The Role of Proper Motivation in Mathematics Education

DANIELA VELICHOVÁ

SLOVAK UNIVERSITY OF TECHNOLOGY IN BRATISLAVA, SLOVAKIA

Introduction

- Role of mathematics in **technical** education
- Motivation – key component
- Active learning strategies
- Attractive applied problems
- Expected didactical outcomes
- Steady and sustainable mathematical knowledge and competencies

Methods

- MiniPBL = adapted PBL approach
- Small-scale real problems
- Mathematical models
- Critical environmental issues
- Inline with EU SDG agenda
  - SDG = Sustainable Development Goals
  - Sustainable education

Methods

- Sustainable development goals

Methods

- Teachers' and Students' miniPBL sheets
  - problem + related mathematical topics
  - respective SDG
  - tasks to be solved (+solution for teachers)
  - expected outcomes
  - guides for teaching/learning
  - forms of assessment
  - resources to be used

Methods

Pythagoras

Funded by the European Union

Mini-PBL example

Teaching Guide for Teachers

Mini-PBL project	
Teacher data sheet: Teaching Guide	
Title	The growth of trees
SDG attended	Using this UN graphics, we mark such SDG which this project works. 
Content units	Differential equations
Sessions	1 sessions of 1h
Hours of autonomous work	1h
Competences to be developed	**Reasoning and modelling** • Develop thinking strategies to solve real life problems • Explore, analyse, and apply mathematical ideas • Estimate reasonably and demonstrate fluent, flexible, and strategic thinking about graphs • Model with mathematics in situational contexts • Think creatively and with curiosity and wonder when exploring problems **Understanding and solving** • Develop, demonstrate, and apply conceptual understanding of mathematical ideas through story, inquiry, and problem solving • Visualize to explore and illustrate mathematical concepts and relationships • Apply flexible and strategic approaches to solve problems • Solve problems with persistence and a positive disposition • Engage in problem-solving experiences connected with real-life examples.

	Communicating and representing • Explain and justify mathematical ideas and decisions in many ways • Represent mathematical ideas in concrete, pictorial, and symbolic forms • Use mathematical vocabulary and language to contribute to discussions in the classroom • Take risks when offering ideas in classroom discourse **Connecting and reflecting** • Reflect on mathematical thinking • Connect mathematical concepts with each other, other areas, and personal interests • Use mistakes as opportunities to advance learning • Incorporate First Peoples worldviews, perspectives, knowledge, and practices to make connections with mathematical concepts
ICT tools to be used	Available Computer Algebra Systems: Mathematica, Maple, Matlab, GeoGebra, etc.
Context: project statement	The climate changes are among other things caused also by the increasing deforestation and decreasing number of forests, trees and green vegetation on the globe. To benefit from the sustainable development and to support vegetation growth it is necessary to understand the growth of individual trees and be aware of the time necessary for the renewal of their missing numbers. To grow each particular tree in a new forest it is essential to know the time interval required for its cultivation.
Tasks and problems	A specific type of tree will grow under suitable conditions with the speed inversely proportional to its height. The tree can grew up to 1 meter during the first three years after planting. A new forest was planted with the particular tree seedlings that were all about 0,5 meter high. The tree growth can be described by a differential equation with separable variables, while general solution of this equation contains constants that can be expressed with respect to the described height decrease under good environment conditions. Tree can grow in good conditions about 7 years till it will reach the average height, while after this period it might beneficially vegetate for about 50 years and still grow, but quite slowly, with the half of the initial growth speed. **Task 1:** Assemble the differential equation describing the growth of trees in a new forest, assuming favorable conditions are secured for their growth. Answer: Let $y(t) > 0$ be the function representing the tree height depending on the time t of its growth, while derivative $y'(t)$ be the speed of the tree growth. This growth is described by the differential equation with separable variables $$y'(t) = \frac{k}{y(t)}, k > 0$$ that can be rewritten as the differential equation with separated variables and solved directly by integration.

Methods

$$y(t).y'(t) - k = 0$$

$$\int y\,dy - \int k\,dt = c$$

$$\frac{y^2}{2} - kt = c$$

$$y^2 = 2kt + 2c$$

Task 2:
Find general solution of this differential equation and particular solution determined by Cauchy initial conditions.

Answer:
General solution of the equation is in the form

$$y(t) = \sqrt{2kt + 2c}$$

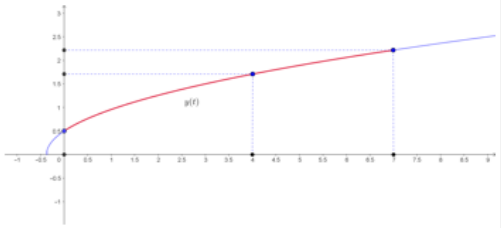
where constants k and c can be found according to the given initial conditions describing the tree growth:

$$y(0) = 0{,}5 \qquad y(3) = 1{,}5$$

$$y(t) = \sqrt{\frac{2}{3}t + 0{,}25}\ , \qquad t \in (0, T), T \in R$$

Task 3:
Sketch respective integral curve of the particular solution representing the tree growth during T years, until it will reach its average height.

Answer:



Tree will grow until it reaches its average height, which is after $T = 7$ years from its planting. The red curve represents the tree growth.

Task 4:
Calculate the average high of trees in this forest after 4 years and after 7 years, when the trees reach their average height.

Answer:

Toolkit 3: One model for mini-PBL

$$y(4) = \sqrt{\frac{8}{3} + \frac{1}{4}} = \sqrt{\frac{32+3}{12}} = \sqrt{\frac{35}{12}} \doteq 1{,}71\ m$$

$$y(7) = \sqrt{\frac{14}{3} + \frac{1}{4}} = \sqrt{\frac{56+3}{12}} = \sqrt{\frac{59}{12}} \doteq 2{,}22\ m$$

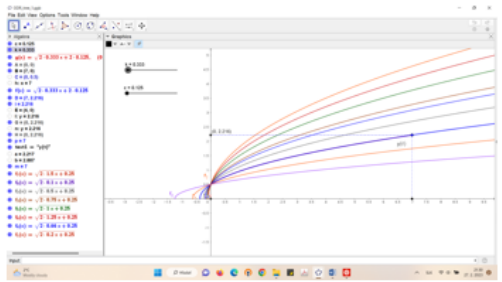
Task 5:
Estimate the height of the tree after 50 years.

Answer:

$$y(50) = \sqrt{\frac{100}{3} + \frac{1}{4}} = \sqrt{\frac{400+3}{12}} = \sqrt{\frac{403}{12}} \doteq 5{,}79\ m$$

Task 6:
Sketch several integral curves of the general solution and investigate their forms determined by different values of the included constants c and k representing the tree growth under different conditions.

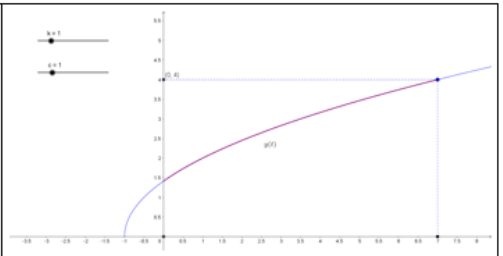
Answer:



Task 7:
Comment on the obtained results from a sustainable point of view. Investigate how the height of the tree seedlings influence the speed of the trees growth.

Answer:
Animation can be obtained easily in the program GeoGebra, with sliders determining the values of constants k and c.

Methods

	
Outcomes expected	- Graphics fitting the solution; - Numerical results explained and put in context; - Capture of ICT tools solutions used; - Sequence of steps followed; - Remark computations done by hand and done by ICT tools; - Provide complete answer to questions; - All the results must be presented in the context of the problem;
Guide for Learning	At the beginning of the course, the students need guides on new activities, and feel your support on a well-structured pack of suggestions on how to address the problems posted. Namely: - Read carefully the problem statement and the tasks posted. Always maintain a global view of all the projects. - Identify, or try to do a first draft match, the content units of your lecture notes involved in every task. - Take your lecture notes open and review before starting to solve the problems. - Match output expected with the tasks posted, at least as first draft approach. - Follow the order of the tasks, try to increase the knowledge of the problem while you are solving the activities. - Always think that maybe there are different ways to solve a problem. - Use ICT tools to avoid hard computations and check your solutions are correct in different ways if possible. - The solutions are always part of a context, expressing such a final solution totally integrated in the problem posted. - Be sure you answer the complete questions. - Always try to solve the questions by yourself. - If the project can be done in groups, discuss with the groups the proposed problem, to confirm and detect fails or weaknesses, confront strategies, discuss presentation format, etc. Working in groups doesn't mean work less but work better.
Guide for Teaching	Some hints needed to present and launch the mini-PBL to students - Do a small Introduction concerning Energy consumption, added to the Climate Change crisis we are currently living in. - Do a small introduction about the relations between power and energy, with the basic equations.

Toolkit 3: One model for mini-PBL

	- Students will form groups of 4 students and solve the mini-PBL using the eduScrum methodology. - The students should do each exercise in a sequential order, starting from Task 1. - The students should be able to thoroughly read and interpret the numerical results from a mathematical and the real-life example point of view. They should include also a discussion of the climate change crisis and enumerate some strategies they could apply at home or even at university to save resources, namely reduce energy consumption. They should also mention how this mini-PBL helps them identify the Sustainable Development Goals 4 And 7.
Assessment	- Final report; - Oral presentation; - Peer-assessment: students will apply peer-assessment for their periodic performance using online peer assessment tools used and available at the respective institution.
Others: References	Active Learning Calculus I (colorado.edu) https://eduscrum.org/about-us-and-how-we-try-to-make-it-happen/ More refs on active-learning tools: https://scholar.google.com/citations?hl=en&user=Aw39XwEAAAAJ&view_op=list_works&sortby=pubdate https://www.youtube.com/watch?v=mQ_mbDAB1us (there are more examples online)

Methods

- Mathematical competencies (SEFI MSIG)
  - Thinking mathematically
  - Reasoning mathematically
  - Posing and solving mathematical problems
  - Modelling mathematically
  - Representing mathematical entities
  - Handling mathematical symbols and formalism
  - Communicating in, with and about mathematics
  - Making use of aids and tools

Methods

- General competencies
  - Reasoning and modelling
    - – explore, analyse, estimate, demonstrate, think creatively
  - Understanding and solving
    - – develop mathematical model, visualize, apply mathematical methods
  - Communicating and representing
    - – explain, justify, use mathematical symbolism, use ICT
  - Connecting and reflecting
    - – see connections, reflect importance of mathematics, learn from mistakes

Methods

- Presenting the project problem
  - The introduction, the core topic and all the information will help students to allocate the tasks and posted problems. Here we include all references, graphics, news from media, official reports (UN, EU, OCDE, UNICEF,...), and any source which helps to acquire a wider idea about the SDG problem we address.
  - The effort to "put in context" the problem could connect mathematics with real-life, with Earth challenges and Humanity needs.
  - By teaching mathematics using SDG we support the reflection of our students, future professionals, on the main challenges of humanity and the Earth, we educate **Global Citizens.**

Methods

- Forms of assessments (related to general competencies)
  - **Written report**
    - solutions of particular tasks with answers and detail calculations supported by illustrations, figures, tables
  - **Oral presentation**
    - performed in front of classmates
    - explanation of solution strategies, justification of chosen mathematical model and methods
    - information about used resource, webpages, AI
    - reflections and interpretation of achieved results
  - **Peer review by students**
    - receiving point score from classmates

Methods

Mini-PBL project	
Student data sheet: Learning Guide	
Title	The growth of trees
SDG attended	Using this UN graphic, we mark such SDG which this project works. 
Content units	Ordinary differential equations of order I
Sessions	1 sessions of 1h + 1h of autonomous work
ICT tools	Available CAS: Mathematica, Maple, Matlab, GeoGebra, etc.
Context: project statement	The climate changes are among other things caused also by the increasing deforestation and decreasing number of forests, trees and green vegetation on the globe. To benefit from the sustainable development and to support vegetation growth it is necessary to understand the growth of individual trees and be aware of the time necessary for the renewal of their missing numbers. To grow each particular tree in a new forest it is essential to know the time interval required for its cultivation.
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Toolkit 3: One model for mini-PBL

	Task 4: Calculate the average high of trees in this forest after 4 years and after 7 years, when the trees reach their average height. **Task 5:** Estimate the height of the tree after 50 years. **Task 6**: Sketch several integral curves of the general solution and investigate their forms determined by different values of the included constants *c* and *k* representing the tree growth under different conditions. **Task 7:** Comment on the obtained results from a sustainable point of view. Investigate how the height of the tree seedlings influence the speed of the trees growth.
Outcomes expected	- Graphics fitting the solution; - Numerical results explained and put in context; - Capture of ICT tools solutions used; - Sequence of steps followed; - Remark computations done by hand and done by ICT tools; - Provide complete answer to questions; - All the results must be presented in the context of the problem;
Guide for learning	- Read carefully the problem statement and the tasks posted. Always maintain a global view of all the projects. - Identify, or try to do a first draft match, the content units of your lecture notes involved in every task. - Take your lecture notes open and review before starting to solve. - Match output expected with the tasks posted. - Follow the order of the tasks, try to increase the knowledge of the problem while you are solving the activities. - Always think that maybe there are different ways to solve a problem. - Use ICT tools to avoid hard computations and check your solutions are correct in different ways if possible. - The solutions are always part of a context, expressing such a final solution totally integrated in the problem posted. - Be sure you answer the complete questions. - Always try to solve the questions by yourself. - If the project can be done in groups, discuss with the groups the proposed problem, to confirm and detect fails or weaknesses, confront strategies, discuss presentation format, etc. Working in groups doesn't mean work less but work better.
Assessment	- Final report; - Oral presentation; - Peer-assessment: students apply peer-assessment for their periodic performance using available assessment tools.
Others: References	https://www.britannica.com/plant/tree/Tree-structure-and-growth https://onetreeplanted.org/blogs/stories/how-trees-grow https://one-more-tree.org/blog/2023/09/01/the-amazing-process-of-tree-growth-from-seedlings-to-towering-heights/

Results

- miniPBL approach was tested within the EU project

www.pythagoras-grant.eu

PROGRAMME Erasmus + ACTION TYPE KA220-HED
Cooperation partnership in higher education
PROJECT NUMBER 2021-1-RO01-KA220-HED-000032258
PERIOD 1. 2. 2022 - 31. 1. 2025
PROMOTING ORGANISATION
Lucian Blaga University of Sibiu (LBUS), Romania
PARTNERS – Denmark, Greece, Slovakia, Spain, Sweden

Results

- Approach to miniPBL implementation
  - individual tasks to be delivered
  - team work on presented problems
- Individual work
  - appreciated by better performing students
- Team work
  - appreciated by almost all students

Results

- Team work - splitting tasks and close cooperation
- Different roles of students in team
  - digging information
  - problem analysis
  - choice of proper mathematical model
  - finding correct solutions
  - interpretation of results
  - development of written report
  - oral presentation

Summary

- MiniPBL learning scenario
  - proved to be a motivating and activating element in teaching mathematics to engineering students
  - a tool engaging students to use mathematics to solve critical real-life (environmental) problems
  - a way to show students benefits that knowledge of mathematical models brings to problem solving
  - an approach to encourage students to communicate, use, and actively present mathematics
  - an opportunity to help students to see connections, learn from mistakes and understand the importance of mathematical competencies and steady knowledge

The miniPBL as **Proper Motivation** in Mathematics Education

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