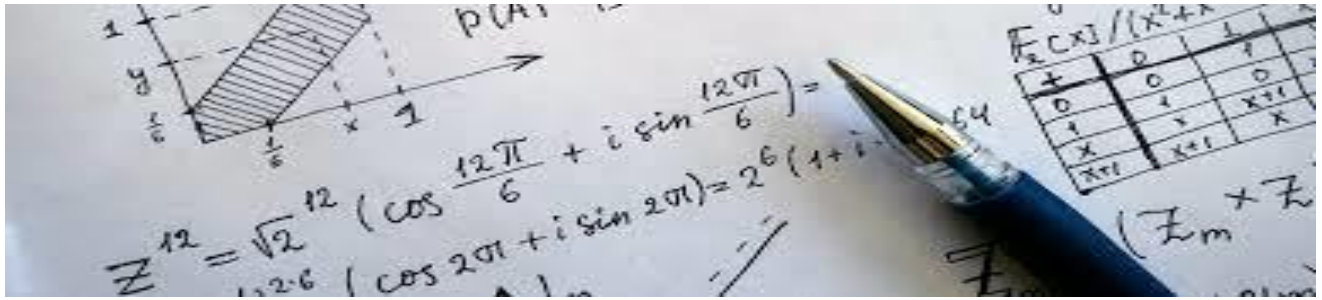

WP3 - Designing digital learning environment by merging a dynamic mathematics system and a computer-aided assessment system

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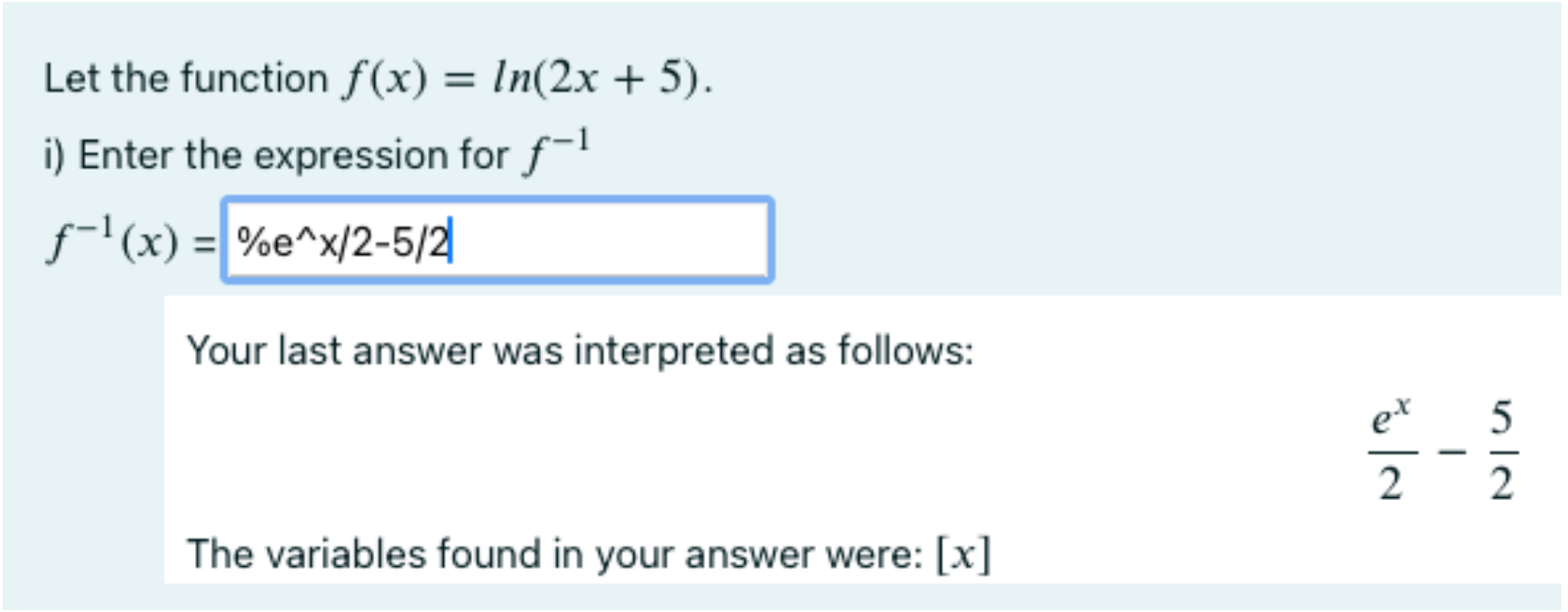
MOODLE STACK:
transforming the assessment of mathematics and sciences into an interactive and personalized experience

STACK online assessment for mathematics and science

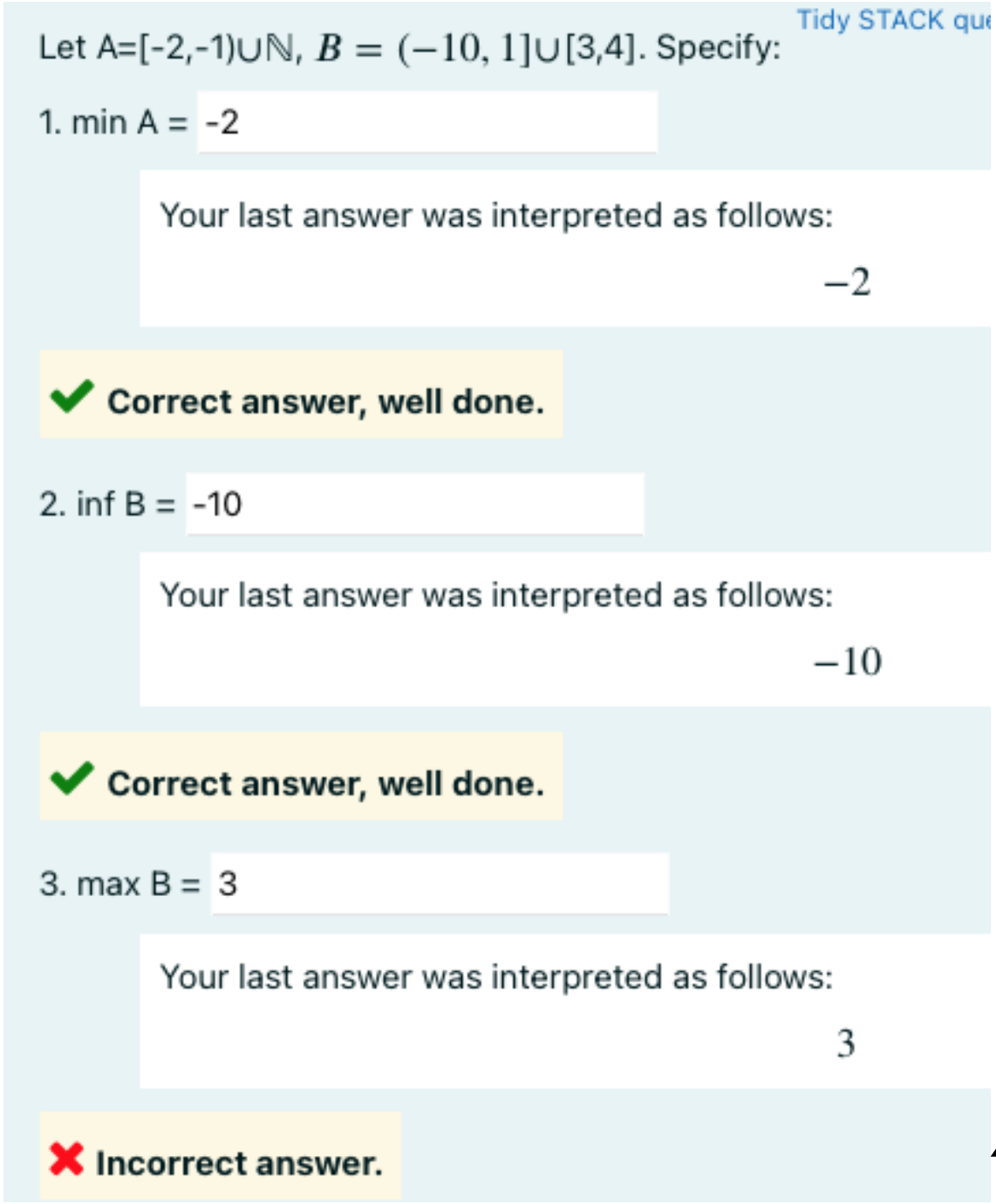
Moodle STACK is a sophisticated assessment system designed for mathematics, scientific, and related disciplines, integrated into the Moodle online learning platform. This complex solution provides an interactive and flexible environment for testing and evaluating students' knowledge in mathematics and sciences, enabling responses that involve mathematical expressions, thereby replacing multiple-choice questions with the ability to input mathematical solutions directly.

To better understand Moodle STACK, it is important to first analyze its essential features and how they contribute to improving the learning and evaluation process in the online environment.

One of the defining characteristics of Moodle STACK is its ability to allow students to input mathematical answers directly within the Moodle platform. This is crucial for accurately assessing knowledge and skills in mathematics and sciences, as some questions require explanations and complex solutions that cannot be reduced to simple multiple-choice selections.

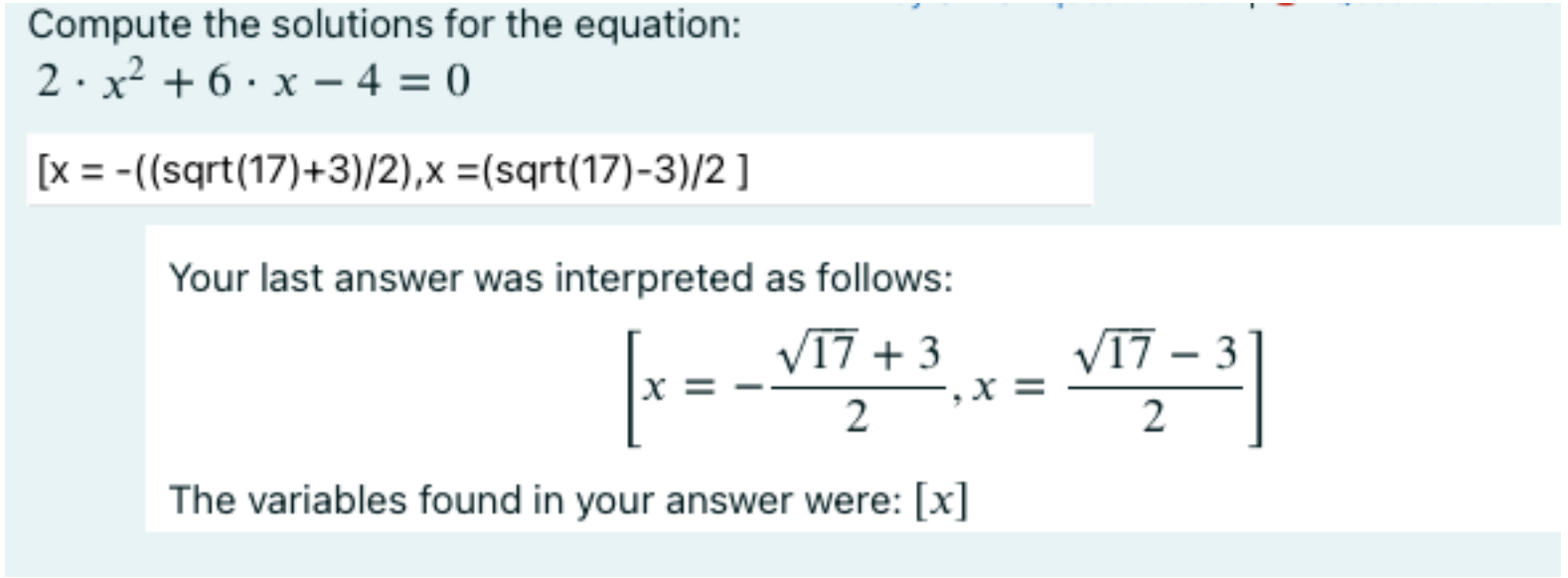


Another important feature of Moodle STACK is the ability to create questions with multiple parts, each part being evaluated separately. This allows for a more detailed assessment of students' competencies and provides a more comprehensive picture of their understanding of the subject.



Additionally, Moodle STACK offers advanced options for generating random components within questions. This is useful for creating a variety of practical questions and for preventing collaboration among students during tests. Randomly generated components ensure that each student receives a unique set of questions, thereby reducing the possibility of copying or cheating.

The coefficients of the equation are randomly generated in an interval set by the teacher for each individual student, or at each viewing of the question.



Question variables

```
z: apply("+", [ ev(rand_with_prohib(-5, 5, [0])*x^2), ev(rand_with_prohib(-5, 10, [0])*x),
ev(rand_with_prohib(-5, 5, [0]))]);

roots: solve(z = 0, x);
```

Another essential aspect of Moodle STACK is the ability to provide personalized and detailed feedback to students. Feedback can be tailored to each student's results and can be used to highlight both their strengths and weaknesses. This approach encourages active student engagement in the learning process and helps them better understand the concepts and issues they are facing.

For example, we have the following equation:

Let $(s_n) = \left\{ \frac{P_k(n)}{Q_i(n)}, n \in \mathbb{N} \right\}$ a sequence such that $P_k(n)$ and $Q_i(n)$ are two polinomyal of degrees $k \leq 3$ respectively.
Give an example of a sequence s_n such that sequence is

a) divergent;

b) convergent to zero;

c) convergent to $\frac{3}{5}$.

Let

$(s_n) = \left\{ \frac{P_k(n)}{Q_i(n)}, n \in \mathbb{N} \right\}$ a sequence such that $P_k(n)$ and $Q_i(n)$ are two polinomyal of degrees $k \leq 3$ respectively.

Give an example of a sequence s_n such that sequence is

a) divergent; {-(2*n^3)+5*n+1}/{n^2-4}

Your last answer was interpreted as follows:

$$\frac{\{-2 \cdot n^3 + 5 \cdot n + 1\}}{\{n^2 - 4\}}$$

The variables found in your answer were: $[n]$

✔ **Correct answer, well done.**

b) convergent to zero; {-(2*n^2)+5*n+1}/{n^3-27}

Your last answer was interpreted as follows:

$$\frac{\{-2 \cdot n^2 + 5 \cdot n + 1\}}{\{n^3 - 27\}}$$

The variables found in your answer were: $[n]$

✔ **Correct answer, well done.**

c) convergent to $\frac{3}{5}$. {6*n^3+1}/{10*n^3-1}

Your last answer was interpreted as follows:

$$\frac{\{6 \cdot n^3 + 1\}}{\{10 \cdot n^3 - 1\}}$$

The variables found in your answer were: $[n]$

✖ **Incorrect answer.**

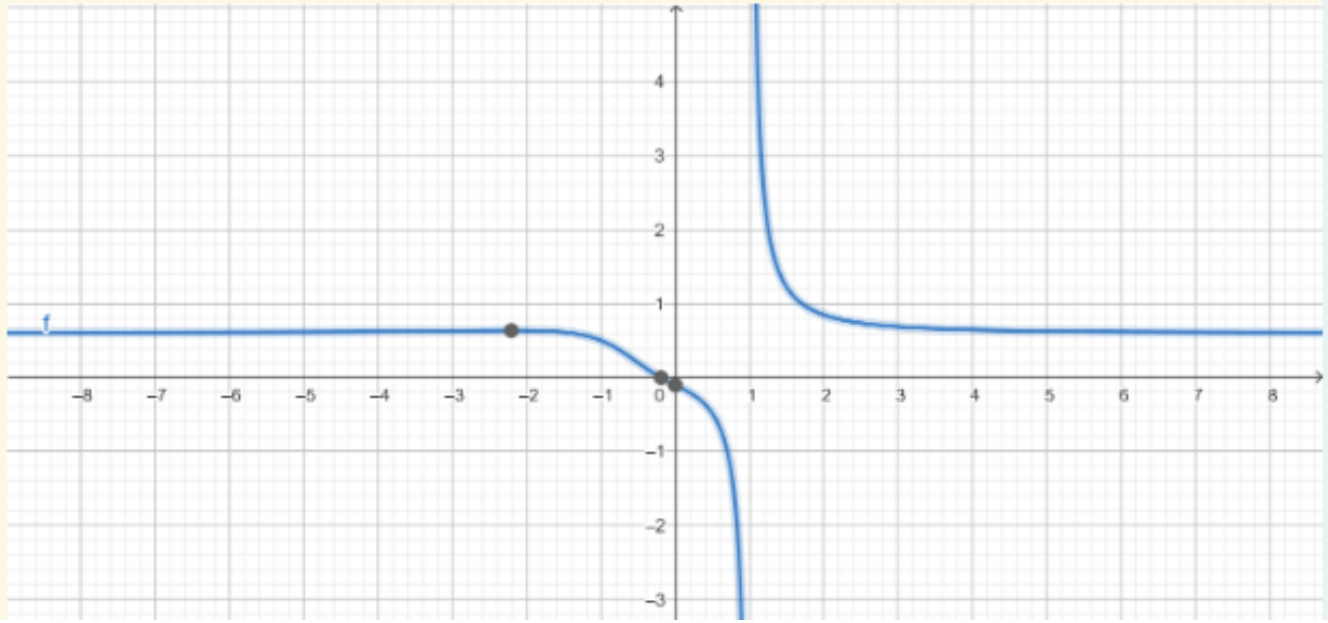
Consider the sequence $(s_n), s_n = \frac{a_k n^k + a_{k-1} n^{k-1} + \ldots + a_1 n + a_0}{b_i n^i + b_{i-1} n^{i-1} + \ldots + b_i n + b_0}$.

(s_n) is convergent to $\frac{3}{5}$ if $k = i$ and $\frac{a_k}{b_i} = \frac{3}{5}$

For example:

$(s_n), s_n = \frac{6n^3+5n+1}{10n^3-1}, n \in \mathbb{N}$ is convergent to $\frac{3}{5}, \lim_{n\to\infty} s_n = \frac{3}{5}$

Using GeoGebra, we can consider the function $f(x) = \frac{6x^3+5x+1}{1-x^3=10}, x \in \mathbb{R}\backslash\{1\}$



- In addition to these core features, Moodle STACK also offers a wide range of additional tools and resources to enhance the learning and evaluation experience. These may include interactive tutorials, sample questions and solutions, practical exercises, and much more, all tailored to the specific needs of each course or discipline.

  - For example, STACK offers the possibility to introduce at the beginning of the evaluation a theoretical part that will give the student a short recapitulation that will help in obtaining a maximum score.

Information

Definition: A sequence of real numbers is a function $f:\mathbb{N}\to\mathbb{R}$, $f(n)=a_n$ or $f:\mathbb{N}\setminus A\to\mathbb{R}$, where $A\subset\mathbb{N}$ finite, $f(n)=a_n$.

Notation: (a_n) is the sequence defined by the function f.

Definition: A sequence of real numbers (a_n) is increasing (decreasing) if $a_n\leq a_{n+1}$ $(a_n\geq a_{n+1})$, $\forall n\geq 0$. If the above inequalities are strictly, then the sequence is called strictly increasing (strictly decreasing).

To study the monotony of a sequence (a_n), the sign of the difference $\Delta a_n = a_{n+1}-a_n$ can be establish or to compare the ratio $\frac{a_{n+1}}{a_n}$ with 1, when $a_n>0, \forall n\geq 0$.

Types of equations in STACK:

- ✓*Multiple choice* – allows the selection of a singular o multiple responses from a pre-defined list
- ✓*True/False* – a simple form of multiple choice question with just the two choices “True” and ”Fals”
- ✓*Matching* – the answer to each of a number of subquestion must be selected from a list of possibilities
- ✓*Essay* – allows a response of a file upload and/or online text. This must then be graded manually
- ✓*Drag and drop into text* – STACK provides mathematical questions for the Moodle quiz. These use a computer algebra system to establish the mathematical properties of the student's responses.
- ✓*Select missing words* – missing words in the question text are filled in using drop-down menus
- ✓*STACK* - STACK provides mathematical questions for the Moodle quiz. These use a computer algebra system to establish the mathematical properties of the student's responses.

This type of question allows students to choose as an answer one or more of the items and has the possibility to **access GeoGebra** to calculate the correct answer.

For this type of question, for each student the items can be generated randomly, so that the items a, b, c, d are always in a different order.

In this example, the correct answer consists of two subpoints, so if the student has selected only one correct answer, they will receive half of the question's score.

Give an example of a sequence:

$(s_n)=\frac{an+b}{cn+d}, n \in \mathbb{N}$, where $a, b, c, d \in \mathbb{R}$ such that the sequence is:

a) increasing and convergent to 3;

b) decreasing and convergent to 3.

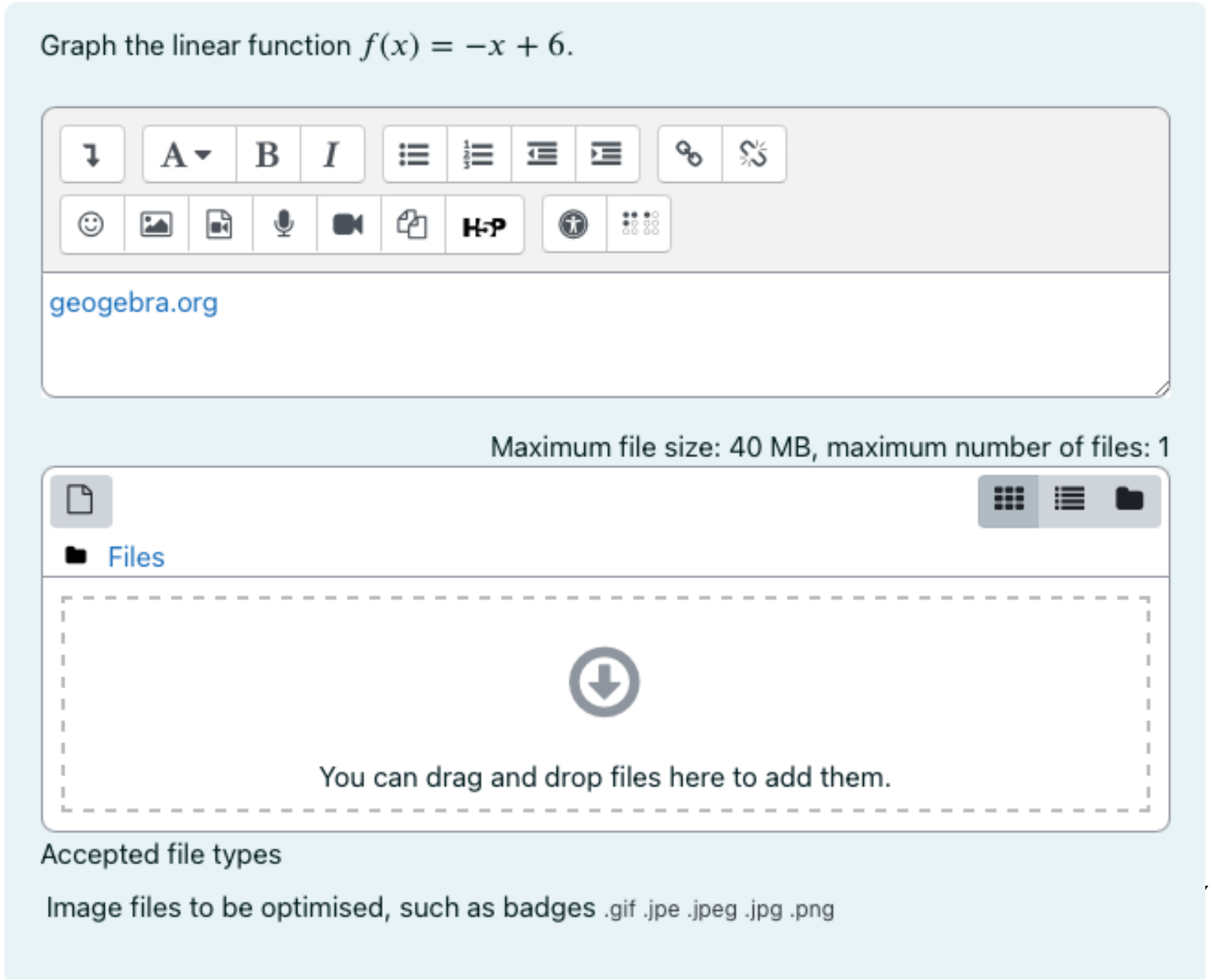
Use GeoGebra to check your sequences before you answer.

https://www.geogebra.org/calculator

Select the correct answer:

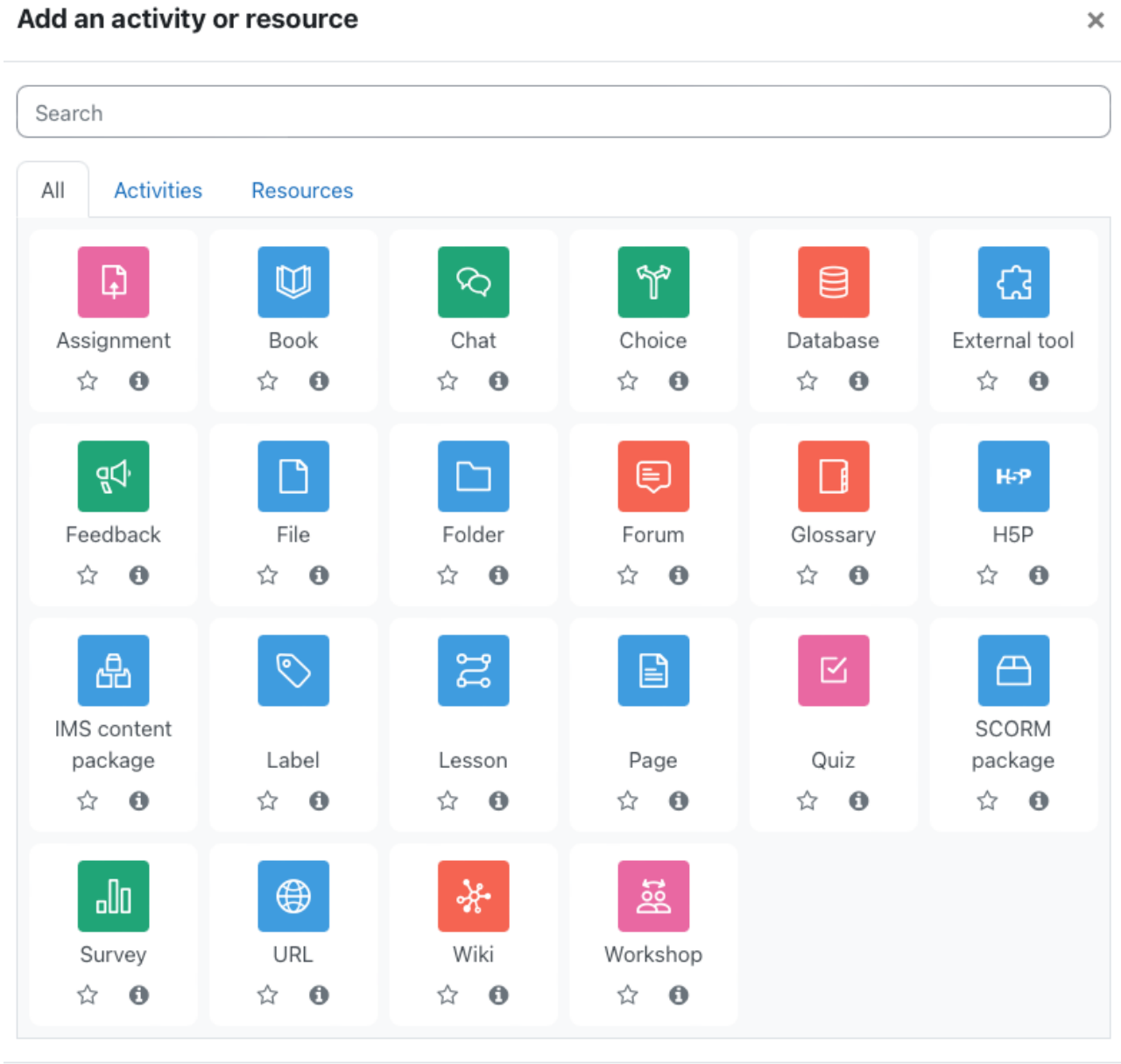
- a. sequence (s_n) is decreasing to 3 if $a = 3c, ad \leq bc$
- b. sequence (s_n) is increasing to 3 if $a = 3c, ad \geq bc, \frac{d}{c} \geq 0$ ✔
- c. sequence (s_n) is decreasing to 3 if $a = 3c, ad \leq bc, \frac{d}{c} \geq 0$
- d. sequence (s_n) is increasing to 3 if $a = 3c, ad \geq bc$ ✖

This type of question is designed to allow students to access GeoGebra for creating the graph related to the question, after which it allows them to upload the file containing the graph generated in GeoGebra.



Create, configure the test and assigning it to students

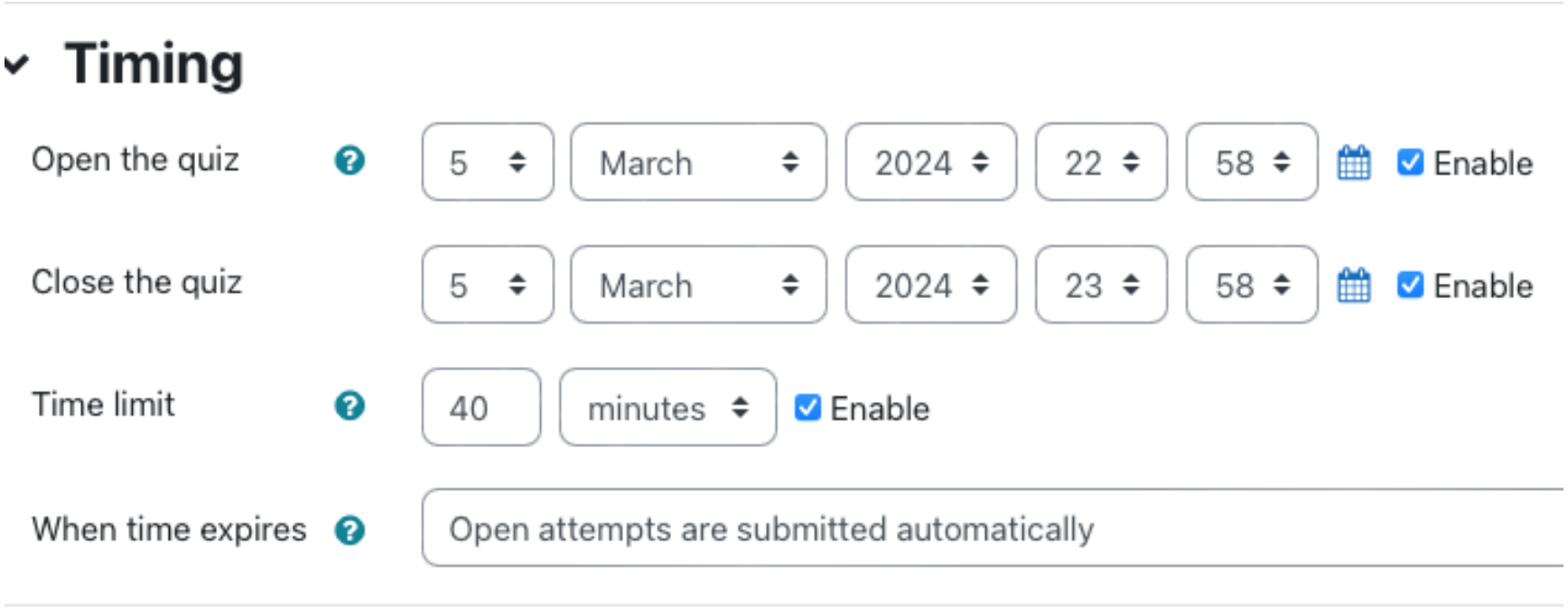
Identify the section where you want to enter the test (for example, the current week) and add a new activity (Add an activity or resource) of the grid test type (Quiz)



Create, configure the test and assigning it to students

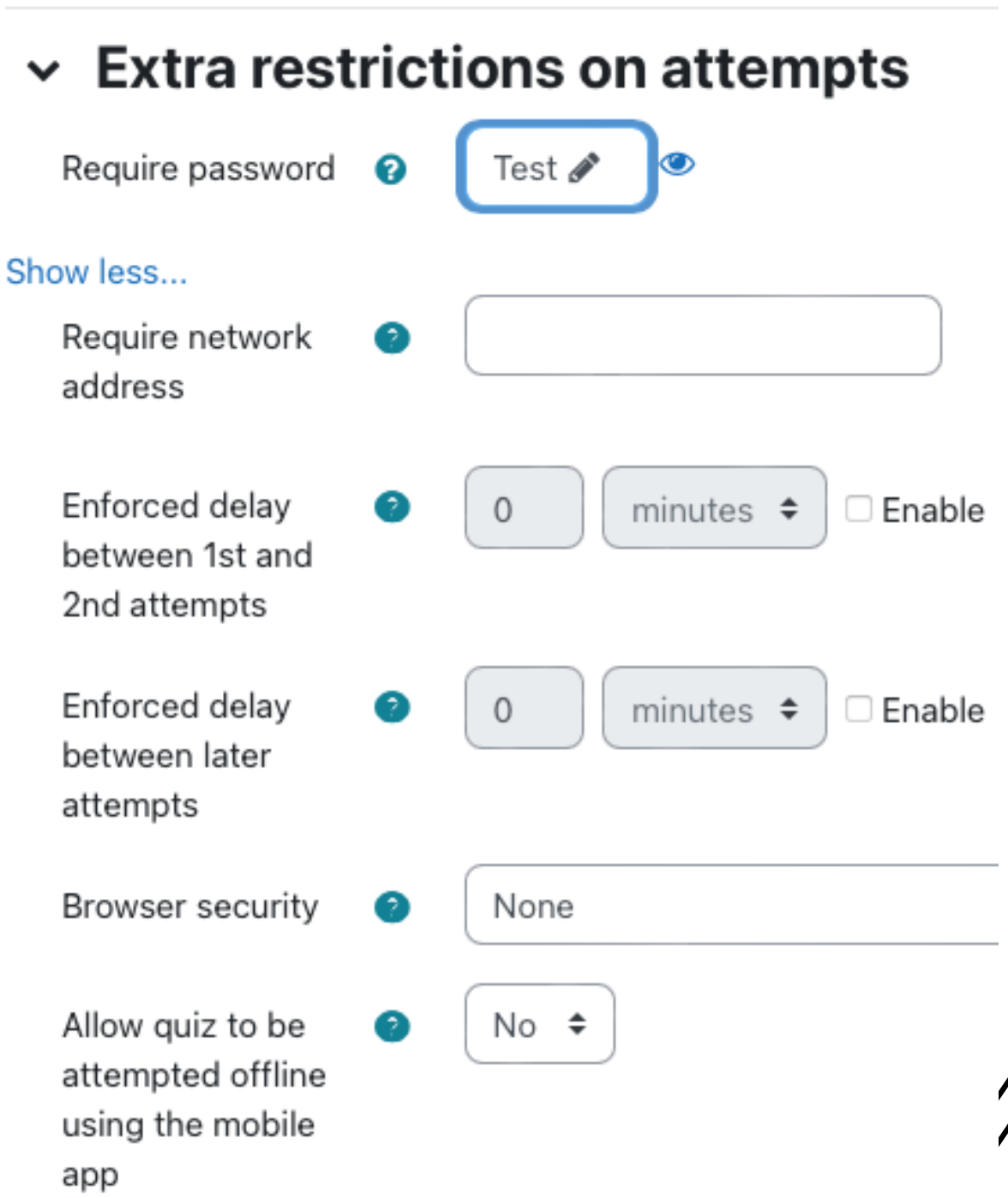
In the Timing section:

- Date and time when the test becomes available to students (Open the quiz);
- Date and time when the test becomes unavailable to students (Close the quiz). Attention: if students are still working at that time, the test will be closed automatically;



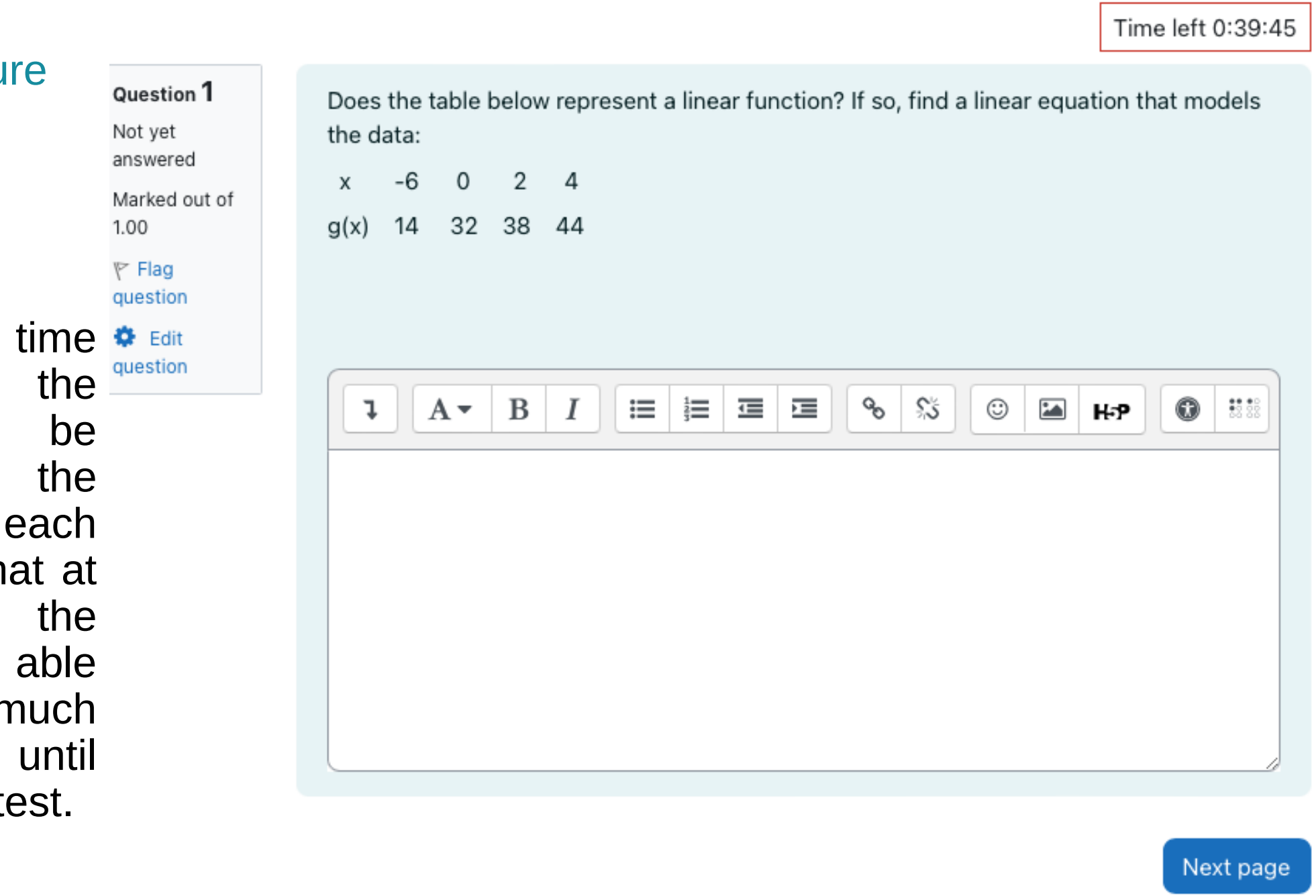
Create, configure the test and assigning it to students

- In the Extra restrictions on attemps section:
  - From this section you can configure a password for accessing the test (to set it, press the Pencil icon);



Create, configure the test and assigning it to students

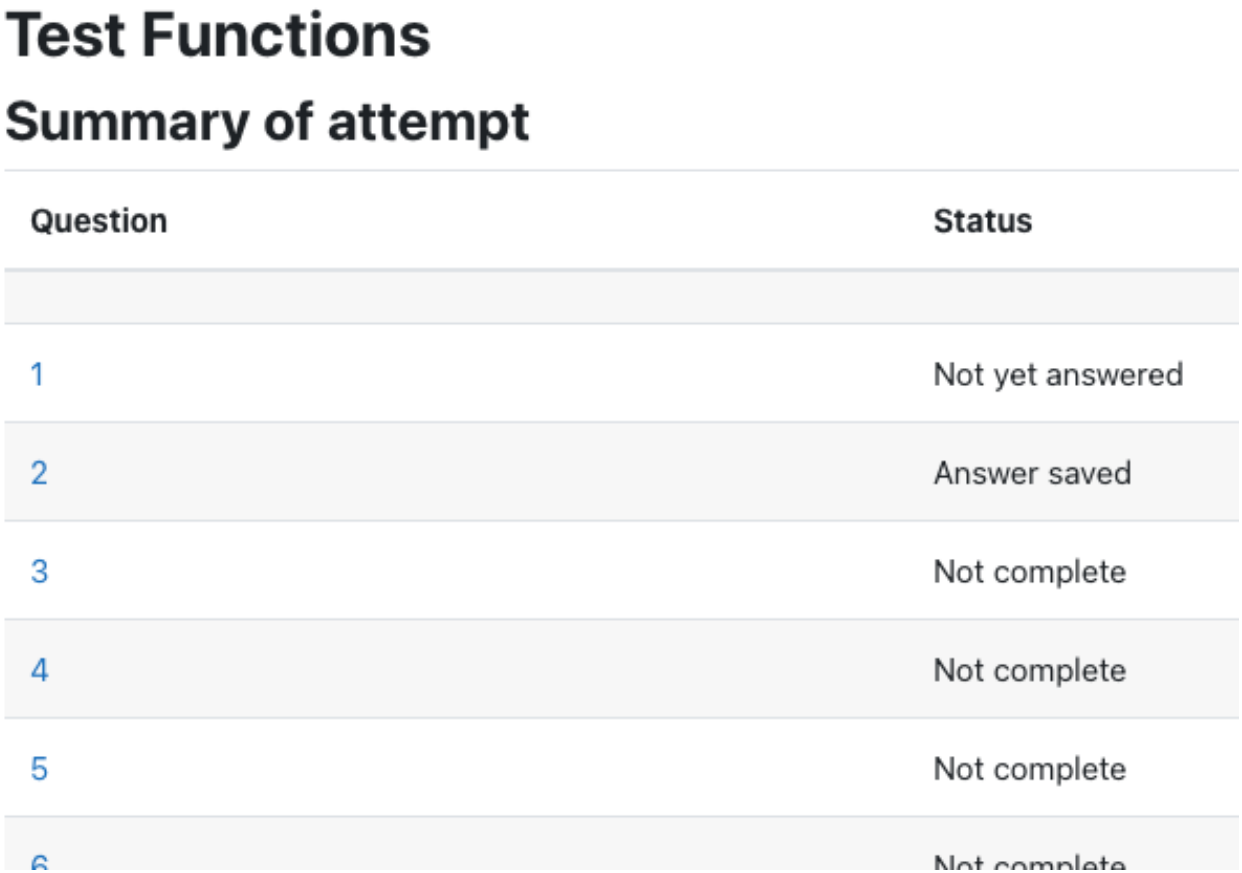
The remaining time for solving the problems will be displayed on the page of each question, so that at any moment the student will be able to see how much time he has until the end of the test.

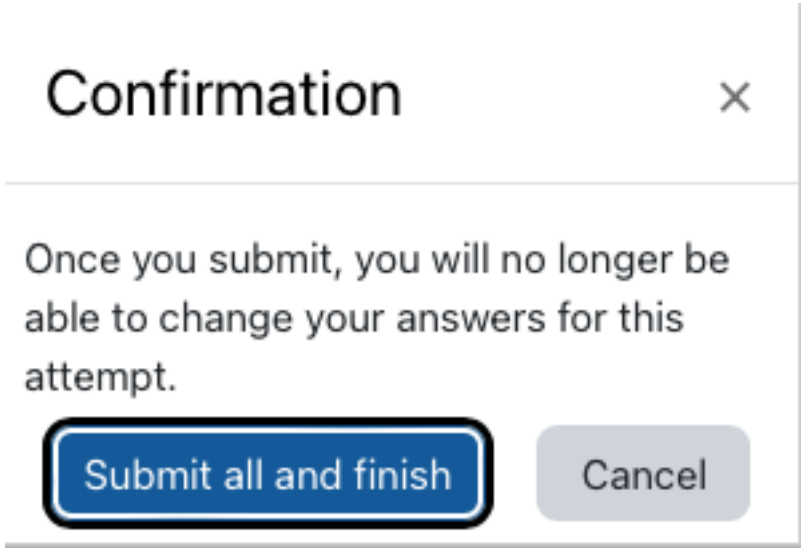


Create, configure the test and assigning it to students

- To complete the test, the student will press Finish attempt... placed in the lower right area of the page.

A summary of the answers is presented before the test is completed; At this point, the student can see the questions he has not answered yet, as well as how much time he has left.





Stack implementation in other areas

The applicability of evaluation systems with integrated learning with feedback shows its benefits, for example, a module that helps students prepare for a license exam. Within it, the test has integrated formative feedback that helps the student to deepen the subject, in cases where the answers are correct, giving details on alternative methods, and in case some notions are not known, to explain in a personalized way what exactly it has to learn, why he had inadvertences in his answers and what connections he needs to make in order to understand the scientific links between the component parts of the subject matter to be studied.

All this transcribes the difficulty of the learning process for content creators for such tests, whose experience is fundamental in creating such quality content.

Question text

What values will the following program display?

```
int x = 9, y = 5, a, b, c;
b = a = x++;
y++;
c = x + y;
b = (++y) + a;
printf("\n%d %d %d", a, ++b, c);
```

a = [[input:ans1]] [[validation:ans1]][[feedback:prt1]]

b = [[input:ans2]] [[validation:ans2]][[feedback:prt2]]

c = [[input:ans3]] [[validation:ans3]][[feedback:prt3]]

Tidy STACK question

What values will the following program display?

```
int x = 9, y = 5, a, b, c;
b = a = x++;
y++;
c = x + y;
b = (++y) + a;
printf("\n%d %d %d", a, ++b, c);
```

a =

b =

c =

Check

Formative Feedback for previous Stack Questions

Standard feedback for correct

✔ Correct answer, well done.
You have carefully analyzed each step of the code and correctly calculated the values of the variables a, b, and c. It is essential to understand how the assignment and increment operators work in the context of complex expressions and to pay attention to the order in which these operations are performed.

An important point that you illustrated with this problem is how the increment (++) and assignment (=) operators can influence the values of variables and the final output of the program.

Standard feedback for partially correct

Your answer is partially correct.

Standard feedback for incorrect

✖ Incorrect answer.
int x = 9, y = 5, a, b, c;

b = a = x++;

y++;

c = x + y;

b = (++y) + a;

printf("\n%d %d %d", a, ++b, c);

In this program, we will analyze the values that the variables a, b, and c will display at the end of execution, taking into account the initial values and the operations performed on them within the code.

Initialization and assignment of values:

int x = 9, y = 5, a, b, c;: We declare the variables x, y, a, b and c, and initialize the variables x and y with the values 9, 5.

b = a = x++;: We assign the values of x to the variables a and b, and then increment x. So a gets the initial value of x, which is 9, and b gets the value of a.

Incrementing y and calculating c:

y++;: We increment y by 1, so y will become 6.

c = x + y;: We calculate the sum of x and y, which are now x=10 and 6, and store it in c. So c will be 16.

Modification of b:

b = (++y) + a;: We increment y by 1 (so y becomes 7) and add the value of a to y. Since a is 9, adding 9 to 7 gives 16.

Display the result:

Finally, we display the values of a, ++b and c, but before displaying the value of b we will increment it by 1, making it 17.

Determination of values:

a: Gets the value of x from the first assignment, so it will be 9.

b: Calculated in the previous step as 17.

c: It was previously calculated as 16.

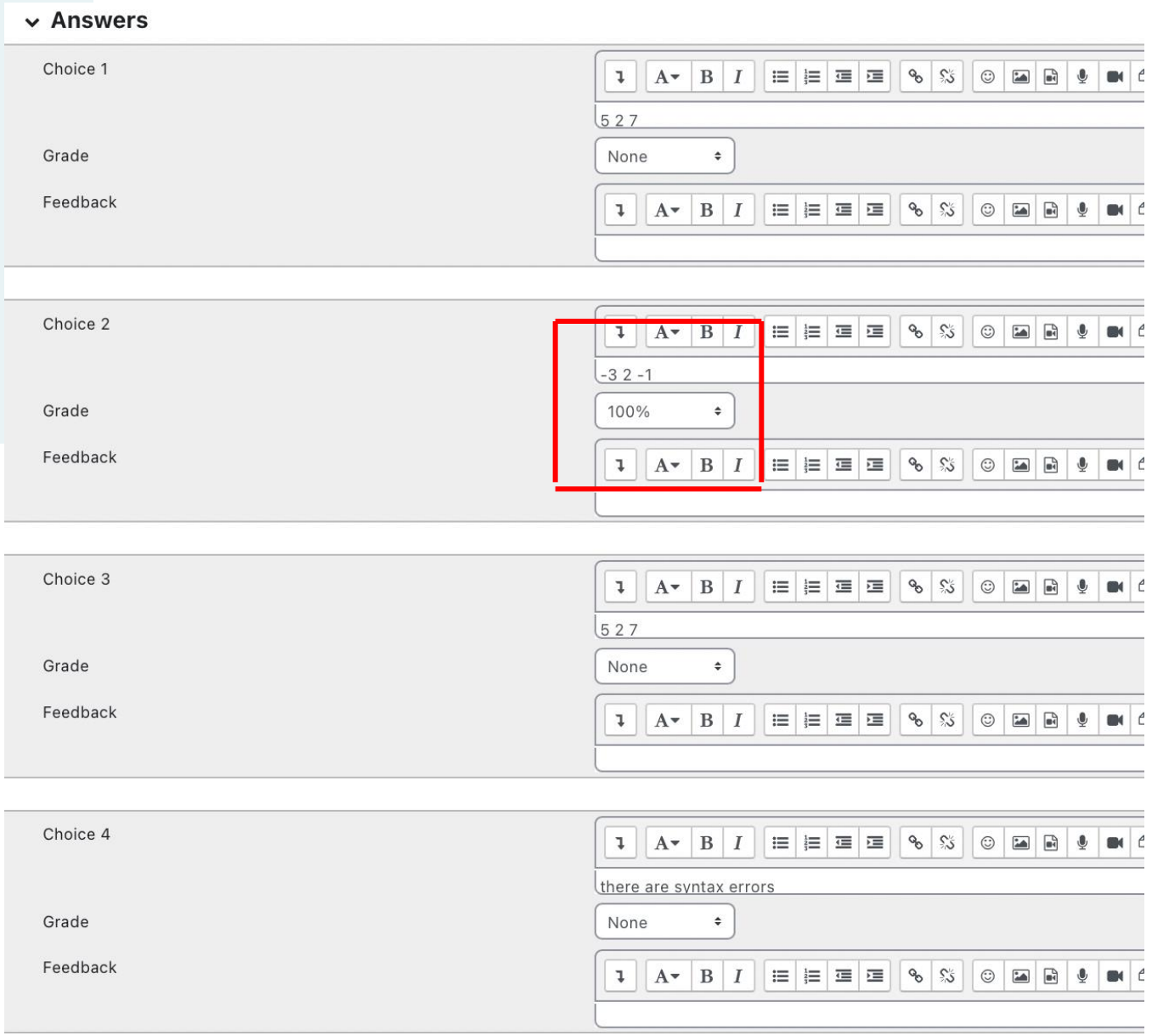
Percentaged multiple type choice tests

Specify the displayed values, if the numerical values 5 2 -3 are to be read in the indicated order:

```
int main() {
    int a, b;
    scanf("%d%d%d", &a, &b, &a);
    printf("%d,", a);
    printf("%d,", b);
    printf("%d", a + b);
}
```

- a. there are syntax errors
- b. 5 2 7
- c. -3 2 -1
- d. 5 2 7

For this example there is only one correct answer, but, if we have more than one answer the grade of the answer will be divided in number of choice.



Combined Feedback for previous Multiple Choice Tests

Combined feedback

For any correct response

Your answer is correct.
You have used the scanf and printf functions correctly to read and display the read values, i.e. the sum of the first two numbers.

Explanations:

Function scanf("%d%d%d", &a, &b, &a); reads three integers from standard input. The last value read to overwrite the previous value of variable a.

The read values are then displayed using the printf function. So:

The variable a will display the last value read for a.

The variable b will display the value read for b.

The sum of the first two numbers, a + b, will display the sum of the values originally read for a and b.

If you enter the values 5, 2 and -3 in this order, the displayed values will be:

a: -3 (last value read for a)

b: 2

a + b: -1 (sum of initially read values for a and b: 5 + 2 = 7, but last read value for a was -3)

You have shown a clear understanding of how to use the scanf and printf functions.

For any incorrect response

Your answer is incorrect.
In this code, a sequence of three integers is read using the scanf function, and then the values read and the sum of the first two numbers are displayed.
Function scanf("%d%d%d", &a, &b, &a); is used to read three integers. Since the variable a is read twice, the last value read to overwrite the previous value of the variable a.

Then the read values are displayed using the printf function. In this case, the a variable, the b variable, and the sum of the first two numbers (a + b) are displayed.

If we enter the values 5, 2 and -3 in this order, following the instructions in the code:

The first value read is 5 and is stored in the variable a.

The second value read is 2 and is stored in the variable b.

The third value read is -3 and is stored again in the a variable, overwriting the previous value.

Thus, the displayed values will be:

a: -3 (last value read for a)

b: 2

a + b: -1 (sum of initially read values for a and b: 5 + 2 = 7, but last read value for a was -3)

Matching questions

We consider the variables a, b, c, d and e. Write the correct version of the statements, so that the following assignments are not accompanied by conversions that modify the assigned values.

c='a' Choose...

a=3 Choose...

d=-3.452 Choose...

b=2.1 Choose...

Check

We consider the variables a, b, c, d and e. Write the correct version of the statements, so that the following assignments are not accompanied by conversions that modify the assigned values.

c='a' char

a=3 int

d=-3.4 ✓ Choose...

b=2.1 char

float

int

Chec

Answers

Available choices

You must provide at least two questions and th

Question 1

a=3

Answer int

Question 2

b=2.1

Answer float

Question 3

c='a'

Answer char

Question 4

d=-3.452

Answer float

For previous question we have implemented the formative feedback for:

- Correct answer
- Partilly correct answer
- Incorrect answer

Combined feedback

For any correct response

Your answer is correct and demonstrates a solid understanding of data types and their correct use in C!
You used the appropriate data types for the variables a, b, c, and d and assigned literal values without requiring implicit conversions to change the assigned values. By using the appropriate data types and values correctly, you ensure accurate data storage in variables and avoid unwanted changes.

For any partially correct response

Your answer is partially correct.
In the C language, data types are used to specify the nature of the data stored in variables and how they are interpreted by the program. Here is a full explanation for each data type mentioned in the given problem:

The char data type is used to store characters and is represented by a single byte in memory. Char variables can store a single ASCII character or a special character. In our case, when we assign the value 'a' to the variable c, we store the ASCII code of the character 'a', which is 97. This ensures that the variable c will contain the ASCII value corresponding to the character 'a'.

The double data type is used to store double-precision floating-point real numbers. Variables of type double are represented on 8 bytes in memory and provide higher precision than single-precision floating-point data types. When we assign the value -3.452 to the variable d, we store this value with double precision in memory.

The int data type is used to store integers. Variables of type int are represented by 4 bytes in memory and can store integer values in the range specified by the data type. When we assign the value 3 to the variable a, we store this value as an integer in memory.

The float data type is used to store single-precision floating-point real numbers. Float variables are represented by 4 bytes in memory and provide lower precision than double data types, but take up less memory space. When we assign the value 2.1 to the variable b, we must add the suffix f to specify that the value is a float, like this: 2.1f.

By using the correct data types and appropriate values, we ensure accurate data storage in variables and avoid implicit conversions that could change assigned values or affect data precision.

Options

Show the number of correct responses once the question has finished

For any incorrect response

Your answer is incorrect.
In the C language, data types are used to specify the nature of the data stored in variables and how they are interpreted by the program. Here is a full explanation for each data type mentioned in the given problem:

The char data type is used to store characters and is represented by a single byte in memory. Char variables can store a single ASCII character or a special character. In our case, when we assign the value 'a' to the variable c, we store the ASCII code of the character 'a', which is 97. This ensures that the variable c will contain the ASCII value

According to those illustrated, the combination of a virtualization system that allows the collaborative integration of software that shares data systems, and the implementation of a STACK system together with the Moodle platform, all these together, constitute a solid base on which systems can be built assisted learning, even in test modules. We return with the clarification that all this transfers the difficulty in the scientific design of the implementation and is based entirely on the ability of the teaching staff to understand the learning process and the technical mastery of pedagogical psychology.

MOODLE STACK:

Implementation example and Small guide for students

STACK online assessment for mathematics and science

Let take an example for implementation and explanations in stack moodle:

Let $(s_n) = \left\{ \frac{P_k(n)}{Q_i(n)}, n \in \mathbb{N} \right\}$ a sequence such that $P_k(n)$ and $Q_i(n)$ are two polinomyal of degrees $k \leq 3$ respectively.
Give an example of a sequence s_n such that sequence is

a) divergent;

b) convergent to zero;

c) convergent to $\frac{3}{5}$.

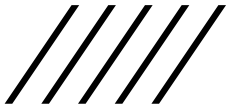
Possible answer:

a) $\frac{\{-2 \cdot n^3 + 5 \cdot n + 1\}}{\{n^2 - 4\}}$

b) $\frac{\{-2 \cdot n^2 + 5 \cdot n + 1\}}{\{n^3 - 27\}}$

c) $\frac{\{6 \cdot n^3 + 5 \cdot n + 1\}}{\{10 \cdot n^3 - 1\}}$

- To reformulate this problem into one that can be implemented in a stack according to the principles of formative feedback, we will have to build it in the following way:
  1. Building a formative feedback for cases in which a student will give incorrect answers, for each subclass of incorrect answers. He will have to take into account the explanation of the knowledge necessary for the student to understand why he made a mistake, where he made a mistake, and to explain the necessary notions so that he can answer this subclass of problems correctly.
  2. Building a formative feedback in case the student chooses a correct answer. He will have to illustrate to the student alternative methods by which he could solve the problem, in this way, expanding his area of knowledge in the field.



For a) – the personalized feedback for incorrect answer

a) divergent; {-(2*n^3)+5*n+1}/{5}

Your last answer was interpreted as follows:

$$\frac{\{-2 \cdot n^3 + 5 \cdot n + 1\}}{\{5\}}$$

The variables found in your answer were: $[n]$

✖ **Incorrect answer.**

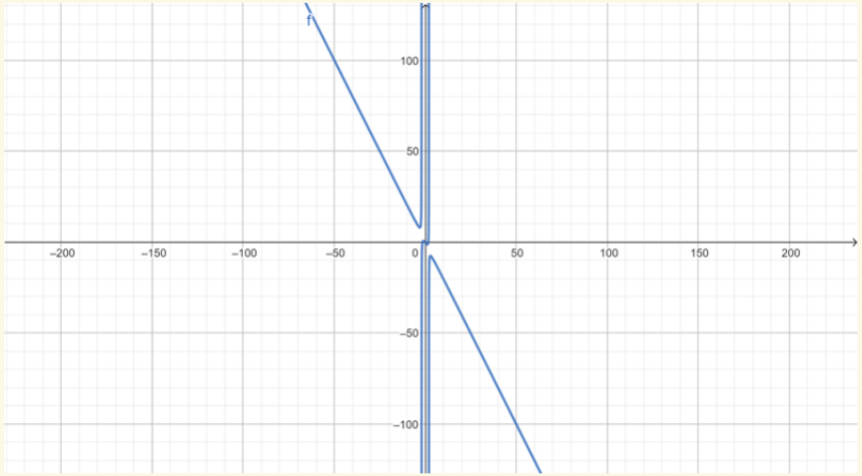
Consider the sequence (s_n), $s_n = \frac{a_k n^k + a_{k-1} n^{k-1} + \ldots + a_1 n + a_0}{b_i n^i + b_{i-1} n^{i-1} + \ldots + b_i n + b_0}$.

(s_n) is divergent if $k > i$

For example:

(s_n), $s_n = \frac{-2n^3+5n+1}{n^2-4}$, $n \in \mathbb{N}$ is divergent, $\lim_{n\to\infty} s_n = -\infty$

Using GeoGebra, we can consider the function $f(x) = \frac{-2x^3+5x+1}{x^2-4}$, $x \in \mathbb{R}\setminus\{-2, 2\}$



For b) – the personalized feedback for incorrect answer

b) convergent to zero; -(2*n^2)+5*n+1

Your last answer was interpreted as follows:

$$-2 \cdot n^2 + 5 \cdot n + 1$$

The variables found in your answer were: $[n]$

✖ **Incorrect answer.**

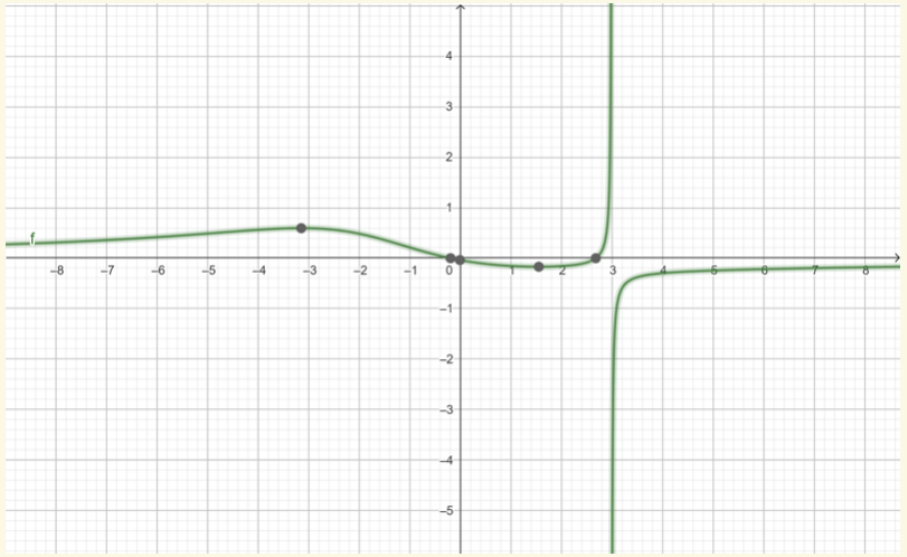
Consider the sequence (s_n), $s_n = \frac{a_k n^k + a_{k-1} n^{k-1} + \ldots + a_1 n + a_0}{b_i n^i + b_{i-1} n^{i-1} + \ldots + b_i n + b_0}$.

(s_n) is convergent to 0 if $k < i$

For example:

(s_n), $s_n = \frac{-2n^2+5n+1}{n^3-27}$, $n \in \mathbb{N}$ is convergent, $\lim_{n\to\infty} s_n = 0$

Using GeoGebra, we can consider the function $f(x) = \frac{-2x^2+5x+1}{x^3-27}$, $x \in \mathbb{R}\setminus\{3\}$



For c) – the personalized feedback for incorrect answer

c) convergent to $\frac{3}{5}$. {6*n^3+5*n+1}/{10}

Your last answer was interpreted as follows:

$$\frac{\{6 \cdot n^3 + 5 \cdot n + 1\}}{\{10\}}$$

The variables found in your answer were: $[n]$

✖ **Incorrect answer.**

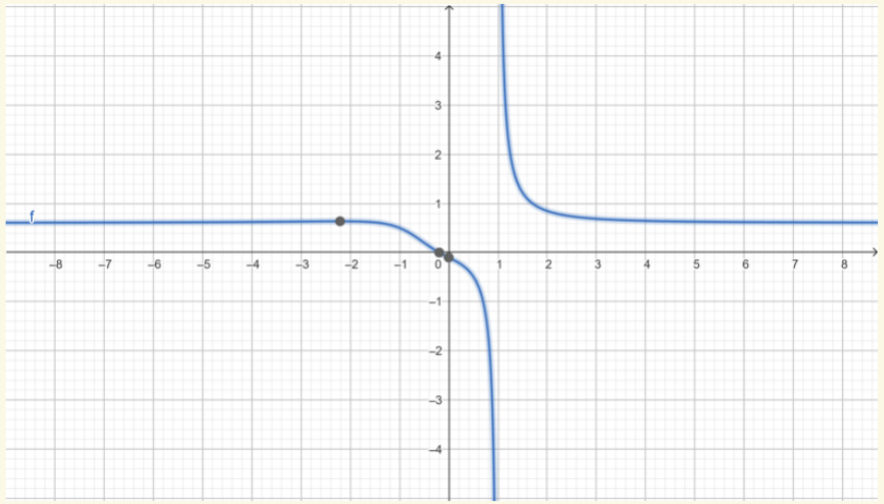
Consider the sequence (s_n), $s_n = \frac{a_k n^k + a_{k-1} n^{k-1} + \ldots + a_1 n + a_0}{b_i n^i + b_{i-1} n^{i-1} + \ldots + b_i n + b_0}$.

(s_n) is convergent to $\frac{3}{5}$ if $k = i$ and $\frac{a_k}{b_i} = \frac{3}{5}$

For example:

(s_n), $s_n = \frac{6n^3+5n+1}{10n^3-1}$, $n \in \mathbb{N}$ is convergent to $\frac{3}{5}$, $\lim_{n\to\infty} s_n = \frac{3}{5}$

Using GeoGebra, we can consider the function $f(x) = \frac{6x^3+5x+1}{1-x^3=10}$, $x \in \mathbb{R}\backslash\{1\}$



For a) – the personalized feedback for correct answer

a) divergent; {-(2*n^3)+5*n+1}/{n^2-4}

Your last answer was interpreted as follows:

$$\frac{\{-2 \cdot n^3 + 5 \cdot n + 1\}}{\{n^2 - 4\}}$$

The variables found in your answer were: $[n]$

✔ **Correct answer, well done.**

Let's analyze if the sequence $s_n = \frac{-2n^3+5n+1}{n^2-4}$ is divergent. Let's look at the behaviour of this sequence as $n \to \infty$.

Consider the polynomials:

- $P_e(n) = -2n^3 + 5n + 1 (degree 3)$
- $Q_2(n) = n^2 - 4 (degree 2)$

The sequence s_n is: $s_n = \frac{-2n^3+5n+1}{n^2-4}$.

To determine the behavior of the sequence, we divide the numerator and the denominator by n^2:

$$s_n = \frac{-2n^2+5n+1}{n^2-4} = \frac{n^2(-2n+\frac{5}{n}+\frac{1}{n^2})}{n^2(1-\frac{4}{n^2})} = \frac{-2n+\frac{5}{n}+\frac{1}{n^2}}{1-\frac{4}{n^2}}$$

As $n \to \infty$, the terms $\frac{5}{n}$, $\frac{1}{n^2}$ and $\frac{4}{n^2}$ tend towards 0. Therefore, the expression becomes:

$s_n \approx \frac{-2n}{1} = -2n$

As $n \to \infty, -2n$ tends towards $-\infty$, which means the sequnce is divergent.

Therefore, the sequence $s_n = \frac{-2n^3+5n+1}{n^2-4}$ is correct example of a divergent sequence.

For b) – the personalized feedback for correct answer

b) convergent to zero; {-(2*n^2)+5*n+1}/{n^3-27}

Your last answer was interpreted as follows:

$$\frac{\{-2 \cdot n^2 + 5 \cdot n + 1\}}{\{n^3 - 27\}}$$

The variables found in your answer were: $[n]$

✔ **Correct answer, well done.**

Yes, the sequence $s_n = \frac{-2n^2+5n+1}{n^3-27}$ is a correct example of sequence that converges to zero. Let's see why.

Consider the polynomials:

- $P_e(n) = -2n^2 + 5n + 1 (degree 2)$
- $Q_2(n) = n^3 - 27 (degree 3)$

The sequence s_n is: $s_n = \frac{-2n^2+5n+1}{n^3-27}$.

To determine the behavior of the sequence as $n \to \infty$, we devide the numerator and the denominator by n^3:

$$s_n = \frac{-2n^2+5n+1}{n^3-27} = \frac{\frac{-2n^2}{n^3}+\frac{5n}{n^3}+\frac{1}{n^3}}{\frac{n^3}{n^3}-\frac{27}{n^3}} = \frac{\frac{-2}{n}+\frac{5}{n^2}+\frac{1}{n^3}}{1-\frac{27}{n^3}}$$

As $n \to \infty$, the terms $\frac{-2}{n}$, $\frac{5}{n^2}$ and $\frac{1}{n^3}$ tend towords 0, and trus the expression becomes: $s_n \approx \frac{0+0+0}{1-0} = 0$

Therefore, the sequence converges to zero as $n \to \infty$.

Thus, the sequence $s_n = \frac{-2n^2+5n+1}{n^3-27}$ is a correct example of sequence that converges to zero.

For c) – the personalized feedback for correct answer

c) convergent to $\frac{3}{5}$. {6*n^3+5*n+1}/{10*n^3-1}

Your last answer was interpreted as follows:

$$\frac{\{6 \cdot n^3 + 5 \cdot n + 1\}}{\{10 \cdot n^3 - 1\}}$$

The variables found in your answer were: $[n]$

Correct answer, well done.

Yes, the sequence $s_n = \frac{6n^3+5n+1}{10n^3-1}$ is a correct example of sequence that converges to $\frac{3}{5}$. Let's see why.

Consider the polynomials:

- $P_e(n) = 6n^3 + 5n + 1(degree3)$
- $Q_2(n) = 10n^3 - 1(degree3)$

The sequence s_n is: $s_n = \frac{-6n^3+5n+1}{10n^3-1}$.

To determine the behavior of the sequence as $n \to \infty$, we devide the numerator and the denominator by n^3:

$$s_n = \frac{6n^3+5n+1}{10n^3-1} = \frac{\frac{6n^3}{n^3}+\frac{5n}{n^3}+\frac{1}{n^3}}{\frac{10n^3}{n^3}-\frac{1}{n^3}} = \frac{6+\frac{5}{n^2}+\frac{1}{n^3}}{10-\frac{1}{n^3}}$$

As $n \to \infty$, the terms $\frac{5}{n^2}$ and $\frac{1}{n^3}$ tend towards 0, and thus the expression becomes:

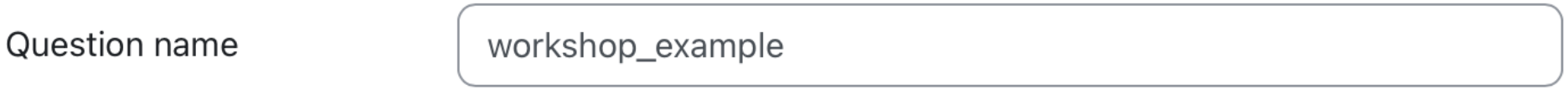
$s \approx \frac{6+0+0}{10-0} = \frac{6}{10} = \frac{3}{5}$

Therefore, the sequence converges to $\frac{3}{5}$ as $n \to \infty$.

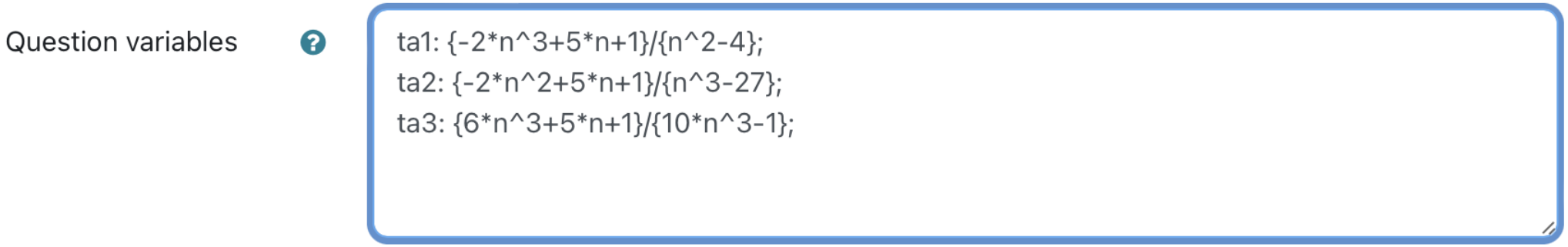
Thus, the sequnece $s_n = \frac{6n^3+5n+1}{10n^3-1}$ is correct example of a sequence that converges to $\frac{3}{5}$.

Steps taken in implementing the question:

- we must enter a name for the question

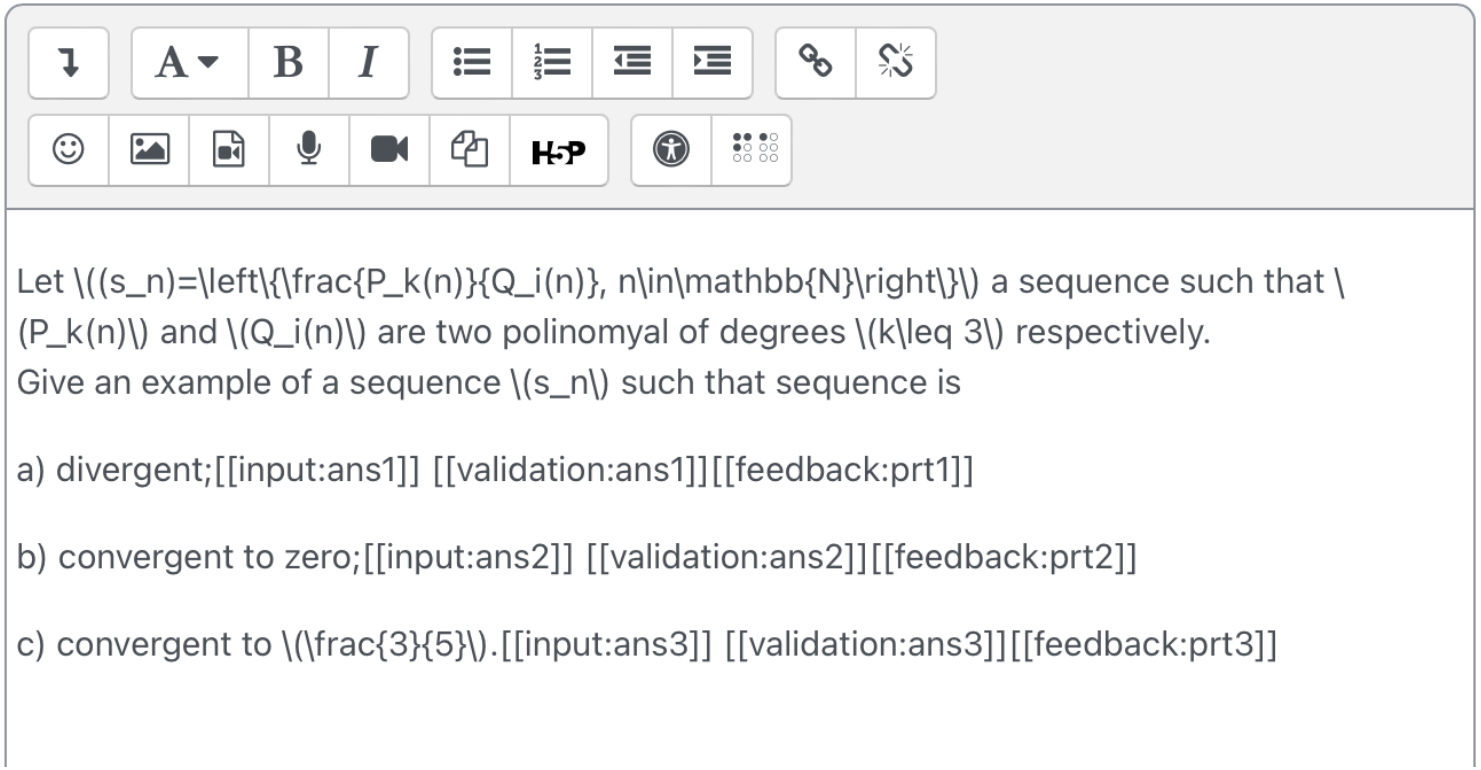


- for an easy use and management of the code in the stack, we have the possibility to define variables and assign them values, like this; if we have a complex answer for the question or we have a frequently used expression within the statement, we can define variables by assigning them those values:



➢introduce the statement:

Question text



- mathematical equations are inserted between \(\)
- for each sub-point in the question we will have one answer, which will have to be validated with the answer given by the teacher, and we can have specific feedback for each sub-point, whether the answer is correct, partially correct or incorrect

➢For each answer we need input a model answer:

Input: ans1

Input type: Algebraic input

Model answer: ta1

Input box size: 15

Input: ans2

Input type: Algebraic input

Model answer: ta2

Input box size: 15

In our example we have defined variables and we used in model answer

Input: ans3

Input type: Algebraic input

Model answer: ta3

Input box size: 15

- ➢We need to input the “Potential response tree” for each sub-point

Potential response tree: prt1

Node 1 — Answer test: AlgEquiv | SAns: ans1 | TAns: ta1 | Test options: | Quiet: No

Node 1 when true — Mod: = | Score: 1 | Penalty: | Next: [stop] | Answer note: prt1-1-T

Node 1 true feedback

Let's analyze if the sequence \(s_n=\frac{-2n^3+5n+1}{n^2-4}\) is divergent. Let's look at the behaviour of this sequence as \(n\rightarrow \infty\).

Consider the polynomials:

- \(P_e(n)=-2n^3+5n+1(degree 3)\)
- \(Q_2(n)=n^2-4(degree 2)\)

The sequence \(s_n\) is: \(s_n=\frac{-2n^3+5n+1}{n^2-4}\).

To determine the behavior of the sequence, we divide the numerator and the denominator by \(n^2\):

\(s_n=\frac{-2n^2+5n+1}{n^2-4}=\frac{n^2(-2n+\frac{5}{n}+\frac{1}{n^2})}{n^2(1-\frac{4}{n^2})}=\frac{-2n+\frac{5}{n}+\frac{1}{n^2}}{1-\frac{4}{n^2}}\)

As \(n\rightarrow\infty\), the terms \(\frac{5}{n},\frac{1}{n^2}\) and \(\frac{4}{n^2}\) tend towards 0. Therefore, the expression becomes: \(s_n\approx \frac{-2n}{1}=-2n\)

As \(n\rightarrow\infty,-2n\) tends towards \(-\infty\), which means the sequnce is divergent.

Therefore, the sequence \(s_n=\frac{-2n^3+5n+1}{n^2-4}\) is correct example of a divergent sequence.

Node 1 when false

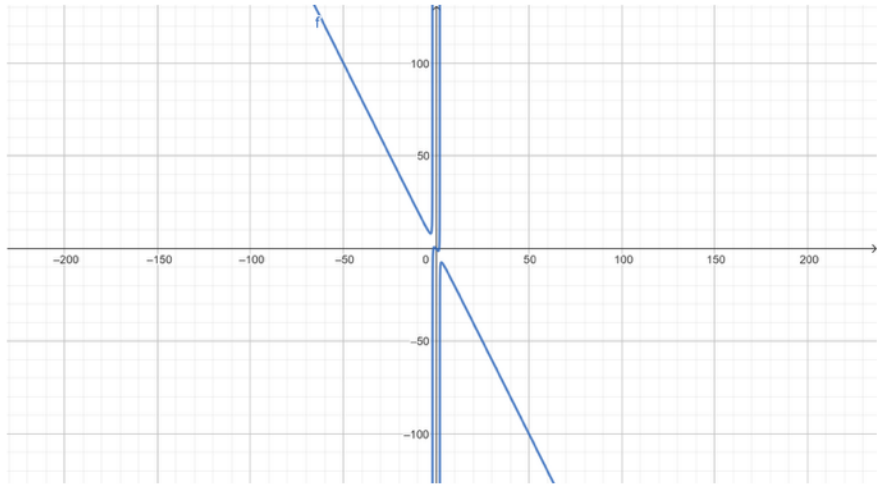
Mod = Score 0 Penalty Next [stop] Answer note prt1-1-F

Node 1 false feedback

Consider the sequence \((s_n),s_n=\frac{a_kn^k+a_{k-1}n^{k-1}+\ldots+a_1n+a_0}{b_i n^i+b_{i-1}n^{i-1}+\ldots+b_in+b_0}\).

\((s_n)\) is divergent if \(k>i\)

For example:
\((s_n),s_n=\frac{-2n^3+5n+1}{n^2-4}, n\in\mathbb{N}\) is divergent, \(\lim_{n\rightarrow\infty}s_n=-\infty\)
Using GeoGebra, we can consider the function \(f(x)=\frac{-2x^3+5x+1}{x^2-4}, x\in\mathbb{R}\backslash\{-2,2\}\)



Potential response tree: prt2

Node 1

Answer test: AlgEquiv | SAns: ans2 | TAns: ta2

Test options: | Quiet: No

Node 1 when true

Mod: = | Score: 1 | Penalty: | Next: [stop] | Answer note: prt2-1-T

Node 1 true feedback

Yes, the sequence \(s_n=\frac{-2n^2+5n+1}{n^3-27}\) is a correct example of sequence that converges to zero. Let's see why.

Consider the polynomials:

- \(P_e(n)=-2n^2+5n+1(degree 2)\)
- \(Q_2(n)=n^3-27(degree 3)\)

The sequence \(s_n\) is: \(s_n=\frac{-2n^2+5n+1}{n^3-27}\).

To determine the behavior of the sequence as \(n\rightarrow\infty\), we devide the numerator and the denominator by \(n^3\):

\(s_n=\frac{-2n^2+5n+1}{n^3-27}=\frac{\frac{-2n^2}{n^3}+\frac{5n}{n^3}+\frac{1}{n^3}}{\frac{n^3}{n^3}-\frac{27}{n^3}}=\frac{\frac{-2}{n}+\frac{5}{n^2}+\frac{1}{n^3}}{1-\frac{27}{n^3}}\)

As \(n\rightarrow\infty\), the terms \(\frac{-2}{n},\frac{5}{n^2}\) and \(\frac{1}{n^3}\) tend towords 0, and trus the expression becomes: \(s_n\approx\frac{0+0+0}{1-0}=0\)

Therefore, the sequence converges to zero as \(n\rightarrow\infty\).

Thus, the sequence \(s_n=\frac{-2n^2+5n+1}{n^3-27}\) is a correct example of sequence that converges to zero.

Node 1 when false

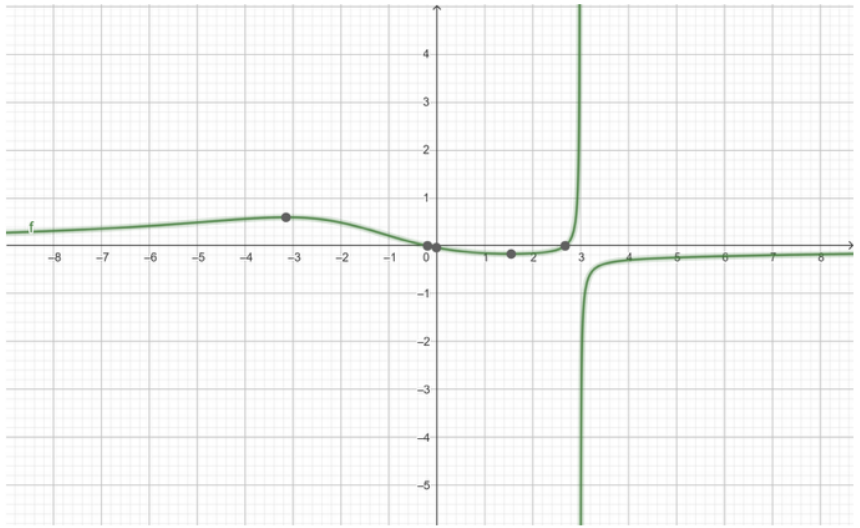
Mod = Score 0 Penalty Next [stop] Answer note prt2-1-F

Node 1 false feedback

Consider the sequence \((s_n),s_n=\frac{a_kn^k+a_{k-1}n^{k-1}+\ldots+a_1n+a_0}{b_i n^i+b_{i-1}n^{i-1}+\ldots+b_in+b_0}\).
\((s_n)\) is convergent to 0 if \(k<i\)
For example:

\((s_n),s_n=\frac{-2n^2+5n+1}{n^3-27}, n\in\mathbb{N}\) is convergent, \(\lim_{n\rightarrow \infty} s_n=0\)

Using GeoGebra, we can consider the function \(f(x)=\frac{-2x^2+5x+1}{x^3-27}, x\in\mathbb{R}\backslash\{3\}\)



Potential response tree: prt3

Node 1

Answer test: AlgEquiv | SAns: ans3 | TAns: ta3

Test options: | Quiet: No

Node 1 when true

Mod: = | Score: 1 | Penalty: | Next: [stop] | Answer note: prt3-1-T

Node 1 true feedback

Yes, the sequence \(s_n=\frac{6n^3+5n+1}{10n^3-1}\) is a correct example of sequence that converges to \(\frac{3}{5}\). Let's see why.

Consider the polynomials:

- \(P_e(n)=6n^3+5n+1(degree 3)\)
- \(Q_2(n)=10n^3-1(degree 3)\)

The sequence \(s_n\) is: \(s_n=\frac{-6n^3+5n+1}{10n^3-1}\).

To determine the behavior of the sequence as \(n\rightarrow\infty\), we devide the numerator and the denominator by \(n^3\):

\(s_n=\frac{6n^3+5n+1}{10n^3-1}=\frac{\frac{6n^3}{n^3}+\frac{5n}{n^3}+\frac{1}{n^3}}{\frac{10n^3}{n^3}-\frac{1}{n^3}}=\frac{6+\frac{5}{n^2}+\frac{1}{n^3}}{10-\frac{1}{n^3}}\)
As \(n\rightarrow\infty\), the terms \(\frac{5}{n^2}\) and \(\frac{1}{n^3}\) tend towards 0, and thus the expression becomes: \(s\approx\frac{6+0+0}{10-0}=\frac{6}{10}=\frac{3}{5}\)
Therefore, the sequence converges to \(\frac{3}{5}\) as \(n\rightarrow\infty\).
Thus, the sequnece \(s_n=\frac{6n^3+5n+1}{10n^3-1}\) is correct example of a sequence that converges to \(\frac{3}{5}\).

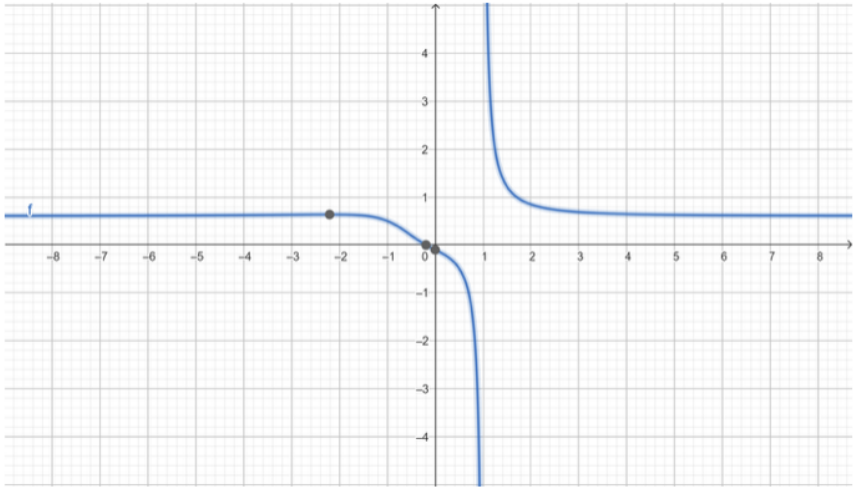
Node 1 when false

Mod = Score 0 Penalty Next [stop] Answer note prt3-1-F

Node 1 false feedback

Consider the sequence \((s_n),s_n=\frac{a_kn^k+a_{k-1}n^{k-1}+\ldots+a_1n+a_0}{b_i n^i+b_{i-1}n^{i-1}+\ldots+b_in+b_0}\).
\((s_n)\) is convergent to \(\frac{3}{5}\) if \(k=i\) and \(\frac{a_k}{b_i}=\frac{3}{5}\)
For example:

\((s_n),s_n=\frac{6n^3+5n+1}{10n^3-1}, n\in\mathbb{N}\) is convergent to \(\frac{3}{5}, \lim_{n\rightarrow\infty} s_n=\frac{3}{5}\)
Using GeoGebra, we can consider the function \(f(x)=\frac{6x^3+5x+1}{1-x^3=10}, x\in\mathbb{R}\backslash\{1\}\)



Let $(s_n) = \left\{ \frac{P_k(n)}{Q_i(n)}, n \in \mathbb{N} \right\}$ a sequence such that $P_k(n)$ and $Q_i(n)$ are two polinomyal of degrees $k \leq 3$ respectively.
Give an example of a sequence s_n such that sequence is

a) divergent; {-(2*n^3)+5*n+1}/{n^2-4}

Your last answer was interpreted as follows:

$$\frac{\{-2 \cdot n^3 + 5 \cdot n + 1\}}{\{n^2 - 4\}}$$

The variables found in your answer were: $[n]$

b) convergent to zero; {-(2*n^2)+5*n+1}/{n^3-27}

Your last answer was interpreted as follows:

$$\frac{\{-2 \cdot n^2 + 5 \cdot n + 1\}}{\{n^3 - 27\}}$$

The variables found in your answer were: $[n]$

c) convergent to $\frac{3}{5}$. {6*n^3+5*n+1}/{10*n^3-1}

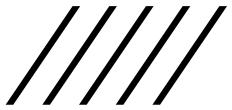
Your last answer was interpreted as follows:

$$\frac{\{6 \cdot n^3 + 5 \cdot n + 1\}}{\{10 \cdot n^3 - 1\}}$$

The variables found in your answer were: $[n]$

Guide for entering the answer

- for $\begin{bmatrix} 1 & 3 \\ 5 & 9 \end{bmatrix}$ should be entered as $matrix([1,3],[5,9])$
- enter $\alpha + \beta$ as $alpha + beta$
- $1 < x$ and $x < 5$, not $1 < x < 5$
- for list 1, 2, 3, 3 type [1, 2, 2, 3]
- for set type {1,2,3}
- $e^x \sin(bx)$ should be entered as $\exp(a * x) * \sin(b * x)$
- i or e is entered as $\%i$ respectively $\%e$
- x^2 is entered as x^2
- $\frac{3}{5x}$ is entered as $3/\{5 * x\}$



THANK YOU FOR YOUR ATTENTION!

Co-funded by the European Union

Pythagoras
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